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VICENTE DA GAMA MACHADO

ESSAYS ON INFLATION AND MONETARY POLICY

Porto Alegre
2011

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Tese a ser submetida ao Programa de Pós-Graduação em Ciências Econômicas da Universidade Federal do Rio Grande do Sul, como requisito parcial para obtenção do título de Doutor em Economia, Área de Concentração: Economia Aplicada

Orientador: Prof. Dr. Marcelo Savino Portugal

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RESUMO

Esta tese é composta de três artigos relacionados à política monetária e inflação e possuem em comum a ênfase na importância das expectativas tanto para o desenho da política monetária como para a dinâmica inflacionária. No primeiro ensaio, contribuímos para o debate sobre a resposta apropriada de política monetária a flutuações de preços de ativos em um contexto de aprendizagem adaptativa. O modelo conta com dois tipos de regras de juros instrumentais como em Bullard e Mitra (2002), porém com um papel adicional para preços de ativos. Do ponto de vista da E-Estabilidade, conclui-se que uma resposta a preços de ativos não é desejável nem com a regra que utiliza expectativas futuras nem com a regra que responde a valores contemporâneos. Crenças heterogêneas a respeito da dinâmica das flutuações de preços de ativos, inflação e hiato do produto são introduzidas. Também é avaliada uma regra de política monetária ótima que inclui um peso para os preços de ativos. De forma geral, conclui-se que o princípio de Taylor é relevante para todas as regras de juros analisadas e que os bancos centrais devem agir com cautela ao considerar a introdução de preços de ativos na política monetária. No segundo ensaio, oferecemos estimativas recentes de persistência inflacionária no Brasil, com uma abordagem multivariada de componentes não-observados, na qual são consideradas as seguintes fontes que impactam na persistência da inflação: desvios das expectativas da meta real de inflação; persistência dos fatores que provocam inflação; e termos defasados da inflação. Dados de inflação, produto e taxas de juros são decompostos em componentes não-observados e, para simplificar a estimativa de um número grande de variáveis desconhecidas, utilizamos análise bayesiana, seguindo Dossche e Everaert (2005). Os resultados indicam que a persistência baseada em expectativas tem grande participação na persistência inflacionária no Brasil, que tem diminuído nos últimos anos. Tal resultado implica que apenas as tradicionais fricções no ajuste de preços usadas nos modelos macroeconômicos não são suficientes para representar a real persistência da inflação. No último capítulo estimamos diversas curvas de Phillips reduzidas com dados brasileiros recentes, numa abordagem de séries de tempo com componentes não-observados, que se apresenta como alternativa às tradicionais estimativas, baseadas em métodos GMM, de curvas de Phillips Novo-Keynesianas (NKPC), que raramente foram bem sucedidas empiricamente. A decomposição em tendência, sazonalidade e ciclo oferece, através do resultado gráfico, interpretação econômica direta. Diferentemente de Harvey (2011), incluímos expectativas de inflação nas estimações, assim como na NKPC habitual. A inflação no Brasil parece ter respondido cada vez menos às medidas de atividade econômica consideradas. Isso consiste em evidência de achatamento da curva de Phillips no Brasil, o que significa por um lado custos de desinflação mais altos, mas por outro lado menores pressões inflacionárias derivadas de crescimento do produto.

Palavras-chave: Política Monetária ótima. Persistência Inflacionária. Curva de Phillips. Aprendizado adaptativo. Modelos de Componentes não-observados.

ABSTRACT

This thesis is composed of three essays on monetary policy and inflation that share particular emphasis on the importance of expectations for both monetary policy design and inflation dynamics. First we contribute to the debate on the appropriate response of monetary policy to asset price fluctuations in an adaptive learning context. Our model accounts for two types of instrumental rules in the spirit of Bullard and Mitra (2002), but with an additional role for asset prices. From the point of view of E-Stability, we find that a response to stock prices is not desirable under both a forward expectations policy rule and an interest rate rule responding to contemporaneous values. Heterogeneous beliefs about the dynamics of asset price fluctuations, inflation and the output gap are introduced. We also evaluate an optimal monetary policy rule including a weight on asset prices. Overall we find that the Taylor principle remain important over all interest rate rules analysed and that central banks should remain cautious when considering the introduction of stock prices in monetary policy. In the second essay, we provide recent estimates of inflation persistence in Brazil in a multivariate framework of unobserved components, whereby we account for the following sources affecting inflation persistence: First, deviations of expectations from the actual policy target; second, persistence of the factors driving inflation; and third, lagged inflation terms. Data on inflation, output and interest rates are decomposed into unobserved components and to simplify the estimation of a great number of unknown variables, we utilize bayesian analysis as in Dossche and Everaert (2005). Our results indicate that expectations-based persistence matters considerably for inflation persistence in Brazil, which has experienced an overall decrease in the last few years. This finding implies that traditional price-setting frictions used in macroeconomic models are not enough to represent actual inflation persistence. In the last chapter we estimate alternative reduced-form Phillips curves with recent Brazilian data, using a framework of time series with unobserved components, as an alternative to traditional GMM estimations of the New Keynesian Phillips Curve (NKPC), which have seldom been empirically successful. The decomposition into trend, seasonal and cycle features offers, through the graphical output, straightforward economic interpretations. Differently from Harvey (2011), we allow for inflation expectations as in the usual NKPC. Inflation in Brazil seems to have responded gradually less to measures of economic activity in recent years. This provides some evidence of a flattening of the Phillips curve in Brazil, which means higher costs of disinflation on the one hand, but also lower inflationary pressures derived from output growth, on the other.

Keywords: Optimal Monetary Policy. Inflation Persistence. Phillips Curve. Adaptive Learning. Unobserved Components Models.

SUMÁRIO

1 INTRODUÇÃO

Esta tese é composta de três ensaios que tratam de duas importantes questões inter-relacionadas em macroeconomia: inflação e política monetária. Ao longo dos capítulos, o ponto de partida é o mesmo ambiente teórico macroeconômico, baseado nos modelos novo-Keynesianos, que são largamente empregados na literatura moderna de economia monetária.

No curso da década anterior à recente crise financeira, a "ciência" da política monetária implícita na teoria e na prática estava relativamente bem definida, dado que as principais estratégias dos formuladores de política haviam sido bem sucedidas em manter inflação baixa e com pouca variabilidade na maioria dos países industrializados. Tal consenso parece ter sido abalado nos anos mais recentes, como salienta Mishkin (2011). Mais especificamente, entre os bancos centrais e o universo acadêmico, novos debates sobre a importância da introdução de preços de ativos na pauta da política monetária têm ganhado fôlego. Inserindo-se nesse debate, no capítulo 2 é desenvolvido um modelo de política monetária que leva em conta a inflação de preços de ativos em um contexto de aprendizagem adaptativa. O modelo se assemelha a Bullard e Mitra (2002), no sentido de que regras de taxas de juros instrumentais são consideradas, e a Carlstrom e Fuerst (2007), devido à inclusão dos preços de ativos. Ao considerar ainda aspectos como crenças heterogêneas e política monetária ótima, esse capítulo contribui tanto para a literatura da resposta apropriada a desequilíbrios nos preços de ativos, como para a crescente literatura de aprendizagem adaptativa em economia monetária.

No processo de formulação e acompanhamento da política monetária, um ingrediente que precisa ser constantemente examinado e entendido é a taxa de inflação. O Brasil representa um ambiente peculiar com um histórico de altas taxas de inflação no final dos anos 80 e início dos 90, taxas de juros relativamente altas, mesmo comparadas a países em desenvolvimento e, ainda assim, crescimento do produto comparativamente alto nos anos mais recentes. Motivados por tais especificidades, nos dois capítulos subsequentes, examinamos a dinâmica da inflação no Brasil, aplicando técnicas empíricas baseadas em modelos com componentes não-observados.

No capítulo 3 são conduzidas medidas empíricas de persistência inflacionária, com uma abordagem multivariada, baseada em Dossche e Everaert (2005). Ao contrário da abordagem tradicional univariada, que se baseia essencialmente na persistência intrínseca, as fontes de persistência inflacionária são discernidas e quantificadas. Para isso são usados dados de inflação, produto e taxa de juros, identificados em um modelo de espaço de estados linear gaussiano, e se utiliza análise bayesiana, tornando possível a estimação de um grande número de variáveis não-observadas a partir de dados a priori de outros estudos brasileiros e estrangeiros. Como subproduto do modelo, obtemos ainda estimativas para a taxa natural de juros, variável chave para a política monetária, que não é observada diretamente.

Dando sequência ao exame da inflação no Brasil, no último capítulo nos dedicamos a uma expressão que tem recebido atenção novamente no período recente. O objetivo é estimar uma Curva de Phillips Novo-Keynesiana com dados brasileiros, porém usando uma decomposição inédita em componentes de tendência, ciclo e sazonalidade. Tal modelo estrutural de séries de tempo oferece o benefício de interpretações econômicas mais diretas. O resultado mais importante é a dinâmica da relação entre a atividade econômica (no nosso caso, medida por diferentes formas de hiato do produto e taxas de utilização da capacidade industrial) e inflação na economia brasileira recente.

Ao longo de toda a tese é reforçado o papel já proeminente das expectativas dos agentes em modelos monetários. No primeiro ensaio isso fica claro de duas formas: primeiro assumimos que os agentes aprendem na medida em que recebem informações, antes de formar suas expectativas. Tal comportamento é considerado como um passo adiante em termos de realismo, comparativamente a expectativas puramente racionais, como bem colocou Evans e Honkapohja (2001). Em segundo lugar, consideramos ainda que os agentes podem se comportar heterogeneamente ao formar expectativas. No segundo ensaio, ainda que se trate com expectativas racionais, as expectativas têm um papel chave em entender a persistência inflacionária agregada no Brasil. Finalmente, no terceiro ensaio, mostramos que as expectativas de inflação baseadas nas pesquisas de expectativas do Banco Central do Brasil definitivamente melhoram o desempenho empírico da curva de Phillips com componentes não-observados.

2 MONETARY POLICY, ASSET PRICES AND ADAPTIVE LEARNING[1]

Abstract

Following recent episodes of financial distress, a prominent topic of debate has been the appropriate response of monetary policy to asset price fluctuations. We assess the role of asset price misalignments in monetary policy in an adaptive learning context. Our model accounts for 2 types of instrumental rules in the spirit of Bullard and Mitra (2002), but with an additional role for asset prices. From the point of view of the E-Stability criterion, commonly used in the learning literature, we find that a response to stock prices is not desirable under both a forward expectations policy rule and an interest rate rule responding to contemporaneous values. Heterogeneous beliefs about the dynamics of asset price fluctuations, inflation and the output gap are introduced. We also evaluate an optimal monetary policy rule including a weight on asset prices. Overall we find that the Taylor principle remains important over all interest rate rules analysed and that central banks should act cautiously when considering the introduction of stock prices in monetary policy.

2.1 Introduction

The issue of what kind of role asset price inflation (or alternatively a measure of asset price gap) should have in the conduct of monetary policy has gained renewed momentum in the aftermath of the recent financial crisis and is still not resolved. After a first wave of contributions to this debate in the beginning of the 90's, mainly following the opposite views of Bernanke and Gertler (2000 and 2001) and Cecchetti et al (2000), there has been resurgent interest both among policymakers and academic researchers.[2] At the same time, there is controversy about the actual behaviour of monetary authorities in the past concerning the

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[1] This essay was prepared while I was a visiting PhD researcher at the University of St. Andrews, UK. I would like to thank my co-supervisor, Professor Kaushik Mitra, and seminar participants at the School of Economics and Finance.

[2] See Greenspan (2005), Trichet (2005), Bullard (2009) and Smaghi (2009) for a general idea of the Federal Reserve and the European Central Bank main views.

importance given to asset price fluctuations, as evidenced by the different empirical conclusions found in the literature.[3]

Forecasts of inflation and output have a key role in the new Keynesian framework of interest rate rules, and it is widely recognized that the expectations of economic agents influence the time path of the economy. Importantly, a great part of this literature still relies on the rational expectations hypothesis. However, as Evans and Honkapohja (2001) argued, the basic assumption implied by rational expectations, that all agents know the true structure of the economy, has proved too strong. The adaptive learning literature instead concedes that agents learn as they are endowed with information from the economy's structure, by updating their forecasting procedures. As a result, some apparently natural policy rules may not result in a stable equilibrium when agents' learning is considered, a point made by Bullard and Mitra (2002) and Evans and Honkapohja (2003a). On top of that, there is substantial evidence of heterogeneity in the formation of expectations of relevant economic variables.[4] Concerning asset prices, there is a particularly vast literature highlighting that agents possess heterogeneous beliefs.[5]

In the present work, we consider the rather realistic points mentioned above. Specifically, we assess the conditions for determinacy and stability under learning (also known as E-Stability[6]) of an extended monetary policy rule that accounts for asset price variations. The starting point is a framework in which expectations are homogeneous and recursive least squares learning prevails, following to some extent Bullard and Mitra (2002) and Carlstrom and Fuerst (2007). In this first step we are able to verify the chief result of Bullard and Mitra (2002) with regard to the Taylor principle in our extended framework. Comparing to Carlstrom and Fuerst (2007), we further investigate an interest rate rule responding to expectations of the key variables and assess E-Stability conditions. Differently from Pfajfar and Santoro (2008) and Assenza, Berardi and Gatti (2009), we focus on a

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[3] Rigobon and Sack (2003) find a substantial policy response to stock prices. On the other hand, Hayford and Maliaris (2004) conclude that, at least during the 90's, monetary policy tended to accommodate the apparent stock prices overvaluation. In the middle, Dupor and Conley (2004) argue that the FED response to stock prices tended to be more relevant in low inflation episodes, than when disinflation was under way.

[4] Branch (2007) and Pfajfar and Santoro (2010) argue that heterogeneity is pervasive in the process of expectation formation, while Milani (2011) identifies a learning process together with expectations shocks as important in economic fluctuations. Wieland and Wolters (2011) also arrive at similar conclusions, adding that heterogeneity of output growth and inflation forecasts tend to vary over time.

[5] Hommes (2006) provides a comprehensive survey of heterogeneous agents in asset pricing and claims that the shift from the representative agent to a behavioural agent-based approach has been motivated, among other factors, by the increasing evidence of bounded rationality and by observations of excess volatility in stock prices.

[6] For a formal definition of E-Stability, see Evans and Honkapohja (2001).

standard new Keynesian macroeconomic model of monetary policy transmission, instead of allowing for cost-channel effects.[7]

We also examine the implications of heterogeneous beliefs about inflation expectations and forecasts of asset market developments, sharing ideas related to Guse (2005). In doing so, we address at the same time two concerns pointed out by Sims (2009): Referring to central banks (CB) research and practice, he criticises the lack of both a consistent treatment of asset markets and of frameworks that depart from the usual rational expectations approach[8]. Finally, we derive optimal monetary policy that contemplates asset prices, extending the expectations-based rule proposed in Evans and Honkapohja (2003a). By using the learning approach to evaluate alternative scenarios, we also provide an additional selection criterion, something particularly important in monetary policy settings, which are acknowledged to often have equilibrium indeterminacy.

The remainder of this work is organized as follows. The next section examines the related literature in two directions: First, the debate on the introduction of asset price inflation in monetary policy and second, determinacy and stability of rational expectations equilibria (REE) under learning in general monetary economics. Section 2.3 describes the small macroeconomic equilibrium model of households and firms. Section 2.4 presents the benchmark learning environment with homogeneous expectations and instrumental rules. In sections 2.5 and 2.6 we allow for extensions such as heterogeneous beliefs and optimal monetary policy, and section 2.7 concludes.

2.2 Literature review

Monetary policy has succeeded reasonably well in stabilizing price index inflation in several countries over the last years, either targeting inflation strictly or a combination of inflation and output variability.[9] At the same time, a significant part of the monetary literature

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[7] The literature has generally found mixed results regarding the importance of cost-channel effects. Using Bayesian methods to estimate parameters, Rabanal (2007) concludes that the traditional demand side effect dominates the supply side effect represented by the cost channel.

[8] Besides the “realistic” argument of modelling heterogeneity, there is also the point that since adaptive learning requires that agents behave as econometricians, they may also be subject to problems of misspecification; see Evans and Honkapohja (2001).

[9] Although Ball and Sheridan (2005) argue that it is not clear whether the improved macroeconomic performance in a cross-country sample can be related to inflation targeting, they point out that targeters and non-targeters seem to have followed similar interest rate policies.

has focused on a new Keynesian approach to monetary rules, as becomes clear in Woodford (2003). The basic approach – which involves nominal rigidities in the behaviour of firms and the short-term interest rate as main policy instrument – has been the workhorse for the analysis of monetary policy both in the academic literature and in the practice of most central banks[10].

Earlier contributions to the debate on the optimal monetary policy response to atypical movements in asset prices dealt mainly with the trade-offs involved in including a weight for the volatility of these prices in a usual central bank reaction function. On the one hand, the rather orthodox view claims that the conduct of monetary policy should only be affected by shifts in asset prices as long as they signal future changes in inflation or output. Bernanke and Gertler (2000, 2001) for example, develop a New-Keynesian model with a role for frictions in the credit market – translated into a financial accelerator – and for financial bubbles, modelled as an endogenous stochastic process. Despite adopting a similar technique, Cecchetti et al (2000) reach opposite conclusions. They explore the fact that a countercyclical attitude from the monetary authority may, under special conditions, soften the impact of abrupt shifts in asset prices and may, therefore, enhance macroeconomic and financial stability.

Building on this first debate, some papers modelled the possibility of a response to asset price deviations in a monetary policy rule. Gilchrist and Saito (2006) develop a dynamic stochastic general equilibrium (DSGE) model with a role for the financial accelerator mechanism, as in Bernanke and Gertler (2000) in which both the private sector and the policymaker are allowed to learn about the trend growth rate of technology. Faia and Monaceli (2007) offer a similar model, but they further account for an agency issue associated with monitoring costs in the lending market, which implies that asset price fluctuations are a symptom of financial distortions. In both these models there is a scope for a response to asset price oscillations in the optimal policy. In the first article, such response is more beneficial when the central bank is more informed about the rate of technology than the private sector, whereas in the second one, it is true as long as the response to inflation is sufficiently small. Other contributions include, for example, Bean (2003), Haugh (2008) and Gruen, Plumb and Stone (2005). Detken and Smets (2004) offer a good survey of this literature.[11]

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[10] Clarida, Gali and Gertler (1999) provide a comprehensive explanation of the standard features of New Keynesian models of monetary policy.

[11] A closely related literature deals with the role central banks should have concerning the development of bubbles in financial markets. See, for example, Kent and Lowe (1997) and Bordo and Jeanne (2002).

Concerning the interactions between asset prices and economic activity (which in turn would justify monetary policy movements), Gilchrist and Leahy (2002) mention three main channels which are explored by different authors in the literature: the wealth channel, Tobin's "q" theory and the financial accelerator.

As previously discussed, the recent crisis has triggered more attention from monetary authorities. The same happened with academic literature, which has been providing innovative approaches to the debate of asset price inflation. More specifically, a view that has been increasingly questioned is that the central banks should not care about asset prices and should instead "clean up the mess" after an asset bubble bursts. Disyatat (2010) argues that a modification in the CB's objective function to include a measure of financial imbalances may be desirable, because it leads to a more practical alternative than introducing an explicit reaction to asset prices, as much of the previous literature had tried to do. By introducing doubts and pessimism in the standard new Keynesian model, Benigno and Paciello (2010) conclude that a flexible inflation targeting policy that includes a reaction to asset prices (represented by Tobin's "q") might be welfare improving in the case when doubts and pessimism about the true model play an important role.[12]

In another strand of the literature of monetary economics, a growing set of studies has been focusing on the criteria of determinacy and stability of rational expectations equilibria under adaptive learning, following Evans and Honkapohja (2001)[13], as confirmed in surveys by Bullard (2006) and Evans and Honkapohja (2008). The rationale for these developments is mainly twofold: First, the acknowledgment that expectations are a central part of monetary theory. As Woodford (2003) points out, "*not only expectations about policy matter, but at least under current conditions, very little else matters*." Second, the need to provide an alternative to the rather strong assumptions implied by the rational expectations theory. Instead of knowing from the beginning the true macroeconomic structure, it is usually assumed that agents form expectations adaptively as they learn the real structure. Moreover, stability of such equilibria may present an alternative to the inherent instability problem of interest rate rules, as noted by Friedman (1968), based on self-fulfilling expectations. The

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[12] Other types of similar departures in the standard macroeconomic model have also been proposed. Kannan, Rabanal and Scott (2009) argue that, provided there is some discretion, a monetary policy response to credit accelerating mechanisms and to distortions in asset prices may be beneficial to macroeconomic stability, together with macroprudential rules. Curdia and Woodford (2010) assess modifications of a Taylor rule to include a reaction to changes either in interest rate spreads or in the aggregate volume of credit.

[13] Evans and Honkapohja (2001) consolidated the theory of adaptive learning focusing mainly on the cobweb model. However, one of the main directions of academic research later proved to be monetary policy, as is evidenced in surveys like Bullard (2006) and Evans and Honkapohja (2008).

central work of Bullard and Mitra (2002) assesses determinacy and E-Stability criteria for different types of interest rate rules. In their view, the so-called learnability of the equilibria (or E-Stability) arises as an additional criterion, which policymakers should take into account. Their results highlight the fact that some interest rate rules that otherwise would be considered desirable, may fail to be optimal under learning dynamics. Evans and Honkapohja (2003a) take a slightly different approach by studying E-Stability conditions for alternative types of interest rate rules derived from optimal monetary policy, for example fundamentals-based and expectations-based rules. McCallum (2008) further cites learnability of the equilibrium as a compelling necessary condition for a REE to be considered plausible.[14] In the words of Bernanke (2007), many of the most interesting issues in contemporary monetary theory require an analytical framework that involves learning by private agents and possibly the central bank as well. Preston (2008) shows that if the CB implements monetary policy under the mistaken assumption that agents have purely rational expectations, severe instability follows. Moreover, Orphanides and Williams (2005) pointed out that imperfect knowledge of the structure may have important implications for monetary policy.[15]

Relatively few studies approached the more specific question of determinacy and stability criteria when a response to asset prices is considered in the interest rate rule, combining both pieces of literature just reviewed. Bullard and Schaling (2002) found that assigning more weight to asset price fluctuations in a Taylor rule, the probability of indeterminacy of the REE increases. Their work relies heavily on the findings of Bullard and Mitra (2002). A similar outcome is obtained by Carlstrom and Fuerst (2007), who furthermore consider patterns of money demand. However, they do not study E-Stability nor deal with a forward-expectations policy rule. Airaudo, Nisticò and Zanna (2007) develop a DSGE model, with overlapping generations that consider agents are non-Ricardian and there is a role for the wealth effect on the demand for consumption. Thus, they assume the supply effects in Bernanke and Gertler (2001) are complemented with demand effects.[16] Airaudo,

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[14] Milani (2007) introduces empirical analysis in an adaptive learning setting and argues that such a setting manages to reproduce important features of observed expectations. More importantly, what he calls "mechanical sources of persistence" - like habit formation in consumption or indexation of past inflation in price-setting - are no longer necessary to match the data when the assumption of rational expectations is relaxed in favour of learning.

[15] While the learning and the imperfect information literature have received much attention, there are further important alternatives to the paradigm of rational expectations in the theory. Sims (2003) focuses on rational inattention by the agents as a more realistic description of their behaviour. Hansen and Sargent (2001) derive important results in the robust control literature.

[16] However, Milani (2008) finds weak evidence for the wealth effect in a both theoretical and empirical model with a role for learning.

Nisticò and Zanna (2007) find a special case where there could be a stable and determinate equilibrium in a rule with positive weight for asset price inflation. Pfajfar and Santoro (2008) also tackle the question of the optimal response to asset price deviations, in a DSGE model where a cost-channel is made explicit. They find that as this channel becomes more important, responding to stock prices increases the regions of determinacy and E-Stability. Assenza, Berardi and Gatti (2009) develop a DSGE model with an augmented Phillips curve – which, in their view, represents more clearly the cost-channel – to account for the impact of asset price misalignments in inflation. It is worth noting that none of these contributions account for heterogeneous expectations.

More recently, there has been some work on important extensions to the learning framework. Backed by recent empirical evidence on expectation formation, heterogeneity in expectations has been a fruitful way of research. As Honkapohja and Mitra (2006) argue, introducing heterogeneity of expectations raises new challenges for policy. Honkapohja and Mitra (2005) focus on a new Keynesian model in which the central bank and private agents form their expectations differently, depending on initial conditions or learning algorithms. In the case of Muto (2010), heterogeneity arises as private agents learn from the central bank forecasts in an interactive way. Guse (2005) analyses heterogeneous expectations and learning in a univariate approach. Other forms of heterogeneity among private agents include, for instance, Branch and Evans (2011)'s predictor selection and Branch and McGough (2009) who assume a fraction of agents are boundedly rational, while the others form expectations rationally. As a general result, the determinacy conditions of the benchmark homogeneous situation are significantly altered. Another example of departure from homogeneous learning is Arifovic, Bullard and Kostyshyna (2007)'s social learning.

2.3 Basic model

As a description of the economy, we adapt the theoretical general equilibrium model of Carlstrom and Fuerst (2007), with the difference that our model is not deterministic. The standard sticky price model economy is populated by households and firms. Households form decisions on consumption, asset holdings and labour supply while firms decide on the pricing of their goods, while using labour as input. Next we separately analyse such decisions and the resulting equilibrium.

2.3.1 Households

The infinitely-lived households are assumed to have an intertemporal discount factor β and decide on the amount of labour supplied N_t, consumption C_t and on their holdings of bonds and shares. The representative household's period-by-period utility function is represented by a CRRA function:

$$\mathrm{U}\left(C_t, N_t, \frac{M_{t+1}}{P_t}\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1-\gamma}}{1-\gamma} + F\left(\frac{M_{t+1}}{P_t}\right) \tag{2.1}$$

where $\sigma > 0$, $\gamma > 0$, $F(\cdot)$ is increasing and concave, P_t denotes price level and $\frac{M_{t+1}}{P_t}$ denotes cash balances at the end of period t. As in Carlstrom and Fuerst (2007), cash balances assumed to enter the household's utility function at t are the cash balances that each household has after finishing period t transactions. Moreover, we assume that at period t each household's portfolio consists of M_t cash balances, B_{t-1} bonds paying r_{t-1} gross interest rate, wage revenue W_tN_t, a monetary injection X_t and S_{t-1} shares of stock that sell at price Q_t and pay D_t dividends, so that the household is subject to the following budget constraint:

$$\begin{aligned} P_tC_t + P_tQ_tS_t + B_t + M_{t+1} \leq P_tW_tN_t + P_tS_{t-1}(Q_t + D_t) + \\ + r_{t-1}B_{t-1} + M_t + X_t \end{aligned} \tag{2.2}$$

The representative household therefore maximises in period t:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1-\gamma}}{1-\gamma} + F\left(\frac{M_{t+1}}{P_t}\right)\right] \tag{2.3}$$

subject to a sequence of budget constraints of the form (2.2). The first-order conditions lead to the usual optimal relations representing the Euler equations for consumption, labour supply and money demand:

$$\left(\frac{E_tC_{t+1}}{C_t}\right)^{\sigma} = \beta \frac{r_t}{E_t(\pi_{t+1})} \tag{2.4}$$

$$C_t^{\sigma}N_t^{\gamma} = W_t \tag{2.5}$$

$$F'\left(\frac{M_{t+1}}{P_t}\right) C_t^{\sigma} = \frac{r_t-1}{r_t} \tag{2.6}$$

and an additional optimal relation for asset prices

$$\left(\frac{E_t C_{t+1}}{C_t}\right)^{\sigma} = \beta \frac{E_t(Q_{t+1}+D_{t+1})}{Q_t} \tag{2.7}$$

Equation (2.4) states that the intertemporal rate of substitution of consumption depends on the discount rate and on the real interest rate, equation (2.5) characterises the usual consumption-leisure trade off and (2.6) is a money demand function.

Finally, equation (2.7) can be understood as an Euler equation for asset prices, which expresses the equality between the utility gains of postponing consumption and the expected relative appreciation of shares and dividends. Note that combining (2.7) and the Euler equation for consumption yields:

$$\frac{r_t}{E_t(\pi_{t+1})} = \frac{E_t(Q_{t+1}+D_{t+1})}{Q_t} \tag{2.8}$$

which denotes a "no-arbitrage condition", since it establishes the equivalence between the return on bonds on the left-hand side and the return on equities on the right-hand side. Equation (2.8) can also be rewritten as to make asset prices explicit:

$$Q_t = \frac{E_t(Q_{t+1}+D_{t+1})E_t(\pi_{t+1})}{r_t} \tag{2.9}$$

2.3.2 Firms

Firms produce differentiated goods that are sold in a monopolistic competition market. Each firm's output is a function of labour input and an aggregate productivity disturbance V_t:

$$Y_t = V_t N_t, \qquad E(V_t) = 1 \tag{2.10}$$

where we also assume firms face constant returns to scale. The firms' cost minimization problem is

$$\min_{N_t} W_t N_t + Z_t(Y_t - V_t N_t)$$

where Z_t is the firm's real marginal cost. The first order condition implies

$$W_t = Z_t \tag{2.11}$$

As in Carlstrom and Fuerst (2007), firms distribute dividends in the same amount of their profits, i.e:

$$D_t = \Pi_t = Y_t - W_t V_t N_t = (1 - Z_t) Y_t \tag{2.12}$$

We consider nominal rigidities in the spirit of Calvo's (1983) staggered price setting. Each period a fraction $0 < \alpha < 1$ of the goods do not have their prices revised, or, in other words, with probability α a given price will be adjusted by the firm. Under reasonable assumptions on the profit function Π_t, Woodford (2003, pg 187) shows that the aggregate inflation rate and the output gap must satisfy the following aggregate-supply relation in any period t:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{2.13}$$

where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\zeta$ measures the degree of price-stickiness[17].

2.3.3 Log-linearized equilibrium

The market clearing conditions are $S_t = 1$, $B_t = 0$ and the resource constraint $C_t = N_t$. Employing standard techniques, it is possible to represent the equilibrium in terms of log deviations from the respective steady state value of each variable.

$$(\sigma + \gamma) c_t = w_t \tag{2.14}$$

$$\sigma(E_t c_{t+1} - c_t) = r_t - r_t^n - E_t \pi_{t+1} \tag{2.15}$$

$$q_t = \beta E_t q_{t+1} + (1 - \beta) E_t d_{t+1} - (r_t - r_t^n - E_t \pi_{t+1}) \tag{2.16}$$

$$d_t = c_t - \frac{z}{1-z} z_t \tag{2.17}$$

$$w_t = z_t \tag{2.18}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{2.19}$$

---

[17] Since κ is negatively related to α, the longer prices are fixed on average, the less sensitive should be inflation to changes in the output gap. Here ζ describes the degree of strategic complementarity between the price-setting decisions of suppliers of different goods and β relates to the discount factor to which profits are discounted. For a more detailed derivation of (2.13) in a sticky-price environment, see Walsh (2003).

The small macroeconomic model can now be formalised. We begin with the new Keynesian Phillips curve (2.19) characterizing the aggregate supply relation. To keep notation more aligned with the learning and monetary policy literature, we substitute the real marginal cost by a measure of the output gap, $x_t = (Y_t - Y_t^n)$. Equation (2.15) and the resource constraint lead to the aggregate demand relationship, also called the intertemporal IS equation:

$$x_t = E_t x_{t+1} - \sigma^{-1}(r_t - r_t^n - E_t \pi_{t+1}) \tag{2.20}$$

where r_t is the nominal interest rate[18]. From (2.14) and (2.17),

$$d_t = -A z_t \tag{2.21}$$

where $A = \frac{z(1+\sigma+\gamma)-1}{(\sigma+\gamma)(1-z)} > 0$ for reasonable calibrations. The negative relationship between dividends and the output gap reflects the typical detrimental effect of marginal costs on firms profitability. Combining (2.16) and (2.21), the result is an equation relating the dynamics of stock price misalignments to their expected values, together with the expected values of inflation and output gap:

$$q_t = \beta E_t q_{t+1} - A(1-\beta) E_t x_{t+1} - (r_t - r_t^n - E_t \pi_{t+1}) \tag{2.22}$$

The structural parameters σ, κ, and β respectively stand for the elasticity of intertemporal substitution, the degree of price stickiness and the discount factor.

The baseline model is complemented with interest rate policy. First we follow Bullard and Mitra (2002) and consider Taylor-type instrumental rules, augmented with a term corresponding to asset price misalignments. The first instrumental rule considers contemporaneous values of the output gap, inflation and asset price deviations:

$$r_t = \varphi_x x_t + \varphi_\pi \pi_t + \varphi_q q_t \tag{2.23}$$

where φ_y is the policy response to the expected future value of variable y. Differently from Carlstrom and Fuerst (2007), we start with a more general rule, which includes the output gap. We also extend Carlstrom and Fuerst (2007)'s analysis by examining an interest rate rule that responds to forward-looking expectations. Such a rule addresses McCallum's (1998) critique that monetary policy based on current values of inflation and the output gap may fail to be

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[18] Following Bullard and Mitra (2002), we assume the natural rate of interest to follow the stochastic process $r_t^n = \rho r_{t-1}^n + \varepsilon_t$, where ε_t is an i.i.d. noise with variance σ_ε^2 and $0 \leq \rho < 1$.

operational due to their non-availability. In our setting, the forward-looking rule can be represented by:

$$r_t = \varphi_x E_t x_{t+1} + \varphi_\pi E_t \pi_{t+1} + \varphi_q E_t q_{t+1} \tag{2.24}$$

In section 2.6 we consider instead targeting rules, and analyse optimal monetary policy, which can be considered an extension to the analysis of Evans and Honkapohja (2003b).

2.4 Determinacy and E-Stability in the benchmark model

As previously mentioned, the adaptive learning literature has been developing considerably fast, following Evans and Honkapohja (2001), especially regarding its applications to monetary policy theory. In this section we present the basic concepts employed throughout the chapter with respect to desirable properties that an REE should have. As a starting point, we will examine determinacy conditions for the rational expectations equilibrium and then, in an adaptive learning environment, we proceed to the E-Stability requirements.

Combining equations (2.19), (2.20), (2.22) and an instrumental interest rate rule such as (2.23) or (2.24) we have the reduced form:

$$y_t = \alpha + B E_t y_{t+1} + \chi r_t^n \tag{2.25}$$

where $y_t = (x_t, \pi_t, q_t)'$ is the vector of endogenous variables forming the system, α and χ are 3×1 parameter vectors and B is a 3×3 parameter matrix, which are properly defined for each type of rule in the next subsections.

Determinacy conditions, or equivalently, conditions for the uniqueness of a REE are largely used as a desirable criterion in the rational expectations literature; see Woodford (2003), among others. A well-known result in Blanchard and Kahn (1980) states that if the number of eigenvalues of B inside the unit circle is equal to the number of non-predetermined variables, the solution to the system is unique. Otherwise, the system may have multiple solutions, which are commonly called “sunspot solutions”. In this case, some of these solutions may still be of interest if they can be learnable in the sense of Evans and Honkapohja (2001) and E-Stability thus arises as a useful additional selection criterion.

As argued before, our focus will be on an environment in which agents form expectations as they learn from the observed values of the system, instead of knowing from the outset the true structure of the economy. As much of the literature, here we suppose agents adopt recursive least squares as a learning rule to update the parameters of their forecasting model.[19]

Let the minimal state solution (MSV) for the system (2.25) be $y_t = \bar{a} + \bar{c} r_t^n$, which stands for the fundamental equilibrium. Suppose agents believe that the solution is of the form

$$y_t = a + c r_t^n \tag{2.26}$$

where now vectors a and c are not known from the beginning but are estimated by private agents. Equation (2.26) is usually called the perceived law of motion (PLM), since it describes the intrinsic beliefs of the private agents about the relevant parameters in each period.

With this PLM, agents then form expectations as

$$E_t y_{t+1} = a + c\rho r_t^n \tag{2.27}$$

Inserting these expectations into the system (2.25), the result is an actual law of motion (ALM):

$$y_t = Ba + (Bc\rho + \chi) r_t^n \tag{2.28}$$

As Evans and Honkapohja (2008) point out, the ALM is a description of the temporary equilibrium for the expectations derived from the PLM.

The mapping from the PLM to the ALM then takes the form:

$$T(a, c) = (Ba, Bc\rho + \chi) \tag{2.29}$$

where the rational expectations solution $(\bar{a}, \bar{c})$ is a fixed point of this map. An important result is the E-Stability principle, as defined in Evans and Honkapohja (2001) and broadly employed in the adaptive learning literature. Given the differential equation

$$\frac{d}{d\tau}(a, c) = T(a, c) - (a, c) \tag{2.30}$$

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[19] Honkapohja and Mitra (2005) assess monetary policy in a context of heterogeneous agents where a fraction of agents use least squares while the other fraction uses the less sophisticated stochastic gradient learning.

if the particular REE $(\bar{a}, \bar{c})$ is locally asymptotically stable under (2.30), than the REE is said to be stable under learning (or E-Stable). Here, τ denotes "notional" or "artificial" time.

It is convenient to detail the exact sequence of events from the point of view of a private agent: at time t, agents have estimates (a_t, c_t) of the parameters of the PLM, computed with data available at $t-1$. Then r_t^n is realised, and agents form expectations according to (2.27). The central bank sets the policy rate r_t following (2.23) or (2.24), which generates the real y_t through the macroeconomic model. The whole process resumes at $t+1$ as agents add new data to their information set to update their estimates (a_{t+1}, c_{t+1}). The desirable E-Stability result is then achieved, if $(a_t, c_t) \rightarrow (\bar{a}, \bar{c})$ as $t \rightarrow \infty$.

Following Evans and Honkapohja (2001), it turns out that E-Stability of the REE obtains provided the eigenvalues of both B and ρB have real parts less than 1. Since $0 \leq \rho < 1$, it suffices for the roots of B to have real parts less than 1. These results apply to the benchmark case, where we consider homogeneous agents. As for the heterogeneous case, section 2.5 presents appropriate determinacy and E-Stability results.

2.4.1 Calibration parameters

Whenever it is feasible and intuitive, our results are expressed in analytical language. Due to the complexity of the calculations involved, we additionally conduct numerical simulations[20], which are illustrated in the figures next. We follow Bullard and Mitra (2002) by adopting Woodford (1999)'s baseline parameters $\sigma = 0.157$, $\kappa = 0.024$ and $\beta = 0.99$. As for the value of A, we follow Carlstrom and Fuerst (2007) for their case of lower marginal cost sensitivity, i.e., $z = 0.85$ and $\gamma = 0.47$, which yield $A = 4.072$. Importantly, alternative calibrations were also tested and proved not to alter significantly the results.

2.4.2 Benchmark case: homogeneous beliefs

We first consider an environment in which all private agents share the same beliefs about the correct form of the solution, which is reflected here as agents following the PLM defined in (2.26). The focus here is the macroeconomic model above comprising instrumental

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[20] For all numerical calculations and graphs, as well as for some cumbersome matrix algebra, the computer software MAPLE 9.5 was used. Codes are available upon request.

interest rate rules. This allows us to better compare our results to Bullard and Mitra (2002) and to papers that consider asset prices in a homogeneous setting, like Pfajfar and Santoro (2008) and Airaudo, Nisticò and Zanna (2007).

2.4.2.1 Contemporaneous data in the interest rate rule

A quite common policy rule, also analysed in Bullard and Mitra (2002), is an instrumental interest rate rule, where the central bank responds to contemporaneous values of inflation and the output gap. In our framework, with the addition of stock price misalignments, this is exactly equation (2.23). Considering the macroeconomic model (2.19)-(2.20) augmented with the stock price equation (2.22) and contemporaneous interest rate rule (2.23), the system can be expressed by $y_t = \alpha + BE_t y_{t+1} + \chi r_t^n$, where the relevant parameters are:

$$\alpha = 0, \chi = \frac{1}{1+\varphi_q}(\sigma^{-1}, \kappa\sigma^{-1}, 1 - \varphi_\pi \kappa \sigma^{-1})' \text{ and}$$

$$B = \frac{1}{\sigma+\sigma\varphi_q+\varphi_x+\varphi_\pi\kappa}\begin{bmatrix} -\sigma-\varphi_q(\sigma + A - A\beta) & -1+\varphi_\pi\beta & -\varphi_q\beta \\ (-\sigma-\varphi_q(\sigma + A - A\beta))\kappa & \kappa\sigma^{-1}(1-\varphi_\pi)+\beta & -\varphi_\pi\beta \\ -\sigma(A - A\beta) - (\varphi_x+\varphi_\pi\kappa)(\sigma + A - A\beta)) & (-1+\varphi_\pi\beta)\sigma & (\varphi_\pi\kappa+\varphi_x+\sigma)\beta \end{bmatrix} \tag{2.31}$$

Next we will show pertinent results supposing two extreme scenarios: first the response to stock price deviations is muted in the reaction function, that is, $r_t = \varphi_x x_t + \varphi_\pi \pi_t$. Then we allow for a response to contemporaneous values of inflation and stock price deviations: $r_t = \varphi_\pi \pi_t + \varphi_q q_t$. Through such simplifications it is possible to analytically assess the questions involved in the introduction of asset prices into rather standard instrumental monetary policy rules.

Proposition 1: *Under the contemporaneous data rule, assume $\varphi_q = 0$, i.e., there is no response to stock price misalignments. Then the necessary and sufficient condition for uniqueness and E-Stability of the MSV REE is*

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) > 0$$

Proof: See Appendix A.

The shaded area in Figure 1 shows the combinations of policy parameters φ_x and φ_π that lead to determinacy and E-Stability of the MSV solution, using well-known calibrations for the structural parameters. The fact that the results on both determinacy and E-Stability are exactly the same as Bullard and Mitra (2002)'s propositions 1 and 2 appears somewhat trivial, since our interest rate rule boils down to theirs. However, it is still an important result, because now there is stock price dynamics involved.

It is easy to see that, provided the response of interest rates to inflation is more than one-for-one, determinacy and E-Stability is achieved, regardless of the policy response to the output gap. This required condition corresponds to the Taylor principle, following the usual term in the literature, as in Woodford (2003). As the Central Bank reacts to the output gap, the required reaction to inflation is loosened and becomes gradually lower than one.

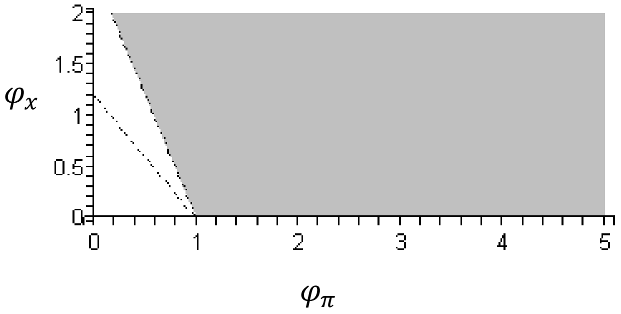

Figure 1: Determinacy and E-Stability regions for rule $r_t = \varphi_x x_t + \varphi_\pi \pi_t$

We now examine a setting where the monetary authority responds to inflation and stock price deviations, but not to the output gap, such that (2.23) boils down to $r_t = \varphi_\pi \pi_t + \varphi_q q_t$. Using the same basic framework, the following result obtains:

Proposition 2: *Under the contemporaneous data rule, Assume $\varphi_x = 0$, i.e, there is no response to the output gap. Then the necessary and sufficient conditions for uniqueness and E-Stability of the REE is*

$$\kappa(\varphi_\pi - 1) - \varphi_q A(1 - \beta) > 0$$

Proof: See Appendix A.

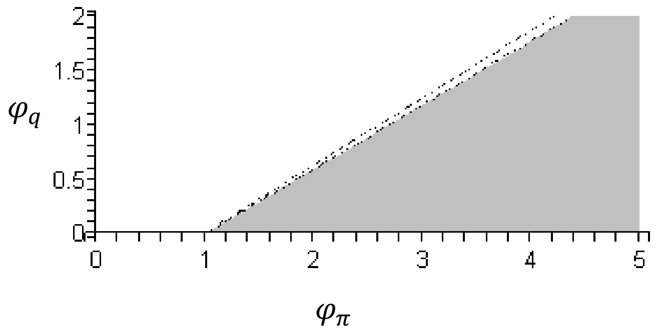

Figure 2: Determinacy and E-Stability regions for rule $r_t = \varphi_\pi \pi_t + \varphi_q q_t$

As expected, our result mirrors Carlstrom and Fuerst (2007)'s proposition 1, since their policy rule does not prescribe a reaction to the output gap. Notice that the result again resembles the Taylor rule, since it requires that nominal interest rates rise by more than the increase in the inflation rate, independently of the response to asset prices. However, as Figure 2 depicts, higher responses to asset prices now *deteriorate* determinacy and E-Stability conditions. This effect can be explained by the usual negative relationship between firm profits (which impact dividends and asset prices) and real marginal costs. As inflation rises, so do marginal costs, and consequently asset prices tend to fall. A response of the policy rate to asset prices in the same direction undermines the CB's response to inflation, as described by Carlstrom and Fuerst (2007).

2.4.2.2 Forward expectations in the interest rate rule

We now turn to a case not studied by Carlstrom and Fuerst (2007), which consists of a forward-looking expectations interest rate rule, where the CB responds to forecasts of output, inflation and asset price deviations. This alternative model which can be understood as an extension to Bullard and Mitra (2002), involves combining (2.19)-(2.20) together with (2.22) and the interest rate rule (2.24), leading to $y_t = \alpha + BE_t y_{t+1} + \chi r_t^n$, where:

$\alpha = 0, \chi = (\sigma^{-1}, \kappa\sigma^{-1}, 1)'$ and

$$B = \begin{bmatrix} 1-\sigma^{-1}\varphi_x & \sigma^{-1}(1-\varphi_\pi) & -\sigma^{-1}\varphi_q \\ \kappa(1-\sigma^{-1}\varphi_x) & \kappa\sigma^{-1}(1-\varphi_\pi)+\beta & -\kappa\sigma^{-1}\varphi_q \\ -A(1-\beta)-\varphi_x & 1-\varphi_\pi & \beta-\varphi_q \end{bmatrix} \tag{2.32}$$

Repeating the same two scenarios studied in the last subsection, we have propositions 3 and 4:

Proposition 3: *Under the forward expectations interest rate rule, assume* $\varphi_q = 0$, *i.e., there is no response to stock price misalignments. Then the necessary and sufficient conditions for uniqueness of the MSV REE are*

$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) < 2\sigma(1+\beta)$$

and

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) > 0$$

Furthermore, the MSV equilibrium is E-Stable if the latter condition is met.

Proof: See Appendix B.

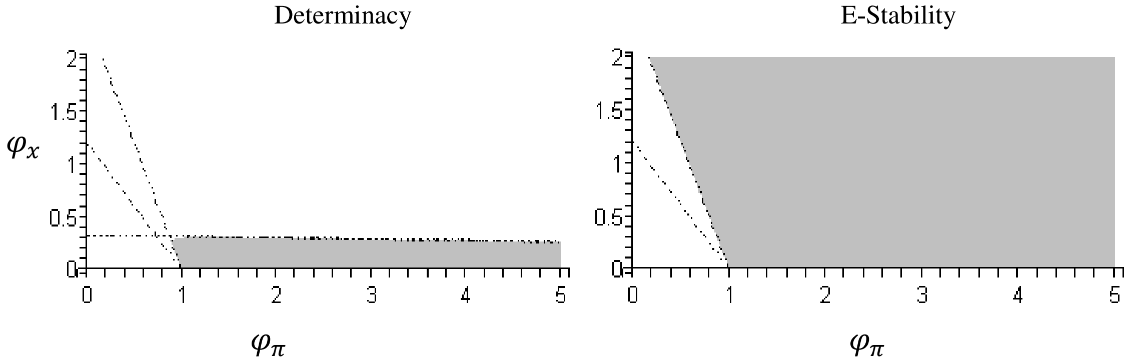

Figure 3: Determinacy and E-Stability regions for rule $r_t = \varphi_x E_t x_{t+1} + \varphi_\pi E_t \pi_{t+1}$

Again, our result is similar to Bullard and Mitra (2002)'s proposition 4. The key message here is that determinacy requirements are stricter under the forward-expectations rule, as becomes clear in Figure 3 above. The first constraint in proposition 3 means that there is an upper bound on both responses φ_x and φ_π.[21] On the other hand, the Taylor principle is enough to guarantee E-Stability and we have the same outcome as proposition 2. As a consequence, there may be some situations in which there is multiplicity of equilibria, but these equilibria may be learned by private agents, a possibility we do not study here.

As in the last subsection, we now examine a setting where the monetary authority responds to inflation and stock price deviations, but not to the output gap, so that (2.24) boils down to $r_t = \varphi_\pi E_t \pi_{t+1} + \varphi_q E_t x_{q+1}$. Using the same basic framework, the following result obtains:

Proposition 4: *Assume $\varphi_x = 0$, i.e., there is no response to the output gap in the instrumental interest rate rule. Then the necessary and sufficient conditions for uniqueness of the MSV REE are*

$$\kappa(\varphi_\pi - 1) + \varphi_q(A(1-\beta) + 2\sigma) < 2\sigma(1+\beta)$$

and

$$\kappa(\varphi_\pi - 1) - \varphi_q A(1-\beta) > 0$$

Furthermore, the MSV equilibrium is E-Stable if the latter condition is met.

Proof: See Appendix B.

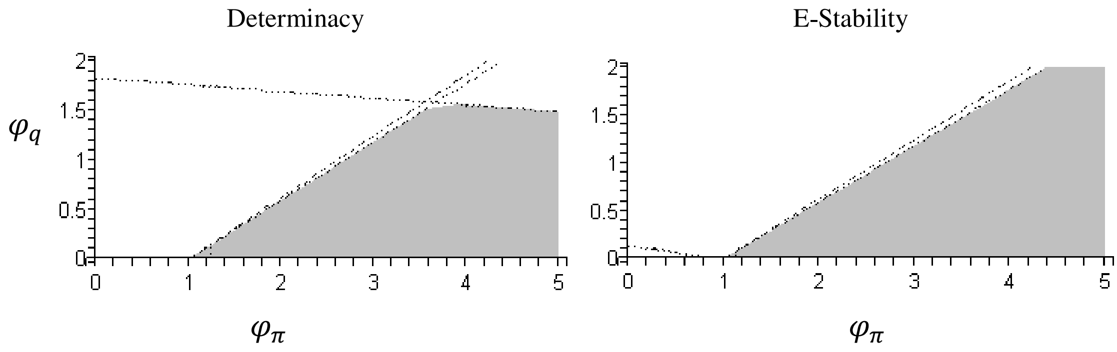

Figure 4: Determinacy and E-Stability regions for rule $r_t = \varphi_\pi E_t \pi_{t+1} + \varphi_q E_t q_{t+1}$

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[21] Using the calibrated parameters, indeterminacy of the REE ensues for any $\varphi_x > 0.32$ or $\varphi_\pi > 27$.

Similarly, when the central bank has to choose on the appropriate level of response between expected inflation and asset price deviations, an upper bound on φ_q emerges, represented by the first constraint on proposition 4. As for E-Stability, again the result we interpret as similar to the Taylor rule suffices.

A key difference between the rules is that, under the forward-expectations rule, equilibrium determinacy guarantees E-Stability, but the converse is not always true; on the other hand, under the contemporaneous rule, both regions are always equivalent.

Overall the desired response to inflation is guided by the Taylor principle, as in Bullard and Mitra (2002). However, differently from what follows the usual reaction to the output gap, as the response to asset prices increases the required response to inflation has to be even more aggressive in order to maintain uniqueness and E-Stability of the REE. These findings are further supported by numerical simulations showing that, under both rules (2.23) and (2.24), as φ_q increases, the area of determinacy and E-Stability in the $\varphi_\pi \, X \, \varphi_x$ space shrinks.[22]

2.5 Heterogeneous beliefs

Maintaining the same macroeconomic framework with the forward expectation policy rule, we now turn to a more realistic assumption that not all private agents form their beliefs in the same fashion. Assume there are two types of agents in the economy: A fraction μ of type 1 agents form expectations according to the MSV perceived law of motion, exactly as in the benchmark case of section 2.3, that is:

$$PLM_1\colon y_t = a_1 + c r_t^n \tag{2.33}$$

The remaining $(1-\mu)$ agents use a different form of PLM, which includes a lagged component. We call this overparameterized law of motion the AR(1) PLM:

$$PLM_2\colon y_t = a_2 + b_2 y_{t-1} + c r_t^n \tag{2.34}$$

where the estimated parameters a_1, a_2 and c are 3×1 vectors and b_2 is a 3×3 matrix.

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[22] Figures and calculations are available from the author on demand.

Taking expectations at $t + 1$ yields:

$$Ey_{t+1} = \mu a_1 + (1 - \mu)[(I + b_2)a_2 + b_2^2 y_{t-1} + b_2 c r_t^n] + \ c\rho r_t^n \qquad (2.35)$$

Substituting it into system (2.25) and reorganising the terms:

$$y_t = \alpha + B[\mu a_1 + (1 - \mu)(I + b_2)a_2 + (1 - \mu)b_2^2 y_{t-1}] +$$

$$+B[(1 - \mu)b_2 c + c\rho]r_t^n + \chi r_t^n \qquad (2.36)$$

This equation is precisely the ALM, which represents the stochastic process followed by y_t, given agents perceived law of motions. The next step is to construct the mapping from the PLM to the ALM, as in Evans and Honkapohja (2001). However, since there are 2 types of PLM, we follow closely Guse (2005). For type 1 agents, he assumes a "projected ALM", which is constructed by the mean of the implied y_t process. On the other hand, the ALM parameters for type 2 agents derive from the intercept and slope parameters of the implied y_t process.

Thus, the mapping from the PLMs to the ALM can be expressed by:

$$T\begin{pmatrix} a_1 \\ a_2 \\ b_2 \\ c \end{pmatrix} = \begin{pmatrix} [I - (1 - \mu)Bb_2^2]^{-1}[\alpha + \mu B a_1 + (1 - \mu)B(I + b_2)a_2] \\ \alpha + \mu B a_1 + (1 - \mu)B(I + b_2)a_2 \\ (1 - \mu)Bb_2^2 \\ (1 - \mu)Bb_2 c + \rho B c + \chi \end{pmatrix} \qquad (2.37)$$

The rational expectation equilibria are fixed points of this mapping and satisfy:

$$a_1 = [I - (1 - \mu)Bb_2^2]^{-1}[\alpha + \mu B a_1 + (1 - \mu)B(I + b_2)a_2]$$

$$a_2 = \alpha + \mu B a_1 + (1 - \mu)B(I + b_2)a_2$$

$$b_2 = (1 - \mu)Bb_2^2$$

$$c = (1 - \mu)Bb_2 c + \rho B c + \chi$$

Following Evans and Honkapohja (2001), E-Stability is then determined by the differential equation:

$$\frac{d}{d\tau}(a_1, a_2, b_2, c) = T(a_1, a_2, b_2, c) - (a_1, a_2, b_2, c) \qquad (2.38)$$

As for the MSV equilibria, since $b_2 = 0$, (2.37) boils down to:

$$T\begin{pmatrix} a_1 \\ c \end{pmatrix} = \begin{pmatrix} \mu B a_1 + (1-\mu) B a_2 \\ \rho B c + \chi \end{pmatrix},$$

so that $\frac{d}{d\tau}(a_1, c) = (\mu B a_1 + (1-\mu) B a_2 - a_1, \rho B c + \chi - c)$.

Regarding AR(1) equilibria, the equilibrium value for b_2 must be $\overline{b_2} = (1-\mu)^{-1} B^{-1}$, so that the differential equation (2.38) conditioned on (2.37) provide the following E-Stability result.

Proposition 5: *Under heterogeneous beliefs, the macroeconomic model with the forward-expectations rule has E-Stable MSV equilibria when the same conditions of Propositions 3 and 4 are met. However, AR(1) equilibria are not E-Stable, regardless of the proportion of agents of each type.*

Proof: See Appendix C.

As expected, MSV equilibria give rise to the same E-Stability results as in the case of propositions 3 and 4, where all agents homogeneously form beliefs. On the other hand, AR(1) equilibria are not E-Stable.

Even when some of the agents form beliefs that are not in accordance with the minimal state variable solution, the equilibrium will tend to converge to the MSV REE, since it leads to E-Stability over a quite broad range of parameters, as seen on the homogeneous case. However, AR(1) equilibria may also be reached and when it happens, no stability under learning at all is guaranteed.

The E-Instability result is unsurprisingly similar to the one represented in Honkapohja and Mitra (2004)'s proposition 3, since in both cases the same class of non-fundamental solutions was studied, that is, autoregressive solutions. This failure is related to the strong influence of past values of the endogenous variables on their current and expected values.

2.6 Optimal monetary policy with asset prices

An interesting further step is to analyse the implications of the introduction of asset price misalignments in an optimised monetary policy setting. For this task, we borrow some ideas of Evans and Honkapohja (2003a,b). In their study of adaptive learning applied to monetary policy in a standard new-Keynesian model, fundamental-based rules arguably entail instability under learning. Conversely, their proposal of expectations-based rules leads to convergence of the optimal REE to E-Stability results. This is valid not only when private expectations are observable, but also when key structural parameters can be estimated and used in the interest rate rule, supposing simultaneous learning by private agents and policymakers. Extending Evans and Honkapohja (2003a,b)'s main results, we focus here on the case of expectations-based rules.

Interest rate policy now consists of the optimisation of an intertemporal objective function. As usual it is assumed that the central bank faces the problem of minimising a quadratic loss function, adapted to account for asset price deviations, that is:

$$\min E_t \sum_{s=0}^{\infty} \beta^s \left[\pi_t^2 + \lambda_x {x_t}^2 + \lambda_q {q_t}^2\right] \tag{2.39}$$

The novelty here, compared to most of the literature on optimal monetary policy, is the CB's additional preference for stock price gap stabilization. We also assume the usual Phillips curve:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \tag{2.40}$$

The cost-push shock u_t follows a random walk, that is, $u_t = \rho u_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$.[23]

The first-order condition of (2.39) subject to the Phillips curve results in the following implied reaction function:

$$\kappa \pi_t + \lambda_x x_t + \lambda_q q_t = 0 \tag{2.41}$$

which can also be interpreted as a targeting rule, because it contains the desired relationship between inflation, the output gap and the stock price gap. Combining the IS relation, the

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[23] As Clarida, Gali and Gertler (1999) point out, this shock enables the model to generate variation in inflation that evolves independently of movement in excess demand, so that a trade-off between inflation and output stabilization arises.

Phillips curve in (2.40) and the implied reaction function (2.41), after some algebra we arrive at:

$$r_t = \delta_x E_t x_{t+1} + \delta_\pi E_t \pi_{t+1} + \delta_q E_t q_{t+1} + \delta_u u_t + r_t^n \tag{2.42}$$

where the coefficients are:

$$\delta_x = \sigma(\sigma+\gamma)^{-1}[\gamma - A(1-\beta)] \tag{2.43}$$

$$\delta_\pi = 1 + \frac{\kappa\beta}{\lambda_q}\sigma(\sigma+\gamma)^{-1} \tag{2.44}$$

$$\delta_q = \beta\sigma(\sigma+\gamma)^{-1} \tag{2.45}$$

$$\delta_u = \frac{\kappa}{\lambda_q}\sigma(\sigma+\gamma)^{-1} \tag{2.46}$$

and $\gamma = \frac{\kappa^2+\lambda_x}{\lambda_q}$. Here, (2.42) is the expectations-based optimal rule, that implements the targeting rule (2.41) in every period.

An important result of the expectations-based rule is that a variant of the Taylor principle is again guaranteed, since $\delta_\pi > 1$ for plausible calibrations and for either λ_x or λ_q strictly positive.

2.6.1 Learning analysis

It is also interesting to check the resulting conditions of stability under adaptive learning, as we conducted for instrumental rules before. Taking into account the macroeconomic model above and the expectations-based rule, the following proposition states the desirable response to stock prices under optimal monetary policy.

Proposition 6: *Consider the expectations-based rule (2.42) together with the IS equation (2.20) and the Phillips curve (2.40). The REE of the obtained system is E-Stable if and only if* $0 < \lambda_q < \frac{1}{A}\left(\lambda_x + \frac{\kappa^2}{A(1-\beta)}\right)$.

Proof: See Appendix D.

According to this result, as the response to stock prices λ_q rises, λ_x has to rise even more for E-Stability to obtain. In other words, CB's preference for stock price gap stabilization is limited by its preference for output gap stabilization. As a consequence, if the Central Bank is rather inflation-targeter, it should respond very carefully to stock prices.

A relevant result is that, contrarily to Evans and Honkapohja (2003a,b), an expectations-based rule is no longer a guarantee that E-Stability of the REE will be reached. Confirming the previous results on instrumental interest rate rules, introducing asset prices is clearly not desirable under an optimal policy perspective as well.

2.7 Conclusions

We have assessed the conditions for determinacy and stability under learning of an extended monetary policy rule that accounts for asset price variations. Since we adopt instrumental interest rate rules in a normative way, our framework is closely related to Bullard and Mitra (2002) and Carlstrom and Fuerst (2007). Nevertheless, we depart from them in various ways: with respect to the latter, we consider both E-Stability of the REE in a learning environment as defined by Evans and Honkapohja (2001) and a forward expectations policy rule. We also take into account heterogeneous beliefs about the dynamics of variables, in a similar way to Guse (2005), and an optimal monetary policy rule, extending Evans and Honkapohja (2003a,b).

In the homogeneous expectations case, we have shown that introducing a response to stock prices in a contemporaneous interest rate rule does not generally lead to desirable outcomes with respect to determinacy, for the same reason as Carlstrom and Fuerst (2007) had already pointed out. The negative relationship between firm profits (and consequently dividends and asset prices) and marginal costs means that, as inflation rises, so do marginal costs, and consequently asset prices tend to fall. A response of the policy rate to asset prices in the same direction undermines the CB's response to inflation, leading to indeterminacy of REE. It turned out that this effect also threatens E-Stability, as we have shown both in a contemporaneous and in a forward expectations interest rate rule.

Heterogeneity leads to MSV equilibria being E-Stable at least over the same regions as in the homogeneous case, whereas AR(1) equilibria are unstable over all regions of the central bank responses. As an implication, if there is a considerable fraction of agents in the economy who form expectations looking at past values of key variables (in our case, stock price developments, inflation and output gap), non-fundamental equilibria which are not learnable may arise.

Our analysis of optimal interest rate policy in the presence of stock price misalignments showed that the Taylor principle is also important, at least when an expectations-based monetary policy rule in the sense of Evans and Honkapohja (2003a,b) is considered.

Given that stock price booms and busts have caused considerable damage to the financial stability of many industrialised countries over the last years, and since no clear-cut stance of monetary policy is to be advised, further measures could also be taken into account. Indeed some authors and policymakers have recently argued that macroprudential regulation should be the best alternative to cope with financial imbalances and the possibility of booms and busts, see IMF (2011). Less discussed are the potential negative side effects of increased regulation. Further research could also extend on the ideas of heterogeneity in various ways. Agents may be allowed to switch between beliefs, depending on some measure of past performance of their forecast, as in Guse (2005), or Branch and Evans (2011). Alternatively, some agents may also form rational expectations, while others behave adaptively, as in Branch and McGough (2009).

2.8 Appendix: Proofs

A. Determinacy and E-Stability conditions: Contemporaneous rule

Proof of proposition 1

As we mentioned in section 2.3.1, for determinacy of the REE of a system like (2.25), all of the 3 eigenvalues of B must lie inside the unit circle. Following LaSalle (1986, pg.28), given the characteristic polynomial of B, $p_B(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$, its roots lie inside the unit circle if and only if the following inequalities hold:

$$|a_0 + a_2| < 1 + a_1 \text{ and } |a_1 - a_0 a_2| < 1 - a_0^2$$

where:

$$a_0 = \frac{\beta^2\sigma}{\sigma + \varphi_x + \varphi_\pi\kappa}$$

$$a_1 = \frac{\beta(2\sigma + \beta\sigma + \beta\varphi_x + \kappa)}{\sigma + \varphi_x + \varphi_\pi\kappa}$$

$$a_2 = -\frac{\beta(2\sigma + \varphi_\pi\kappa + 2\varphi_x + \kappa) + \kappa + \sigma}{\sigma + \varphi_x + \varphi_\pi\kappa}$$

The first inequality produces the result of the proposition, while the second one does not bind, as is shown in Figure 1 with numerical calculation using calibrated values.

For E-Stability of the REE we need all the eigenvalues of B to have real parts less than 1, which is equivalent to the condition that all eigenvalues of $C = B - I$ should have negative real parts (where I is an identity matrix). For the given characteristic polynomial of C, $p_C(\lambda) = \lambda^3 + a_{E2}\lambda^2 + a_{E1}\lambda + a_{E0}$, the necessary and sufficient conditions follow from the Routh-Hurwitz Theorem (Gandolfo, 1997 pgs. 221-223):

$$a_{E1} > 0$$

$$a_{E0} > 0$$

$$a_{E2} > 0$$

$$a_{E1}a_{E2} - a_{E0} > 0$$

Note that either the first or the second constraint can be suppressed, since either one is implied by the remaining three.

The polynomial coefficients are then:

$$a_{E0} = (1-\beta)\frac{\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta)}{\sigma + \varphi_x + \varphi_\pi\kappa}$$

$$a_{E1} = \frac{\kappa\big((3-2\beta)\varphi_\pi - (2-\beta)\big) + \sigma(1-\beta)^2 + \varphi_x(1-\beta)(3-\beta)}{\sigma + \varphi_x + \varphi_\pi\kappa}$$

$$a_{E2} = \frac{\kappa((3-\beta)\varphi_\pi - 1) + 2\sigma(1-\beta) + \varphi_x(3-2\beta)}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa}$$

Condition $a_{E0} > 0$ implies the Taylor Principle result $\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) > 0$. The remaining are non-binding conditions, as we again confirmed through numerical analysis. Note that the dotted lines on the plots represent these non-binding constraints.

Proof of proposition 2

We follow the proof of proposition 1. Coefficients a_0, a_1 and a_2 are derived in the same way, with the difference that $\varphi_q = 0$ instead of $\varphi_x = 0$:

$$a_0 = \frac{-\beta^2\sigma}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa}$$

$$a_1 = \beta\frac{\varphi_q A(1-\beta) + \sigma(2+\beta) + \varphi_q\sigma + \kappa}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa}$$

$$a_2 = \frac{\varphi_q A(1-\beta) + \sigma(1+2\beta) + \varphi_q\sigma(1+\beta) + \kappa(\beta\varphi_\pi + 1)}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa}$$

Here, algebraically solving $|a_0 + a_2| < 1 + a_1$ leads to the result of the proposition, while $|a_1 - a_0 a_2| < 1 - a_0^2$ turn out not to bind.

An analogous result is reached for the E-Stability conditions. Given the coefficients

$$a_{E0} = (1-\beta)\frac{\kappa(\varphi_\pi - 1) + \varphi_q A(1-\beta)}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa}$$

$$a_{E1} = \frac{\kappa((3-2\beta)\varphi_\pi - (2-\beta)) + \sigma(1-\beta)^2}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa} +$$

$$+\frac{\varphi_q\sigma(1-\beta) + \varphi_q A\beta(3-\beta) + 2\varphi_q A}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa}$$

$$a_{E2} = \frac{\kappa((3-\beta)\varphi_\pi - 1) - 2\sigma(1-\beta) + \varphi_q\sigma(2-\beta) - \varphi_q A(1-\beta)}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa}$$

it is easy to show that $a_{E0} > 0$ results in the corresponding Taylor principle like equation. At the same time, conditions $a_{E1} > 0$ and $a_{E2} > 0$ are not binding.

B. Determinacy and E-Stability conditions: Forward-looking rule

Proof of proposition 3

Using the same idea behind the proof of Proposition 1, under the forward-expectations interest rate rule the following coefficients of the characteristic polynomial of *B* obtain:

$$a_0 = \frac{\beta(\beta\varphi_x - \beta\sigma)}{\sigma}$$

$$a_1 = \frac{2\beta\varphi_x - \beta\kappa(\varphi_\pi - 1) - \sigma(\beta^2 + 2\beta)}{\sigma}$$

$$a_2 = \frac{\kappa(\varphi_\pi - 1) + \varphi_x + 2\sigma(1 - \beta)}{\sigma}$$

Solving for $|a_0 + a_2| < 1 + a_1$ we have both conditions shown in the text. The constraint $|a_1 - a_0 a_2| < 1 - a_0^2$ does not bind (see Figure 3).

For E-Stability of the REE again all eigenvalues of $C = B - I$ should have real negative parts. Given the characteristic polynomial of C, $p_C(\lambda) = \lambda^3 + a_{E2}\lambda^2 + a_{E1}\lambda + a_{E0}$, we showed the required conditions are

$$a_{E0} > 0$$

$$a_{E2} > 0$$

$$a_{E1}a_{E2} - a_{E0} > 0$$

where:

$$a_{E0} = (1 - \beta)\frac{\kappa(\varphi_\pi - 1) + \varphi_x(1 - \beta)}{\sigma}$$

$$a_{E1} = \frac{\kappa(\varphi_\pi - 1)(2 - \beta) + \sigma(1 - \beta)^2 + 2\varphi_x(1 - \beta)}{\sigma}$$

$$a_{E2} = \frac{\kappa(\varphi_\pi - 1) + \varphi_x + 2\sigma(1 - \beta)}{\sigma}$$

Condition $a_{E0} > 0$ implies $\kappa(\varphi_\pi - 1) + \varphi_x(1 - \beta) > 0$. The other conditions are not binding, as we again confirmed through numerical analysis.

Proof of proposition 4

As for determinacy, we follow the proof of proposition 1. The coefficients a_0, a_1 and a_2 are derived in the same way, with the difference that $\varphi_q = 0$ instead of $\varphi_x = 0$:

$$a_0 = \beta \frac{\varphi_q A(1 - \beta) + \varphi_q \sigma - \sigma\beta}{\sigma}$$

$$a_1 = \frac{\sigma\beta(\beta + 2) - \kappa\beta(\varphi_\pi - 1) - \varphi_q A(1 - \beta) - \varphi_q \sigma(\beta + 1)}{\sigma}$$

$$a_2 = \frac{\kappa(\varphi_\pi - 1) + \varphi_q \sigma - \sigma(2\beta + 1)}{\sigma}$$

Here, algebraically solving $|a_0 + a_2| < 1 + a_1$ leads to both results of the proposition, while $|a_1 + a_0 a_2| < 1 - a_0^2$ does not to bind (see boundaries on Figure 4).

A similar pattern marks the E-Stability conditions. Given the coefficients

$$a_{E0} = (1 - \beta) \frac{\kappa(\varphi_\pi - 1) + \varphi_q A(1 - \beta)}{\sigma}$$

$$a_{E1} = \frac{\kappa(\varphi_\pi - 1)(2 - \beta) + \sigma(1 - \beta)^2 \varphi_q (1 - \beta)(\sigma - A)}{\sigma}$$

$$a_{E2} = \frac{\kappa(\varphi_\pi - 1) + \varphi_q \sigma + 2\sigma(1 - \beta)}{\sigma}$$

it is easy to show that $a_{E0} > 0$ results in the corresponding Taylor principle like equation. At the same time, conditions $a_{E2} > 0$ and $a_{E1} a_{E2} - a_{E0} > 0$ do not bind under the calibrated parameters, as shown in Figure 4.

C. Heterogeneous expectations

Proof of proposition 5

Let us first concentrate on MSV equilibria. Using the same notation as Evans and Honkapohja (2001), we have:

$$DT_{a_1}(\overline{a_1}, \overline{a_2}) = \mu B$$

$$DT_c(\bar{c}) = \rho B$$

$$DT_{a_2}(\overline{a_1}, \overline{a_2}) = DT_c\left(\bar{c}, \overline{b_2}\right) = 0$$

Since $0 < \rho \leq 1$ and $0 \leq \mu \leq 1$, E-Stability obtains whenever the eigenvalues of B have real parts less than one, or alternatively, the eigenvalues of $B - I$ have negative parts less than one. Note that this is equivalent to the conditions in Propositions 3 and 4. Moreover, when $\rho < 1$ and/or $\mu < 1$, E-Stability conditions for MSV equilibria are even less strict. Note also that since there is a discontinuity when $\mu = 0$, a strictly positive number of type 1 agents is required for the MSV REE to be E-Stable.

Now, turning to AR(1) equilibria, after solving for the differential equation (2.38) and substituting $\overline{b_2} = (1-\mu)^{-1}B^{-1}$, we arrive at:

$$DT_{a_1}\left(\overline{a_1}, \overline{b_2}\right) = [I - (1-\mu)Bb_2^2]^{-1}\mu B = [I - (1-\mu)^{-1}B^{-1}]^{-1}\mu B$$

$$DT_{a_2}(\overline{a_2}, \overline{b_2}) = (1-\mu)B(I + b_2) = (1-\mu)B + I$$

$$DT_{b_2}\left(\overline{b_2}\right) = (1-\mu)(b_2^T \otimes B + I \otimes B b_2) =$$

$$= (B^{-1})^T \otimes B + (1-\mu) I \otimes (1-\mu)^{-1} I$$

$$DT_c\left(\bar{c}, \overline{b_2}\right) = (1-\mu)Bb_2 + \rho B = I + \rho B$$

For E-Stability we need the eigenvalues of DT_{a_1}, DT_{a_2}, DT_{b_2}, and DT_c to simultaneously have real parts less than one. Using the property that the eigenvalues of a Kronecker product of two matrices A and B are the individual products of the eigenvalues of each matrix (Lancaster and Tismenetski, 1985), there is an unstable root, with value 2, among the eigenvalues of DT_{b_2}, independently of the proportion of agents of each type.

D. Expectations-based rule

Proof of proposition 6

Following the basic ideas explained in section 2.4, combining the expectations-based rule (2.42) and the IS and Phillips curve leads to $y_t = \alpha + BE_t y_{t+1} + \chi r_t^n$, where the relevant parameters are:

$$\alpha = 0, \chi = \tfrac{1}{1+\delta_q}(\sigma^{-1}, \kappa\sigma^{-1}, 1 - \delta_\pi \kappa\sigma^{-1})' \text{ and}$$

$$B = \begin{bmatrix} 1 - \sigma^{-1}\delta_x & \sigma^{-1}(1 - \delta_\pi) & -\sigma^{-1}\delta_q \\ \kappa(1 - \sigma^{-1}\delta_x) & \kappa\sigma^{-1}(1 - \delta_\pi) + \beta & -\kappa\sigma^{-1}\delta_q \\ -A(1 - \beta) - \delta_x & 1 - \delta_\pi & \beta - \delta_q \end{bmatrix}$$

Substituting (2.43)-(2.45) and solving for the condition that the roots of B must have real parts less than 1, it turns out that E-Stability obtains whenever

$$\frac{\kappa^2 - \beta\kappa^2 + 2A\beta\lambda_q - 2\beta\lambda_x - A\lambda_q + \lambda_x + \beta^2\lambda_x - A\beta^2\lambda_q}{\kappa^2 + \sigma\lambda_q + \lambda_x} > 0$$

which leads to

$$\lambda_q < \tfrac{1}{A}\left(\lambda_x + \tfrac{\kappa^2}{A(1-\beta)}\right)$$

Finally, $\lambda_q > 0$ is mathematically required in order to have the term $\gamma = \frac{\kappa^2+\lambda_x}{\lambda_q}$ defined.

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3 MEASURING INFLATION PERSISTENCE IN BRAZIL USING A STRUCTURAL TIME SERIES MODEL[1]

Abstract

Most of the literature treats estimates of inflation persistence as unconditional measures, since they are usually based on univariate autoregressive equations. In this article, we provide recent estimates of inflation persistence in Brazil in a multivariate framework of unobserved components, whereby we account for the following sources affecting inflation persistence: First, the fact that it may derive from deviations of expectations from the actual policy target. Second, persistence of the factors driving inflation, such as the behaviour of interest rate and output may also affect the dynamics of inflation. Third, the usual intrinsic measure of persistence is evaluated through lagged inflation terms. Data on inflation, output and interest rates are decomposed into unobserved components, which are identified in a linear Gaussian state-space model. To simplify the estimation of a great number of unknown variables, we employ Bayesian analysis as in Dossche and Everaert (2005). Our results indicate that expectations-based persistence matters considerably for inflation persistence in Brazil, which has experienced an overall decrease in the last few years. These findings imply that traditional price-setting frictions used in macroeconomic models may be misleading in terms of representing real inflation persistence.

3.1 Introduction

Vestiges of inflationary memory, particularly in developing countries, which experienced decades of high inflation levels, may still be an important obstacle in the process of price stabilization. In the 1980s and early 1990s, Brazil underwent a period of high inflation rates. The inertial component of inflation was admittedly strong in that period. Although the Real Plan in 1994 managed to reduce inflation rates, it is not yet clear whether a substantial decrease in inflation persistence has followed, according to the Central Bank of Brazil’s Inflation Report of December 2008, see BCB (2008).

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[1] This study benefitted from comments at the XIV ESTE (Escola de Séries Temporais), in Gramado, 2011 and at the Macro and Financial Econometrics Conference at the University of Heidelberg in September, 2011.

Inflation persistence is commonly associated with high levels of inflation and inflationary uncertainty, as well as with high stabilization costs. While none of these features is desirable in an inflation targeting economy, it is important to accurately measure the persistence of inflation. Most of the literature on measures of inflation persistence is based on inflation data only, usually building on univariate autoregressive equations. Examples include Pivetta and Reis (2007), Cogley and Sargent (2005) and Levin and Piger (2004), as well as Petrassi and Oliveira (2010) in the Brazilian case. These measures are said to represent unconditional inflation persistence, since they do not consider the underlying inflation generating process.

In this article, we use a model based on Dossche and Everaert (2005)[2], recognizing that there are specific effects on the process of inflation other than past inflation values, which have a considerable impact on inflation persistence. We first estimate univariate inflation persistence for Brazil with a slightly different approach, by introducing an expectations-based source of persistence, in the sense of Angeloni et al (2004). Then, we provide estimates of inflation persistence in a multivariate framework of unobserved components, dealing with the following sources of inflation persistence: First, the fact that it may derive from deviations of expectations from the actual policy target. This source is known as expectations-based persistence, and it may be understood as similar to the “sticky information measure”, as in Mankiw and Reis (2002). According to these authors, firms gather information on prices slowly because of the costs incurred in acquiring and in reoptimizing it.[3] If this is really the case, then substantial differences between private agents’ expected targets and central bank policy targets have a potential influence on inflation persistence that should not be overlooked. In fact, both Caetano and Moura (2009) and Guillen (2008) found evidence that agents update information in Brazil in a similar frequency as found in studies for the USA and Europe, such as Mankiw, Reis and Wolfers (2003). Second, persistence of the factors driving inflation, such as the stance of interest rate and of potential output may also affect the persistence of inflation. According to Dossche and Everaert (2005), persistence of output gaps in response to business cycle shocks add to the persistence of inflation, referred to by them as

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[2] Their work reflects research conducted in the context of the Eurosystem Inflation Persistence Network (IPN) of the European Central Bank, which aimed to explain price setting and inflation dynamics in order to address patterns, causes and policy implications of inflation persistence in the euro area.

[3] A similar argument is found in Sims (2003). According to his theory of “rational inattention”, it may happen that people simply have limited ability to acquire and process information. Other possibilities are the assumptions that the central bank has imperfect credibility, as in Kozicki and Tinsley (2005) or that agents are uncertain about central bank preferences, as in Cukierman and Meltzer (1986), or also that agents are learning about the true model of the economy, as Milani (2007) argued.

"extrinsic persistence." Third, the usual "intrinsic persistence" related to the nature of the price-setting mechanism is also evaluated as we account for lags in the inflation equation.

Data on inflation, output and interest rates are decomposed into unobserved components, forming a linear Gaussian state-space model. Due to the sizeable number of unknown components, to simplify the estimation, we use data from other studies of the Brazilian economy (or from other countries whenever Brazilian data are not available) as priors in a Bayesian analysis. Put differently, the various unobserved coefficients in our basic equations are assigned coherent distributions, in order to make the whole analysis possible.

The main outcomes are the time distributions of state variables such as the perceived and actual inflation target and coefficients that represent the sources of persistence. Another important outcome of the estimation is the natural rate of interest. Besides playing a role in inflation dynamics, this variable is useful to check whether interest rates have departed from the estimated natural stance, in order to empirically assess, for example, arguments that the Central Bank of Brazil has been too conservative in its interest rate policy.

This article is organized into five sections. After this introduction, we present the related literature, concerning both empirical and theoretical contributions to inflation persistence. In Section 3.3, we present the model and discuss the inputs and the estimation strategies adopted. Section 3.4 contains the main results and section 3.5 concludes.

3.2 Inflation persistence revisited

Modern literature on inflation persistence follows two major paths. The first one deals with macroeconomic approaches as an alternative to the new Keynesian model described in Calvo (1983), whose dynamics does not include the inflation persistence observed in the real world. Cogley and Sbordone (2008) use a dynamic stochastic general equilibrium (DSGE) model and advocate that there is room for persistence in the original purely forward-looking model provided that the inflationary trend is slightly changed. In general, the proposed change is concerned with microeconomic details, as in Fuhrer and Moore (1995), who argue for changes in the design of price and wage contracts. Erceg and Levin (2003) deal with the problem of agents' information on inflation by using a signal extraction approach. Mankiw and Reis (2002) introduce the notion that, aside from prices, information is also sticky. This change in the price-setting behaviour of firms makes persistence more apparent in a Phillips curve model and theoretically explains the commonly observed lagged response of inflation to monetary policy shocks. If inflation persistence is in fact empirically important, it must be taken into account in macroeconomic models and the contributions of this strand of literature should be more important for policy. On the other hand, models that generate excessively high inflation persistence relying purely on “intrinsic” frictions may lead to misleading implications.

The second path, which is also the focus of the present study, seeks to empirically measure inflation persistence. A common practice is to adopt univariate time series approaches, in which persistence is represented by the sum of autoregressive coefficients of an AR model of inflation. Examples include Pivetta and Reis (2007) and Cecchetti and Debelle (2006). Pincheira (2009) estimates inflation persistence for Chile, and concludes that it has decreased in the past few years. This is, however, a simpler form of analysing the variable, which does not contemplate the full dynamics of inflation, since it only captures the intrinsic persistence derived from price and wage inflation.

In Brazil, some recent papers, such as Petrassi and Oliveira (2010) and Rebelo, Silva and Lopes (2009), provide estimates of inflation persistence and compare them to those obtained for other countries. The former paper computes the autoregressive component of inflation in different univariate equations, from a simple AR specification to more elaborate reduced-form Phillips curves and confirms the idea that industrial economies tend to exhibit lower levels of inflation persistence than do developing countries. On the other hand, Rebelo, Silva and Lopes (2009) use an ARFIMA approach to conclude that inflation persistence in Brazil is not so high compared with that of other developing countries, advocating a policy of lower interest rates. However, in these models, they neither account for expectations nor for output deviations. Therefore, despite different conclusions, these models elaborate on unconditional estimates of inflation persistence.

Controversy still exists over whether persistence has decreased or not in the past few years, as in these approaches, this strongly depends on how inflationary trend is modeled, as pointed out by Marques (2004). When structural breaks or Markov shifts are allowed, autoregressive coefficients naturally tend to decrease, indicating that inflation persistence becomes more constant, which does not necessarily apply to the type of modeling we propose here.

Some works investigate persistence using an impulse-response approach. In this case, the focus lies on the half life of an inflation shock. Examples include Dixon and Kara (2010) and Dossche and Everaert (2005). Cogley and Sargent (2005) estimate persistence based on the normalized spectrum of inflation in a Bayesian setting, and conclude that it has decreased in recent decades in the USA.

The paper by Dossche and Everaert (2005) is the main reference used in the present study. By decomposing the inflation generating process into unobserved variables, using the Kalman filter, the authors measure inflation persistence in the Euro area and in the USA and identify types of persistence at different levels. According to them, measures of persistence have often been overvalued by emphasising on intrinsic persistence.

Laubach and Williams (2003) jointly estimate the natural rate of interest and the potential output trend growth rate as unobserved components. Despite the fact that the analysis conducted by Laubach and Williams (2003) focuses on the dynamics of the natural rate of interest, we understand – as Dossche and Everaert (2005) – that the introduction of similar relationships allows for improved measurement of actual inflation persistence, controlling for natural variations in the interest rate.

3.2.1 The Brazilian Context

In the 1980s and in the early 1990s, Brazil underwent a period of high inflation rates. In some episodes, inflation persistence was a key component that further fueled inflation rates, as demonstrated by constant and deliberate price indexation and packages of sweeping economic reforms. With the implementation of the Real Plan, the inflationary spiral was broken. Nevertheless, in recent periods, as occurred at the beginning of 2003, there were high levels of persistence, as described in the Open Letter of the Brazilian Central Bank (BCB, 2003) addressed to the Brazilian finance minister. In the presence of an inflation rate that was well above the target for 2002, being aware that inflationary inertia would increase in subsequent months, and that a tighter monetary policy would be too costly, the Central Bank decided to adopt a so called “adjusted target” as benchmark. Such target, laid out in BCB (2003), established that the BCB should control some of the generated inflationary inertia, in order for the convergence of actual inflation to its target to take place only in 2004, thus averting a tighter monetary policy in 2003. What happened back then is a clear example of the effect of a higher inflation rate in inflationary inertia, which means soaring inflation in the future, leading to a vicious cycle. This cycle was later interrupted, when inflation levels met the official targets that had been set. In recent years, inflation persistence in Brazil has apparently decreased, but it is still a cause for concern. BCB (2008), for example, argued that "[...] it is not clear whether the decrease in inflation rates in these countries was followed by a reduction in the persistence of inflation […]", when referring to emerging countries, such as Brazil, Argentina and Turkey. Specifically in regard to Brazil, it reads: "[...] given the extremely high rates of inflation the country had experienced for decades, inflationary memory *might still be of relevance*."

Consequently, attempts to accurately measure inflation persistence seem to be at least as important in Brazil as they were in the Euro area by the time the Inflation Persistence Network was established.

3.3 Methodology

3.3.1 Macroeconomic model

As previously argued, a considerable part of the literature measures inflation persistence by focusing on one of the following equivalent values: Either the coefficient for the sum of lagged terms φ_i in a simple univariate decomposition of inflation such as:

$$\pi_t = \mu + \sum_{i=1}^{k} \varphi_i \pi_{t-i} + \nu_t \tag{3.1}$$

$$\nu_t \sim N(0, {\sigma_\nu}^2)$$

or the largest autoregressive root (LAR), on the other hand, which is computed, for example, by Pivetta and Reis (2007) and Petrassi and Oliveira (2010), and represented by ρ in an equation such as:

$$\pi_t = \mu + \rho\pi_{t-1} + \sum_{i=1}^{k} \phi_i \Delta\pi_{t-i} + \nu_t \tag{3.2}$$

In both cases, the dynamics of inflation relies on a measure of its average, μ, and on its autoregressive components. Therefore, any estimate of persistence based on these equations is an unconditional measure.

Our model begins with a slight modification of this univariate setting. According to Dossche and Everaert (2005) and Kozicki and Tinsley (2005), inflation is allowed to follow a stationary AR process around the perceived inflation target π_t^P:

$$\pi_t = (1 - \sum_{i=1}^{k} \varphi_i)\pi_t^P + \sum_{i=1}^{k} \varphi_i \pi_{t-i} + \nu_{1t} \tag{3.3}$$

$$\nu_{1t} \sim N(0, {\sigma_{\nu_1}}^2)$$

where π_t^P is treated as an unobserved component and represents what agents expect inflation target to be. We assume π_t^P is related to the actual inflation target π_t^T, which is also an unobserved component here[4], in the following way:

$$\pi_{t+1}^P = (1 - \delta)\pi_t^P + \delta\pi_{t+1}^T + \eta_{1t} \tag{3.4}$$

$$\eta_{1t} \sim N(0, {\sigma_{\eta_1}}^2)$$

The so-called intrinsic persistence is represented in (3.3) by $\sum_{i=1}^{k} \varphi_i$, since it indirectly measures the speed at which shifts on π_t^P have an impact on observed inflation. On the other hand, $(1 - \delta)$ is a good approximation of the expectations-based source of persistence. Clearly, if δ is close to 1, private agents perfectly predict the actual inflation target, so there is no effect on the persistence of inflation derived from erroneous expectations. The error term η_{1t} represents shocks to the perceived inflation target, and only have a short-run impact on π_t^P. The actual inflation rate target is modelled as a random walk, that is,

$$\pi_{t+1}^T = \pi_t^T + \eta_{2t} \tag{3.5}$$

$$\eta_{2t} \sim N(0, {\sigma_{\eta_2}}^2)$$

Shifts in the inflation target are meant to represent changes in central bank preferences, for instance, due to changes in the composition of the Monetary Policy Committee or as a consequence of modifications in the economic outlook. Therefore, η_{2t} represents shocks to the Central Bank's inflation target that have a long-run impact on π_t^P. Using the simplification that $\eta_{1t} = 0$, equation (3.4) thus becomes:

$$\pi_{t+1}^P = (2 - \delta)\pi_t^P + (\delta - 1)\pi_{t-1}^P + \delta\eta_{2t} \tag{3.6}$$

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[4] While in an inflation targeting system, the target is itself an essential component, usually made available publicly, the variable π_t^T represents here what one could call a target effectively pursued by the Central Bank, which is obviously not observable in the economy.

Consequently, adding to the direct impact of the lagged coefficients on inflation, there is an indirect effect derived from imperfect information about the actual target, which also translates into inflation persistence. This differentiates our model from the usual univariate approaches. However, in our univariate setting, it is still not possible to disentangle these types of persistence from extrinsic persistence, since the level of output and interest rates do not play any role.

In order to do so, we further consider a structural macroeconomic model based on Rudebusch and Svensson (1999) and in line with an inflation targeting regime[5]. Differently from Dossche and Everaert (2005), we further consider alternative observation periods, in order to capture shifts in inflation persistence. The basic model consists of three observation equations, using the terminology of state-space models. The first one is a conversion of equation (3.3) into a new Keynesian Phillips curve, by adding a lagged output gap term h_{t-1}, that is:

$$\pi_t = (1 - \textstyle\sum_{i=1}^{k} \varphi_i)\pi_t^P + \sum_{i=1}^{k} \varphi_i \pi_{t-i} + \phi_1 h_{t-1} + \nu_{1t} \qquad (3.7)$$

$$\nu_{1t} \sim N(0, {\sigma_{\nu_1}}^2)$$

The second observation equation is a Central Bank's reaction rule, under which interest rates respond to an inertial component i_{t-1}, to a neutral position of the interest rate $(\pi_t^P + r_t^*)$ and to deviations of inflation from its target $(\pi_{t-1} - \pi_t^T)$:

$$i_t = \rho_2 i_{t-1} + (1 - \rho_2)(\pi_t^P + r_t^*) + \rho_1(\pi_{t-1} - \pi_t^T) + \nu_{2t} \qquad (3.8)$$

$$\nu_{2t} \sim N(0, {\sigma_{\nu_2}}^2)$$

where $(\pi_t^P + r_t^*)$ can be understood as the nominal natural rate of interest. This rule meets general theoretical principles for economies that adopt inflation targets and allows extracting information about shifts in inflation targets.

Finally, the following equation contemplates the aggregate demand side. As usual, we assume real output is decomposed into potential output and output gap, $y_t^r = y_t^P + h_t$, while the latter is explained by lagged terms and by a monetary policy transmission mechanism $(i_t - \pi_{t-1}^P - r_{t-1}^*)$, corresponding to an IS relationship:

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[5] Bogdanski *et al.* (2000) describe the main features of the model used for the Brazilian economy, which are similar to Rudebusch and Svensson (1999).

$$h_t = \phi_2 h_{t-1} + \phi_3 h_{t-2} + \phi_4 (i_t - \pi_{t-1}^P - r_{t-1}^*) + \nu_{3t} \tag{3.9}$$

$$\nu_{3t} \sim N(0, {\sigma_{\nu_3}}^2)$$

Following Dossche and Everaert (2005), extrinsic persistence can be represented by the sum of terms ϕ_2 and ϕ_3, which clearly include the persistence of output deviations from their potential level. The unobserved variables in equations (3.7) through (3.9) have the following behaviour over time:

$$y_{t+1}^P = y_t^P + \lambda_{t+1} + \eta_{3t} \tag{3.10}$$

$$\lambda_{t+1} = \lambda_t + \eta_{4t} \tag{3.11}$$

$$r_{t+1}^* = \gamma \lambda_{t+1} + \tau_{t+1} \tag{3.12}$$

$$\tau_{t+1} = \theta \tau_t + \eta_{5t} \tag{3.13}$$

in addition to equation (3.6) for the perceived inflation target. As usual, η_{3t}, η_{4t} and η_{5t} are normally distributed with zero mean and corresponding variances equal to ${\sigma_{\eta_3}}^2$, ${\sigma_{\eta_4}}^2$ and ${\sigma_{\eta_5}}^2$.

Note that potential output follows a random walk with drift. The drift term λ_t represents the trend growth of potential output, which changes according to technological improvements.

Equations (3.11) through (3.13) are based on Laubach and Williams (2003), who focus on measuring the natural rate of interest. Such rate (r_t^*) depends on the trend growth of potential output and other determinants, such as households' rate of time preference (τ_t). The central argument of Laubach and Williams (2003) is that the natural rate of interest in the USA has varied significantly and thus should be taken into consideration in monetary policy decisions. Since these variables are jointly estimated in the present model, and the natural rate of interest is of paramount importance to the conduct of monetary policy, we provide estimates for the Brazilian economy in the analysed period.

3.3.2 Estimation strategy

The Kalman filter was used due to its superiority in estimating models with time-varying parameters. Instead of taking for granted that economic agents instantly recognize the true model, it is assumed that they learn about it (and, especially, about the changes), using new information efficiently. The Kalman filter algorithm is implemented with the appropriate state-space representation of the time series model.

First, a measurement (or observation) equation relates a vector y_t of N known variables with a state vector α_t, of dimension $M \times 1$, as follows:

$$y_t = Z\alpha_t + Ad_t + \varepsilon_t \tag{3.14}$$

where Z and A are coefficient matrices, d_t is a vector of exogenous variables $K \times 1$, and ε_t is an error vector, such that $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = H$.

The vector of unobserved components, α_t, behaves dynamically following the state equation[6]

$$\alpha_t = T\alpha_{t-1} + R\eta_t \tag{3.15}$$

where T and R are coefficient matrices, and η_t is an error vector with zero mean and $Var(\eta_t)= Q$. Here, an analogous state equation is used, with a slight change in the timing of the state variable, which does not alter the Kalman filter result, provided that ε_t and η_t are uncorrelated:

$$\alpha_{t+1} = T\alpha_t + R\eta_t. \tag{3.16}$$

The algorithm also requires an initial state vector α_0, for which $E(\alpha_0) = a_0$, such that $E(\varepsilon_t\alpha_0') = 0$ e $E(\eta_t\alpha_0') = 0$ and $Var(\alpha_0) = P_0$, where P_0 is positive semidefinite.

In practice, as observations of y_t become available, the algorithm updates the mean and variance of the state vector up to period t. It is as

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[6] Note that, in their general forms, state equations should neither contain lagged variables in more than one period nor other state variables. However, these restrictions can be overcome in the state-space specification, as shown further ahead.

sumed that the information about $t-1$ is known, that is, α_{t-1} is normally distributed with known mean a_{t-1} and variance P_{t-1}. According to this procedure, the mean of the state vector at t is:

$$a_{t|t-1} = T\alpha_{t-1} \tag{3.17}$$

which is commonly referred to as a forecasting equation. The distribution predicted by the algorithm for the next observation, y_t, is normally distributed with mean

$$\hat{y}_{t|t-1} = Z\alpha_{t|t-1} + Ad_t \tag{3.18}$$

and covariance matrix

$$F_t = ZP_{t|t-1}Z' + H_t. \tag{3.19}$$

When the observation of y_t is known, the mean and variance of the updated conditional distribution of the state variable are obtained, which correspond, respectively, to:

$$a_t = a_{t|t-1} + P_{t|t-1}Z'F_t^{-1}(y_t - Z\alpha_{t|t-1}Ad_t) \tag{3.20}$$

and

$$P_t = P_{t|t-1} - P_{t|t-1}Z'F_t^{-1}ZP_{t|t-1} \tag{3.21}$$

This process is repeated up to the last period $t = T$ for which information on y_T is available. These general specifications allow us to express the univariate model (equations 3.3 through 3.6) in state space:

$$y_t = [\pi_t]$$

$$Z = \left[\left(1 - \sum_{i=1}^{4} \varphi_i\right) \quad 0\right]$$

$$\alpha_t = [\pi_t^P \quad \pi_{t-1}^P]'$$

$$A = [\varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \varphi_4]$$

$$d_t = [\pi_{t-1} \quad \pi_{t-2} \quad \pi_{t-3} \quad \pi_{t-4}]'$$

$$\varepsilon_t = [\nu_{1t}]$$

$$H = [\sigma_{\nu_1}^2]$$

$$T = \begin{bmatrix} 2-\delta & \delta-1 \\ 1 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} \delta \\ 0 \end{bmatrix}$$

$$\eta_t = [\eta_{2t}]$$

$$Q = [\sigma_{\eta_2}^2]$$

Analogously, the equations of the multivariate model can be represented in state space form, as follows:

$$y_t = [\pi_t \;\; i_t \;\;\; y_t^r \;]'$$

$$Z = \begin{bmatrix} 0 & (1-\sum \varphi_i) & 0 & 0 & -\phi_1 & 0 & 0 & 0 & 0 & 0 \\ \rho_1 & (1-\rho_2) & 0 & 0 & 0 & 0 & (1-\rho_2)\gamma & 0 & (1-\rho_2) & 0 \\ 0 & 0 & \phi_4 & 1 & -\phi_2 & -\phi_3 & 0 & \phi_4\gamma & 0 & \phi_4 \end{bmatrix}$$

$$\alpha_t = [\pi_t^T \;\; \pi_t^P \;\; \pi_{t-1}^P \;\; y_t^P \;\; y_{t-1}^P \;\; y_{t-2}^P \;\; \lambda_t \;\; \lambda_{t-1} \;\; \tau_t \;\; \tau_{t-1}]'$$

$$A = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 & \phi_1 & 0 & 0 \\ \rho_1 & 0 & 0 & 0 & 0 & 0 & \rho_2 \\ 0 & 0 & 0 & 0 & \phi_2 & \phi_3 & -\phi_4 \end{bmatrix}$$

$$d_t = [\pi_{t-1} \;\; \pi_{t-2} \;\; \pi_{t-3} \;\; \pi_{t-4} \;\; y_{t-1} \;\; y_{t-2} \;\; i_{t-1}]'$$

$$\varepsilon_t = [v_{1t} \;\; v_{2t} \;\; v_{3t}]'$$

$$H = \begin{bmatrix} \sigma_{v_1}^2 & 0 & 0 \\ 0 & \sigma_{v_2}^2 & 0 \\ 0 & 0 & \sigma_{v_3}^2 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & (1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\eta_t = [\eta_{1t} \;\; \eta_{2t} \;\; \eta_{3t} \;\; \eta_{4t} \;\; \eta_{5t}]'$$

$$Q = \begin{bmatrix} \eta_{1t} & 0 & 0 & 0 & 0 \\ 0 & \eta_{2t} & 0 & 0 & 0 \\ 0 & 0 & \eta_{3t} & 0 & 0 \\ 0 & 0 & 0 & \eta_{4t} & 0 \\ 0 & 0 & 0 & 0 & \eta_{5t} \end{bmatrix}$$

Operationalization of the usual algorithm of the Kalman filter requires that matrices Z, A, H, T, R and Q be known. As they depend on a vector of unknown parameters ψ, it is necessary to additionally make an inference about this vector. Including data obtained from other studies, it is possible to treat ψ as a vector of random parameters with prior density $p(\psi)$ and to estimate the posterior density $p(\psi|y)$ using Bayesian inference[7]. Alternatively, the goal is to find the posterior mean $\bar{g}$ given by:

$$\bar{g} = E[g(\psi|y)] = \int g(\psi)p(\psi|y)d\psi. \tag{3.22}$$

Following Dossche and Everaert (2005) and using Bayes' theorem, this turns into:

$$\bar{g} = \frac{\int g(\psi)z^g(\psi,y)g(\psi|y)d\psi}{\int z^g(\psi,y)g(\psi|y)d\psi} \tag{3.23}$$

where

$$z^g(\psi,y) = \frac{p(\psi)p(y|\psi)}{g(\psi|y)}. \tag{3.24}$$

Departing from a sample of n independent choices of ψ , called $\psi^{(i)}$, we obtain a $\bar{g}_n$ estimator of $\bar{g}$:

$$\bar{g}_n = \frac{\sum_{i=1}^{n} g(\psi^{(i)})z^g(\psi^{(i)},y)}{\sum_{i=1}^{n} z^g(\psi^{(i)},y)}. \tag{3.25}$$

In practice, we introduce estimators of the posterior mean of $\bar{\psi} = E(\psi|y)$. This is possible by admitting $g\left(\psi^{(i)}\right) = \psi^{(i)}$ and $\tilde{\psi} = \bar{g}_n$ in the previous equation, where $\tilde{\psi}$ is the estimator of $\bar{\psi}$. The 5th and 95th percentiles are also computed. Briefly, an estimate of the 5th percentile, $\tilde{\psi}^{5\%}$, derives from $F(\tilde{\psi}^{5\%}|y) = 0.05$, where $F(\tilde{\psi}|y) = Prob(\psi_j^{(i)} \leq \psi_j)$ denotes the jth element in ψ. The same applies to the 95th percentile.

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[7] An alternative approach is the optimization of the diffuse likelihood function from the exact Kalman filter. However, in a framework containing so many unobserved components, this turns out to be unfeasible, as Dossche and Everaert (2005) pointed out.

3.3.3 Data and prior information

In this study, we used observed Brazilian data on inflation, output and interest rate. For inflation, a quarterly series of seasonally adjusted consumer price index (IPCA) was utilized as reference for the targeting regime. For the output, we employed seasonallly adjusted quarterly GDP, in constant prices, obtained from the Brazilian Institute of Geography and Statistics (IBGE). Finally, for the interest rate, we used for each quarter the average of daily frequencies of the SELIC rate in the corresponding quarter. The sampling period for the univariate and multivariate cases stretches from the first quarter of 1995 to the first quarter of 2011, totaling 65 observations. Additionally, we estimate two models that are identical with the multivariate one with respect to the equations, but include 49 observations, since they consider the alternative periods 1995-2007 and 1999-2011. In such way, we are able to assess the dynamic behaviour of persistence measures. On the final graphs, all values are annualized for the sake of illustration.

In order to simplify the Bayesian procedure in the presence of several unobserved series, we decided to use the values of some studies carried out for the Brazilian economy and those of some international studies, as prior information. Tables 1 and 2 summarize the values of the distributions used respectively for the univariate and multivariate cases, along with the estimated posteriors.

In the case of the univariate estimation, more specifically with regard to intrinsic persistence, the major reference was Petrassi and Oliveira (2010), as previously commented. In their study, the mean of ρ values for the different models was $\rho = 0.46$, which, in our case,[8] corresponds to $\sum_{i=1}^{4} \varphi_i$, as in equation 3.3. This value was distributed among φ_i, so as to provide more recent lags with a heavier weight. As prior information for the parameter that measures expectations-based persistence, δ, we used the figures proposed by Guillen (2008) and Caetano and Moura (2009), which quantify the presence of sticky information in Brazil, following Mankiw and Reis (2002). The value of $\delta = 0.16$ means that, on average, 84% of the inflation target perceived by agents results from the target perceived in the previous quarter and only 16% originates from the actual target used by the Central Bank. This is easily

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[8] As shown by Levin and Piger (2004), one can compute intrinsic persistence either in terms of the sum of the AR coefficients or by the largest AR root, since both measures are equivalent. Therefore, our prior mean for $\sum_{i=1}^{k} \varphi_i$ is considered to equal an average ρ from Petrassi and Oliveira (2010).

seen from equation (3.4). The remaining coefficients required for the univariate estimation were obtained from foreign studies. The variances of errors ν_1 and η_2 derive respectively from Dossche and Everaert (2005) and from a combination of Kozicki and Tinsley (2005) and Smets and Wouters (2005). The variances for the construction of prior and posterior distributions of the parameters followed Dossche and Everaert (2005).

To initialize the multivariate estimation, we first used some posterior means obtained for the variables estimated in the univariate model $(\varphi_i, \delta, \sigma_{\nu_1}^2, \sigma_{\eta_2}^2)$. Thus, the univariate estimation was also useful for adjusting and refining the values to be used in the main model. For the coefficient of the output gap in the Phillips curve (equation 3.7), we used the value of the linear curve introduced by Muinhos (2004), $\phi_1 = 0.28$. With respect to the IS curve parameters, the results obtained by Aragón and Portugal (2009), which take into account changes in Central Bank's policy preferences, were used as reference. The relevant values for equation (3.9) are: $\phi_2 = 0.5$, $\phi_3 = -0.18$ and $\phi_4 = 0.008$. For the interest rule parameters (equation 3.8), our reference was Sin and Gaglianone (2006), who estimated a DSGE model for Brazil, based on Smets and Wouters (2005). The prior means $\rho_1 = 0.3$ and $\rho_2 = 0.7$ originate from their work. From Dossche and Everaert (2005) we obtain $\sigma_{\nu_2}^2 = 0.3$. Finally, Laubach and Williams (2003) was our source of information for the coefficients related to equations (3.10) through (3.13). Their figures for parameters γ, θ, $\sigma_{\nu_3}^2$, $\sigma_{\eta_3}^2$, $\sigma_{\eta_4}^2$ and $\sigma_{\eta_5}^2$, as well as the other parameters of the multivariate estimation are shown in Table 2.

For the estimation of alternative periods (1995/1-2007/1 and 1999/1-2011/1), the prior distributions were exactly the same as those of the multivariate estimations, in order not to interfere with the coefficient results.

3.4 Results

For both the univariate and multivariate estimations, we show two sets of results. First, in Tables 1 and 2, the prior means and distributions of the unobserved coefficients and then their respective posteriors are shown. Second, in Figures 1 and 2 we illustrate key state variables relating to the estimated and perceived targets in comparison to the Brazilian inflation rate.

As previously highlighted, the univariate case was first estimated, with the decomposition of inflation persistence into intrinsic persistence, derived from the sum of AR coefficients, and into expectations-based persistence, represented by $1 - \delta$.

The posteriors obtained for the unobserved components were relatively similar to the values from Brazilian studies used as priors. However, the variances $\sigma^2_{v_1}, \sigma^2_{\eta_2}$ proved to be quite different from Dossche and Everaert (2005) figures. This is probably due to the different nature of shocks hitting inflation in Brazil and in the euro area.

Table 1: Prior and posterior distributions - Univariate model

Variable	Prior Mean	5%-95%	Posterior Mean	5%-95%
φ_1	0.23	[0.15, 0.31]	0.262	[0.18, 0.34]
φ_2	0.10	[0.02, 0.18]	0.079	[0.00, 0.16]
φ_3	0.08	[0.00, 0.16]	0.082	[0.01, 0.16]
φ_4	0.05	[-0.03, 0.13]	0.051	[-0.02, 0.13]
δ	0.16	[0.08, 0.24]	0.169	[0.09, 0.25]
$\sigma^2_{v_1}$	1.3	[0.36, 2.73]	0.859	[0.64, 1.17]
$\sigma^2_{\eta_2}$	0.12	[0.03, 0.25]	0.053	[0.01, 0.17]

Figure 1 shows the behaviour of state variables in the univariate case compared to IPCA observed inflation. In the first observations, the values are erratic, as a consequence of the filter algorithm. After that, the estimated inflation target had its highest rates around 2001, reaching a peak of yearly 8.25% in the third quarter of 2001, and then dropped vigorously until the 1st quarter of 2006 to 5.15%. After that it has rather been slowly increasing again. As for the target perceived by agents, there was a mean-reverting behaviour and, in more recent years, a greater stability and adherence to the target we estimated to be the actual target followed by the Central Bank.

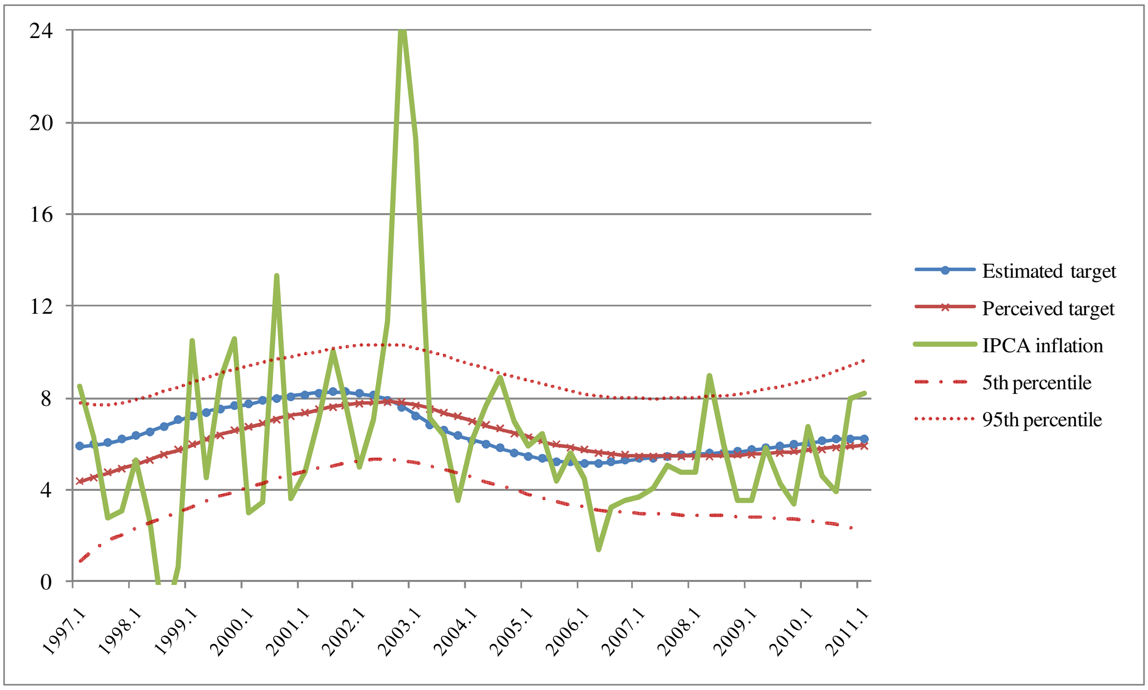

Figure 1: Unobserved targets and inflation (yearly, %) - Univariate model

In regard to the results of the multivariate estimation (see Table 2), we have a broader scenario that now includes extrinsic persistence, as mentioned before. The priors generally proved to result in good approximations for most coefficients. The initial profile of the autoregressive coefficients slightly changed, since the 2nd inflation lag (φ_2) is oddly lower than the 3rd and 4th lags (φ_3, φ_4). Dossche and Everaert (2005) found a lower 3rd lag among the results of the euro area. Both their result and ours reflect the importance of the choice of several lags as a measure of intrinsic persistence, as opposed to simply choosing the first lag. More importantly, the sum of autoregressive components resulted in greater intrinsic persistence (0.62 instead of 0.47). On the other hand, lower expectations-based persistence was found (0.77 instead of 0.83). Extrinsic persistence, which is specific to the multivariate setting, summed 0.44.

Together these figures suggest that, on the one hand, extrinsic persistence matters, since its introduction impacts the results of other sources of persistence, but on the other hand, it has a lower share on aggregate persistence than the other two drivers.

Table 2: Prior and posterior distributions - Multivariate model

Variable	Prior Mean	5%-95%	Posterior Mean	5%-95%
$*\varphi_1$	0.26	[0.10, 0.43]	0.369	[0.30, 0.46]
$*\varphi_2$	0.08	[-0.09, 0.24]	0.030	[-0.01, 0.07]
$*\varphi_3$	0.08	[-0.08, 0.25]	0.108	[0.06, 0.16]
$*\varphi_4$	0.05	[-0.11, 0.22]	0.110	[0.10, 0.12]
$**\phi_1$	0.28	[-0.12, 0.44]	0.264	[0.20, 0.33]
$**\phi_2$	0.5	[0.34, 0.66]	0.461	[0.43, 0.50]
$**\phi_3$	-0.18	[-0.34, -0.02]	-0.018	[-0.03, -0.01]
$**\phi_4$	0.008	[-0.40, 0.42]	0.0004	[0, 0.001]
$***\gamma$	1	[0.92, 1.08]	0.994	[0.93, 1.06]
$**\rho_1$	0.3	[0.26, 0.34]	0.26	[0.26, 0.26]
$**\rho_2$	0.7	[0.66, 0.74]	0.685	[0.68, 0.69]
$*\delta$	0.17	[0.09, 0.25]	0.227	[0.21, 0.24]
$***\theta$	0.97	[0.95, 0.99]	0.970	[0.97, 0.97]
$*\sigma^2_{v_1}$	0.86	[0.23, 1.78]	0.827	[0.78, 0.86]
$***\sigma^2_{v_2}$	0.3	[0.21, 0.41]	0.377	[0.34, 0.41]
$***\sigma^2_{v_3}$	0.16	[0.11, 0.22]	0.168	[0.16, 0.18]
$***\sigma^2_{\eta_1}$	0.0001	[0.00, 0.0001]	0.0001	[0.00, 0.0001]
$*\sigma^2_{\eta_2}$	0.05	[0.04, 0.07]	0.048	[0.04, 0.05]
$***\sigma^2_{\eta_3}$	5.86	[4.10, 7.91]	7.058	[5.81, 8.39]
$***\sigma^2_{\eta_5}$	0.10	[0.07, 0.14]	0.111	[0.10, 0.12]
$***\sigma^2_{\eta_4}$	0.01	[0.00, 0.01]	0.006	[0.00, 0.01]

*Denotes variables, for which univariate posterior means were used as prior means.

**Variables, for which figures from other Brazilian studies were used.

***Variables, for which figures from foreign studies were used.

Likewise, the behaviour of the main state variables (Figure 2) confirms that inflation persistence is partially explained by the difference between the target perceived by agents and the actual inflation target. Despite the similar mean-reverting tendency of the target perceived by agents, expectations distortions decreased in the multivariate case, i.e., the expected inflation target followed the estimated target more closely. This suggests that expectations-based persistence is less present than in the univariate case, but this is probably because in the multivariate model we can disentangle extrinsic persistence from the other two sources.

Therefore, this result actually means that persistence of expectations plays a major role on aggregate persistence.

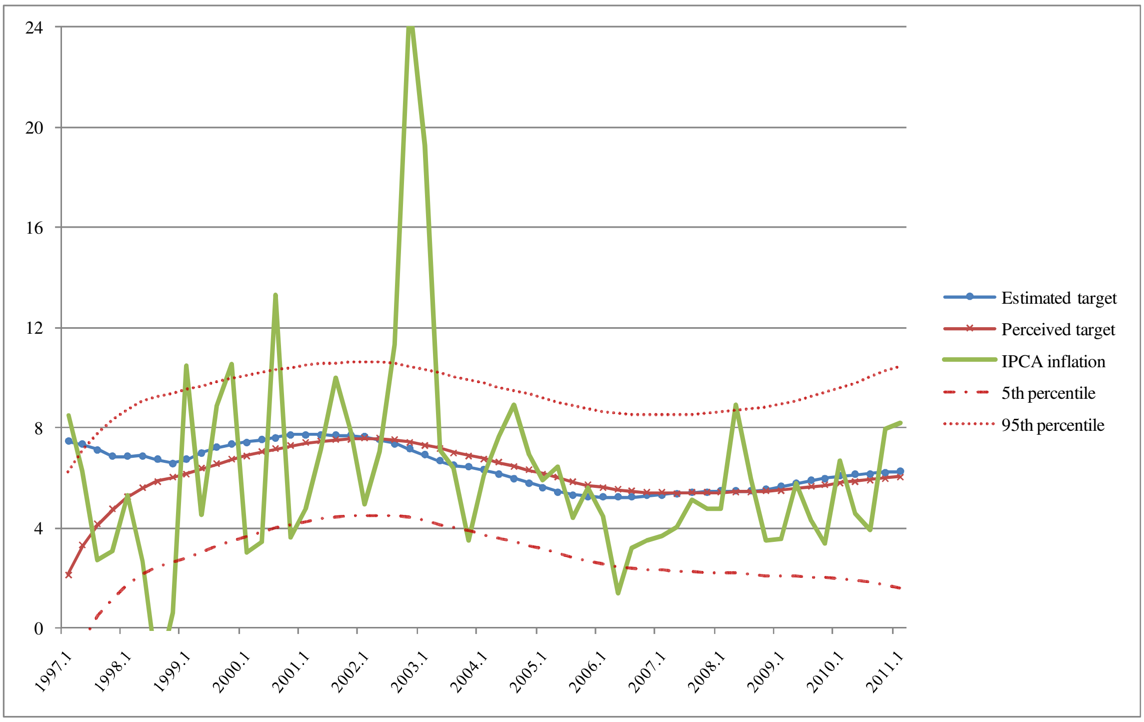

Figure 2: Unobserved targets and inflation (yearly %) - Multivariate model

It is interesting to compare the estimated central bank targets with actual policy followed by the Central Bank of Brazil. Figure 3 shows the estimated target π_t^T and the lower and upper tolerance bounds defined by the Brazilian National Monetary Council (CMN) during the inflation targeting regime. It should be underscored that the years in which estimated targets were above the tolerance region were also those years in which headline inflation did not manage the predicted target. From the second half of 2000 until the beginning of 2002, for example, although the preset tolerance bound showed a falling trend, our estimated targets suggest lenient policy towards price stabilization has been carried out. After that, following the election of President Lula da Silva, the Central Bank needed to tighten the monetary policy to curb inflation and its inertia or persistence as well.[9] Not until 2004 could the estimated targets return to the tolerance region, where it remained comfortably

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[9] In BCB (2003) the Central Bank of Brazil announced adjusted targets for 2003 and 2004 headline inflation. The respective revised target limits can be seen in Figure 3. Moreover, in BCB (2004), persistence concerns are evident, being based on the finding that the inertial component had contributed 63.7% to inflation in 2003.

until the beginning of 2009. Clearly, after the improvement of the inflation targeting regime, with credibility gains and anchoring of expectations, the Central Bank could put into practice an inflation target that is more centred inside the tolerance region. However, the recent financial crisis pushed up estimated targets, and our result for the beginning of 2011 (6.24%) is close to the upper announced bound (6.5%). Such recent evolution has prompted the Central Bank to repeatedly raise interest rates during the first half of 2011.

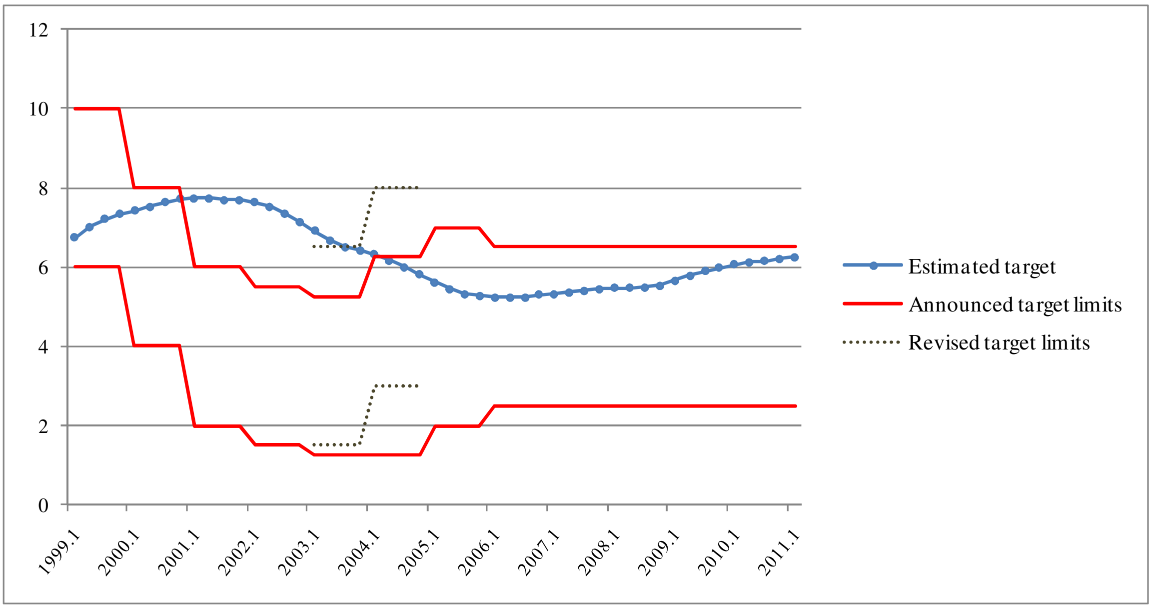

Figure 3: Estimated Central Bank targets, announced tolerance limits (yearly, %)

Consolidated estimates of the relevant parameters for the sources of persistence are shown in the first columns of Table 3. Departing from the univariate to the multivariate approach, there is a decrease in expectations-based persistence and an increase in intrinsic persistence. The reduction in expectations-based persistence is probably an effect of the introduction of extrinsic persistence, as already mentioned. The result $\delta = 0.23$ demonstrates that the speed of expectations updating is higher than what would be expected from "sticky information" estimates produced by Guillén (2008) for Brazil.

We also estimate the same model, considering two alternative periods. The periods were formed removing 16 quarters from each extremity. Thus, the first period starts in 1995 and ends in 2007, while the second term consists of 1999-2011. In the choice of the number of observations we had in mind the accuracy of estimations; obviously the division into two separate periods would generate inaccurate results, due to the number of unobserved variables. Moreover, the second period coincides with the inflation-targeting era in Brazil. It should be noted that both intrinsic and expectations-based persistence decreased over time, as expected. With the adoption of the inflation targeting regime in Brazil in the second period, the Central Bank could anchor inflation expectations more strongly to its actual targets, as a result of credibility improvements. Nevertheless, as already pointed out, expectation distortions have clearly been a key source of persistence over all periods, as opposed to the traditional view of intrinsic persistence. This finding stays in line with the Euro area and US results from Dossche and Everaert (2005).

Table 3: Summarized measures of persistence

Variable	Univariate (1995-2011)	Multivariate (1995-2011)	Multivariate 1st period (1995-2007)	Multivariate 2nd period (1999-2011)
$\sum \varphi_i$	0.47	0.62	0.60	0.38
$(1-\delta)$	0.83	0.77	0.81	0.78
$\phi_2 + \phi_3$	-	0.44	0.33	0.35

Extrinsic persistence, on the other hand, has not quite changed over the periods considered and was considerably lower than the one found in the Euro area and in the US. Therefore, persistent deviations of output from its natural level may be more present in these developed economies than in Brazil.

Most of the Brazilian literature on persistence measures (for example, Campelo and Cribari-Neto (2003) and Durevall (1999)) focuses on periods of higher inflation levels, that is, before 1994, so that a direct comparison to our results is unfruitful. Our univariate intrinsic persistence measures match closely Petrassi and Oliveira (2010) in a similar period.[10] Dossche and Everaert (2005) also reported similar findings when comparing their intrinsic persistence

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[10] Their intrinsic persistence estimates for the 1995 to 2009 period range from 0.416 to 0.509, whereas our main result for the univariate model was 0.47.

results with the ones from univariate frameworks in the USA and in the euro zone. However they do not consider alternative periods.

Finally, the natural rate of interest is another estimation result with an important economic value. It informs the monetary authority about the natural stance of the interest rate over time. In our model, in particular, its introduction prevented shifts in the actual inflation target from being affected by changes in the benchmark that represents the natural rate of interest. Equations (3.11) through (3.13) determine the evolution of the equilibrium interest rate over time, together with the trend growth rate and the parameter that measures time preferences. The average natural rate of interest for the period between the third quarters of 1999 and 2005 was 10.5%, slightly higher than figures found by Portugal and Neto (2009) for the same period, 9.6%. The extreme values found in the estimations are 19.8% for the 4th quarter of 1997 and 5.9% for 2009/3. The declining trend is in line with recent estimations from the Central Bank of Brazil (BCB, 2010).[11] As for its determinants, shifts in the trend growth rate did not seem to significantly affect the recent behaviour of the natural rate of interest, as did shifts in time preferences. The same pattern was observed in the data obtained by Dossche and Everaert (2005) for the Euro zone and the USA.

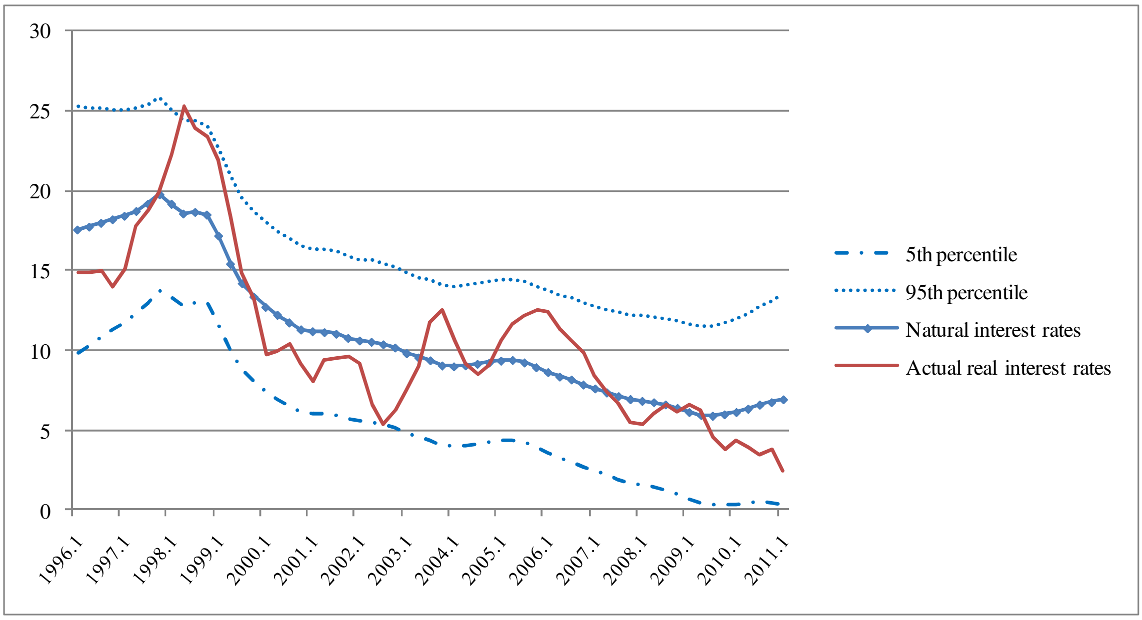

Figure 4: Natural vs. actual real interest rates (yearly, %)

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[11] The Central Bank of Brazil reports the recent reduction in sovereign debt risk, as a consequence of positive developments in national accounts, as the main driver for this decline in the natural rate of interest. For a review on alternative approaches to the measurement of natural rates of interest, see Giammarioli and Valla (2004).

We also compared the estimated natural rates of interest with actual real interest rates, computed by simply subtracting the observed IPCA inflation from nominal interest rates for each quarter, that is, $r_t = \pi_t - i_t$. If the monetary authority follows a neutral stance regarding policy rate adjustment, both rates should ideally evolve similarly. Considering the whole period, this is certainly not the case, but if we take into account only the period under the inflation targeting regime, interest rate gaps are relatively smaller considering historical interest rate figures.

3.5 Conclusions

In spite of the fundamental importance of measuring inflation persistence and including it in general equilibrium models for the formulation of monetary policy and of empirical regularities, there is no consensus agreement on how to do that. The aim of this chapter was to measure inflation persistence for a recent time period in Brazil, and also to identify the types of inflation persistence using three major sources: intrinsic, extrinsic, and expectations-based.

Aside from the usual intrinsic persistence, the impact of macroeconomic shocks, such as output deviations, on persistence, which lead to extrinsic persistence, was also determined. It was also assumed that shifts in the inflation target (which are not always known and perceived by all agents) may result in permanent changes in mean inflation. Private agents' perception of the policy target sometimes differ considerably from the actual target followed by the central bank, which also induces inflation persistence. These facts are often not considered by conventional empirical models when measuring persistence.

Following Dossche and Everaert (2005), the inflation generating process was decomposed[12] into unobserved variables using the Kalman filter. The main contributions were the following: first, this type of estimation is novel for Brazil, because the dynamics of inflation and of its determinants are jointly assessed in a model that contemplates not only the sticky price behaviour, but also the idea of sticky information. Second, by clarifying inflation persistence in Brazil and simultaneously adding more information on the natural rate of

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[12] In the present study, we only consider the IPCA (broad consumer price index). However, a possible extension could deal with administered prices separately in order to quantify persistence in these prices and its importance to monetary policy.

interest, this study provides important data that may enrich the framework of monetary policy in the country.

The following major results were obtained. First, the role of inflation expectations, of shifts in inflation targets, and of significant deviations of output and of the natural rate of interests in inflation persistence should not be neglected in any representation of inflation dynamics. Not surprisingly, the impact of accounting for such factors in measures of inflation persistence is that the usual intrinsic persistence is quite different from traditional figures from univariate approaches. These factors are especially important for Brazil, which has gone through the implementation of an inflation targeting regime and episodes of inflation during foreign crises and the domestic crisis in 2002/2003. Second, although intrinsic inflation persistence has significantly decreased in the past few years, the other two sources did not experience such a trend. Hence, one should cautiously interpret literature findings that point to an undisputable decrease in inflation persistence, especially if they are based on intrinsic measures of persistence. As we particularly showed, expectations-based persistence proved to be both high and almost unchanged over recent years. Also, according to Portugal and Neto (2009), estimates suggest that interest rates have fluctuated around the estimated natural stance, contradicting arguments that the Central Bank of Brazil has been too conservative in its interest rate policy. Of course one should be cautious since we deal with a restricted period and the natural rate of interest entails a long-run definition. Finally, this study indicates that persistence is less distressing for stable inflationary settings (such as the period between 2006-2009 in Brazil), but that in unstable ones, the cost of disinflation in terms of output loss, or the sacrifice ratio, tends to be even higher. This is due to agents' perceptions not being easily anchored, when the central bank inflation targets change more often.

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4 PHILLIPS CURVE IN BRAZIL: AN UNOBSERVED COMPONENTS APPROACH[1]

Abstract

This study estimates a reduced-form Phillips curve for recent Brazilian data, using a framework of time series with unobserved components. The decomposition into trend, seasonality and cycle offers, through the graphical output, straightforward economic interpretations. Furthermore, our framework represents an alternative to traditional GMM estimations of the new Keynesian Phillips curve (NKPC), which have seldom been empirically successful. Differently from Harvey (2011), we allow for expectations to play a key role as in the usual NKPC. Besides GDP, we also use capacity utilization rate and an unexplored time series (IBC-Br), as measures of economic activity. A multivariate unobserved components model of inflation and output is then fitted, assuming these variables follow similar cycles. Our findings support the view that Brazilian inflation targeting has been successful in reducing the variance of both the seasonality and level of the inflation rate, at least until before the subprime crisis. Furthermore, inflation in Brazil seems to have responded gradually less to measures of economic activity in recent years. This provides some evidence of a flattening of the Phillips curve in Brazil, a trend also shown by Tombini and Alves (2006), which translates into higher costs of disinflation on the one hand, but also lower inflationary pressures derived from output growth, on the other hand.

4.1 Introduction

The Phillips curve has been a recurrent subject of debate in macroeconomics, ever since the first approaches developed by A. W. Phillips (1958) and Samuelson and Solow (1960). Its implicit formulation encompasses an important trade-off between inflation rate and unemployment rate or alternatively between inflation rate and output gap. Numerous countries use this aggregate supply relation when formulating and implementing monetary policy, often jointly with an aggregate demand equation (IS) and an interest rate rule. The

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[1] A preliminary version of this chapter benefitted from comments at the XXV Jornadas Anuales de Economia, Central Bank of Uruguay, 2010.

Phillips curve is also frequently employed in inflation forecasting models, as reviewed by Stock and Watson (2008).

Over the past few years the new Keynesian Phillips curve (NKPC) has become popular in its various forms. The initial formulation, mainly attributed to Calvo (1983), consisted of a connection between inflation and real marginal cost plus an inflation expectations component. The driving force was the observed behaviour of sticky price adjustment by some firms. This sort of rigidity accounted for the designation “new Keynesian”. Over time, some changes occurred, often to make up for flaws in microeconomic (approach to price adjustments) or empirical aspects (i.e. persistence of observed inflation, not contemplated in the original equation). On the macroeconomic side, it has been argued that, under some circumstances, the coefficient of real marginal cost may even be negative, which is economically counterintuitive, see Rudd and Whelan (2007). Even the widely adopted hybrid new Keynesian Phillips curve, which includes lagged inflation, has not been successful in adequately explaining inflation dynamics. According to Rudd and Whelan (2007), this occurs whether output gap or labour income share is used as a proxy for real marginal cost. Mavroeidis (2005) assesses a few methodological problems with estimating forward-looking rational expectations models by GMM. By focusing mainly on the model proposed by Gali and Gertler (1999), the author concludes that limited information methods, such as GMM, usually render identification issues. In other words, despite some recent renewed interest in the Phillips curve, supported by the new Keynesian literature, its dynamics of the relationship between inflation and economic activity still lacks strong empirical foundations. The success of its estimations for the recent past of the Brazilian economy is equally questionable, as pointed out by Sachsida, Ribeiro and Santos (2009).

In the present study we verify the dynamics of inflation in an estimation with unobserved components for the Brazilian economy, following to some extent the approach proposed by Harvey (2011). A simple relationship is established between monthly inflation and output data, in which inflation is explained by a set of unobserved components (UC), in addition to the usual output gap and expected inflation terms. As a first step, output gap is identified by extracting the cycle from the appropriate output series, also by the UC method.[2]

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[2] Some authors (Gali and Gertler, 1999 and Schwartzman, 2006 and Sachsida, Ribeiro and Santos, 2009 in the Brazilian case) suggest that the output gap has not been a significant measure of inflationary pressures in GMM estimations. On the other hand, measures such as labour income share, or unit labor cost are also criticized for producing a countercyclical pattern in the analysis of U.S. data (Rudd and Whelan, 2007). As in the present

Differently from Harvey (2011), the inflation expectations term is introduced in the Phillips curve, bringing the model closer to a standard hybrid NKPC. This way, notions about price rigidity and inflationary inertia are taken into account, but at the same time, we depart from standard econometric approaches. The stochastic trend component, modeled as a random walk, is regarded as core inflation, successfully substituting the lagged term of the hybrid NKPC. Thereafter, a multivariate estimation is used, in which the appropriate output gap is implicitly present in the output equation, instead of being inserted exogenously, which is an advantage as it precludes the previous estimation of an additional unobserved component. Put differently, the Philips curve parameters are found without having to first estimate the output gap. The "similar cycles" approach follows naturally since the cyclical movements of the two series are assumed to arise as a result of a common business cycle.

The aim of the present model is therefore to parsimoniously reproduce the stylized facts of the relation between inflation and output gap, providing more recent estimates of this important interaction and its dynamics in Brazil. A further contribution concerns the use of output gap obtained from a trend-cycle decomposition of the Brazilian Central Bank's index of economic activity (IBC-Br), still unexplored in academic works. Notwithstanding the relatively small sample size, interesting conclusions can be drawn from this series, which leads to some reflection about the contemporaneous evolution of economic activity in Brazil.[3] By considering time variation of the output gap parameter, which is new in comparison to Harvey (2011), we also test its linearity for Brazil. Some studies on developed countries, as the one conducted by Kuttner and Robinson (2010), advocate a recent flattening of the Phillips curve in the US, in the sense that the output gap coefficient has become gradually smaller. This behaviour has important macroeconomic implications, as we discuss later. Moreover, an analysis of the forecasting power is carried out by comparing our models with a simple forecasting model in order to test the assumption that Phillips curves may provide good inflation forecasts (STOCK; WATSON, 2008).

Much of the literature that focus on estimating the new Keynesian Phillips curve considers the inflation trend to be stationary, as reviewed by Rudd and Whelan (2007) and

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study we introduce a different method for Brazilian series, we prefer to test the output gap, which is also an important policy variable for most central banks. However, capacity utilization rate and newly developed IBC-Br series are also considered.

[3] This index was adopted by the Central Bank of Brazil in 2009, in order to follow up economic activity in a more tempestive fashion, due to its low occurrence of lags and to its monthly periodicity. According to the Brazilian Central Bank's Inflation Report of March 2010, the IBC-Br has evolved in considerably close connection to the GDP series.

Nason and Smith (2008b). On the other hand, recent works have sought to model Phillips curves with a stochastic inflation trend, as done in the present study. Lee and Nelson (2007) propose a bivariate specification between inflation and unemployment, in which inflation trend varies over time. Goodfriend and King (2009) explain the stochastic behaviour of inflation trend based on assumptions about central bank policy.

Dealing more specifically with the Phillips curve with unobserved components, Vogel (2008) uses a modeling strategy that resembles the one utilized in the present study, but instead her focus of inflationary pressures is the unemployment gap. Interestingly, her work combines the idea of Gordon's (1997) "triangle" model of inflation, in which the NAIRU (Non-Accelerating Inflation Rate of Unemployment) varies over time, with the new Keynesian model that focuses on short-term inflation dynamics. Harvey (2011) proposes decomposing recent US inflation into transitory and permanent components, following the methodology of structural time series models, described with more detail in Harvey (1989). However he does not consider inflation expectations, as already argued, under the argument that identification becomes difficult.

With respect to the Brazilian literature on this issue, Sachsida, Ribeiro and Santos (2009) provide a good survey and propose a regime-switching model to account for time-variation in the Phillips curve parameters. Schwartzman (2006) estimates a Phillips curve using industrial capacity utilization data to address the fact that the output gap is not observable. Fasolo and Portugal (2004) adapt a NKPC for Brazil based on the NAIRU, giving a sharper focus on expectations formation. Some works, such as Arruda, Ferreira and Castelar (2008) and Correa and Minella (2005), used Phillips curve versions to assess their inflation forecasting power. To our knowledge, there are no Brazilian studies that investigate inflation dynamics with a primary focus on the decomposition of its factors into permanent and transitory unobserved components.

This chapter is organized into five sections. Section 4.2 deals with the econometric estimation of Phillips curves with exogenous marginal cost measures, and the underlying conceptual issues. Section 4.3 presents our multivariate estimation and its respective results. Section 4.4 describes some extensions to the basic model, and Section 4.5 concludes.

4.2 Basic model with unobserved components

The main reference for the basic model is Harvey (2011), who used a structural time series approach in which the output gap was regarded as explanatory variable in a decomposition of the US inflation rate.

The specification with unobserved components has some advantages over ARMA models. First, unlike the ARMA specification, the components provide a straightforward economic interpretation. More importantly, in ARMA specifications, the model's dynamics relies exclusively upon the dependent variable, whereas in UC models, the dynamics is constantly inferred by observations.[4]

4.2.1 Comments on Harvey (2011) and our divergences

Before moving on to the specification used, some brief comments should be made about Harvey's (2011) model and about the adaptations performed.

A basic structural[5] time series model of inflation can be easily represented by:

$$\pi_t = \mu_t + \psi_t + \gamma_t + \varepsilon_{1t} \tag{4.1}$$

$$\varepsilon_{1t} \sim N(0, {\sigma_{\varepsilon_1}}^2)$$

where the observed series π_t is decomposed into trend (μ_t), cycle (ψ_t), and seasonal (γ_t) components, and into an irregular white noise component (ε_{1t}). In addition to permanent and transitory components, it is possible to add explanatory variables, and also structural breaks, level breaks and outliers, as in a usual regression. The trend component in (4.1), augmented with cycles and seasonals, represents the underlying level of inflation.

Adding an output gap term h_t to equation (4.1), as a measure of inflationary pressure, the result is a Phillips curve similar equation, using unobserved components:

$$\pi_t = \mu_t + \psi_t + \gamma_t + \phi h_t + \varepsilon_{2t} \tag{4.2}$$

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[4] This point was made by Wongwachara and Minphimai (2009).

[5] Models with unobserved components are also known in the literature as structural time series models. See Harvey (1989).

$$\varepsilon_{2t} \sim N(0, {\sigma_{\varepsilon_2}}^2)$$

Harvey (2011) argues that, under some hypotheses, an inflation model with this configuration may simultaneously capture the backward- and forward-looking ideas of the hybrid new Keynesian Phillips curve.[6] Such configuration is based on the following terms explaining current inflation: lagged inflation, output gap and an inflation expectations component, i.e.:[7]

$$\pi_t = \delta_b \pi_{t-1} + \phi h_t + \delta_f E_t(\pi_{t+1}) + \varepsilon_{3t} \tag{4.3}$$

$$\varepsilon_{3t} \sim N(0, {\sigma_{\varepsilon_3}}^2)$$

In other words, Harvey (2011) focuses on estimating (4.2), arguing that this formulation contemplates the notion of a hybrid NKPC as in (4.3).

At least with respect to the lagged term, it is reasonable to affirm that it can be successfully replaced with the specification proposed here. It suffices to observe that a simple model that combines inflation and output gap h_t:

$$\pi_t = \mu_t + \phi h_t + \varepsilon_{4t} \tag{4.4}$$

$$\varepsilon_{4t} \sim N(0, {\sigma_{\varepsilon_4}}^2)$$

can be written as:

$$\pi_t = E_{t-1}(\mu_t) + \phi h_t + \nu_t \tag{4.5}$$

$$\nu_t \sim N(0, {\sigma_\nu}^2)$$

where $\nu_t = \pi_t - E_{t-1}(\pi_t)$ is an innovation and $E_{t-1}(\mu_t)$ is a weighted average of past observations, corrected for the output gap's effect. If we include cycle and/or seasonal components in (4.5), we have the term $E_{t-1}(\mu_t)$ capturing both the past trend, and information on lagged inflation rates, appropriately weighted. This formulation seems to be more realistic than the Phillips curve with a plain lagged inflation term. In addition, as pointed out by Harvey (2011), admitting that h_t is stationary in (4.4), the long-term inflation forecast

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[6] Gali and Gertler (1999) and Christiano, Eichenbaum and Evans (2005) are theoretical references on the treatment of inflation through the hybrid new Keynesian Phillips curve. Their differences basically lie on the way prices are adjusted.

[7] Nason and Smith (2008b) argue that the hybrid NKPC is consistent with a variety of price and information adjustment schemes. Therefore, the focus on reduced-form coefficients, δ_b, ϕ and δ_f, instead of on structural parameters, simplifies the analysis without interfering in the importance of the result.

is the current value of μ_t, i.e., the unobserved term of the structural model becomes a measure of core inflation or underlying rate of inflation.

In regard to the expectations term, our view diverge from that adopted by Harvey (2011), which assumed that the hybrid NKPC is equivalent to an equation relating inflation to core inflation expectation, to expectation on the sum of future output gaps and to the current output gap, i.e.,

$$\pi_t = E_{t-1}(\mu_t{}^*) + \gamma\phi^* \sum_{j=o}^{\infty} \gamma^j E_t\left(h_{t+1+j}\right) + \phi h_t + \nu_t \qquad (4.6)$$

$$\nu_t \sim N(0, \sigma_\nu{}^2)$$

In addition to the need to appeal to several simplifying assumptions, this does not fully solve the problem, i.e., it does not allow, in general terms, modeling the past and future effects of hybrid NKPC as in the equation with unobserved components, or in the present model, equation (4.4). The author acknowledges the difficulty in doing so and places little importance on the future term, citing Rudd and Whelan (2007) and Nason and Smith (2008a).

Unlike Harvey (2011),[8] we included an inflation expectations term in the analysis, as we consider it to be a crucial element when modeling inflation dynamics, in line with most of the new Keynesian literature. Furthermore, it is possible to check whether the future term indeed plays a major role for Brazilian data in our model, as highlighted by Sachsida, Ribeiro and Santos (2009).[9] However, Nason and Smith (2008b) draw attention to the weak identification of traditional GMM-based estimates of the NKPC. More importantly, Orphanides and Williams (2005) point out that instrumental variables methods impose the unrealistic restrictions that monetary policy conduct and the formation of expectations be constant over time. These points motivate the use of survey data in our study[10]. Among studies that used inflation expectations surveys, Basistha and Nelson (2007), for instance, adopt an inverse perspective, in which they estimate the output gap using a forward-looking Phillips curve; Orphanides and Williams (2005) use data from the Survey of Professional

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[8] Vogel (2008) also argues that inflation expectations should not be neglected in the basic equation, while citing the difficulty in the identification of μ_t in Harvey (2011) regarding past or future effects.

[9] According to their model, studies that consider the Phillips curve to be nonlinear underestimate the role of the future term in the Brazilian inflation dynamics.

[10] On the other hand, some authors highlight the drawback of survey-based forecasting bias, as a sign of agents' lack of rationality. However, Araujo and Gaglianone (2010) state that survey series contained in the Central Bank of Brazil's Focus Bulletin do not suffer forecasting bias in the case of expectations over a shorter time horizon (one and three months ahead).

Forecasters to estimate a model of the US economy, supposing private agents constantly learn as they form their forecasts.

4.2.2 A NKPC with unobserved components

The link between resource utilization and inflation is at the heart of the Phillips curve. Therefore, one should begin by considering some real activity variable that represents the inflationary pressure (or the real marginal cost, as in the original NKPC). The most frequent examples include labour income share, deviation from the natural rate of unemployment or output gap. In the present study, the major focus is on the output gap, measured using two indicators of economic activity, GDP and the IBC-Br series, which is published by the Central Bank of Brazil (BCB). Even though some authors such as Schwartzman (2006) and Sachsida, Ribeiro and Santos (2009) support series with larger economic information to the detriment of econometrically detrended output gap series, we use the output gap for the following reasons: it is still an important index used by the BCB for monetary policy formulation as signal of demand pressures on prices. Secondly, as this is a new method for the analysis of Brazilian data, it should be tested in comparison to this index, which is widely used in economic theory and practice. Last but not least, it is expected that with the gradual and larger availability of data after the introduction of the inflation targeting system, the output gap may become more representative of inflationary pressures in Brazil[11]. Nonetheless, for the sake of comparison, we reproduce the same estimations with monthly industrial capacity utilization rate (ICU) series.

A large strand of the literature is devoted to the estimation of the output gap series, which is not directly observed in the economy.[12] Since the primary goal here is not to explore these techniques, we opted for decomposing the logarithm of output into unobserved trend and cycle components, as in Harvey (2011).

$$\log y_t = \mu_t + \gamma_t + \psi_t + \varepsilon_t \tag{4.7}$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} \tag{4.8}$$

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[11] This is true both for the IBC-Br series and for usual quarterly GDP series.

[12] The main tools used in the applied literature are: production function approach, which has the advantage of imposing some economic structure, with information on capital accumulation and on total factor productivity; and the econometric approach, in which the trend of the real GDP series is identified as potential output, a good and useful approximation when reliable macroeconomic data on capital and labor are not available.

$$\beta_t = \beta_{t-1} + \zeta_t \tag{4.9}$$

Simultaneously, we extracted the series seasonal component γ_t and stochastic cycle ψ_t, which is equivalent to the output gap h_t and takes on the following form:

$$\begin{bmatrix}\psi_t \\ \psi_t^*\end{bmatrix} = \rho_\psi \begin{bmatrix}\cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c\end{bmatrix}\begin{bmatrix}\psi_{t-1} \\ \psi_{t-1}^*\end{bmatrix} + \begin{bmatrix}\kappa_t \\ \kappa_t^*\end{bmatrix} \tag{4.10}$$

where ρ_ψ is a damping factor and λ_c in the frequency in radians $(0 \le \lambda_c \le \pi)$. Error terms ε_t and ζ_t are normally independently distributed with variances σ_ε^2 and σ_ζ^2 . κ_t and κ_t^* are mutually uncorrelated disturbances with zero mean and common variances $\sigma_\kappa^2 = \sigma_{\kappa^*}^2$. The dynamics of the stochastic seasonal component γ_t is identical with the one described next in equations (4.14) and (4.15). Note that the expression above indicates a smooth trend which, together with a cyclical component, represents an attractive decomposition for output data, according to Koopman et al (2007). The trend described in (4.8) and (4.9) can also be referred to as an integrated random walk.[13] A traditional tool for trend extraction is the Hodrick-Prescott (HP) filter. However, even if the resulting output gap is similar to the one obtained here, the HP filter tends to be less efficient at the end of the series, as described by Mise, Kim and Newbold (2005).

For the sake of comparison, our estimations begin with a simpler Phillips curve (model I), adapted from Harvey (2011), with the inclusion of interventions in order to capture irregularities in the data:

$$\pi_t = \mu_t + \gamma_t + \phi h_t + \textstyle\sum_{k=1}^{l} d_k \theta_{k,t} + \varepsilon_t \tag{4.11}$$

$$\varepsilon_t \sim N(0, {\sigma_\varepsilon}^2)$$

where μ_t and γ_t follow the same dynamics of equations (4.13) through (4.15).

Adding the expectations term, we obtain the proposed Phillips curve model, which is classified as model II, III or IV in the next section, depending on the variable used as measure of marginal cost:

$$\pi_t = \mu_t + \gamma_t + \phi h_t + \delta_f E_t(\pi_{t+1}) + \textstyle\sum_{k=1}^{l} d_k \theta_{k,t} + \varepsilon_t \tag{4.12}$$

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[13] Portugal (1993) performs a similar estimation for Brazilian annual data from 1920 to 1988, but he considers both a fixed output growth rate and a stochastic one. In the present model, the difference between both is the term ζ_t.

$$\varepsilon_t \sim N(0, {\sigma_\varepsilon}^2)$$

$$\mu_t = \mu_{t-1} + \eta_t, \tag{4.13}$$

$$\eta_t \sim N(0, {\sigma_\eta}^2)$$

$$\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{j,t} \tag{4.14}$$

$$t = 1, \dots, T$$

where each $\gamma_{j,t}$ is generated by:

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^{*} \end{bmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^{*} \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^{*} \end{bmatrix} \tag{4.15}$$

$$j = 1, \dots, [s/2]$$

In the above expression for trigonometric seasonality, $\lambda_j = 2\pi j/s$ is the seasonal frequency in radians, and $\omega_{j,t}$, $\omega_{j,t}^{*}$ are normally independent distributed seasonal disturbances with zero mean and common variance σ_ω^2. To choose the intervention dummy variables $\theta_{k,t}$ we analysed the auxiliary residuals, which are smooth estimates of the disturbances of irregular, level and slope components.[14]

Equation (4.12) is also called measurement or observation equation containing variables that explain the observed inflation. Equations (4.13) through (4.15) form the state equations that characterize the dynamics of unobserved variables. Note that inflation trend is dealt with by using a local level approach, compatible with nonstationarity, which is common in the literature. As to seasonality, component γ_t can be seen as the sum of time-varying trigonometric cycles.

For the implementation of the Kalman filter algorithm, it is necessary that the model's equations are expressed in state-space form, i.e.:

$$\pi_t = (1 \quad 1 \quad 0)\boldsymbol{\alpha_t} + \phi h_t + \delta_f E_t(\pi_{t+1}) + \boldsymbol{d_k \theta'_{k,t}} + (\sigma_\varepsilon \quad 0 \quad 0 \quad 0)\boldsymbol{u_t}$$

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[14] The inclusion of the cyclical component ψ_t was also tested, but it was found to incorrectly capture some typical outlier episodes such as peaks or troughs found in the cycles. A larger amount of years would probably minimize this problem. Thus, the component was not considered at this estimation stage.

$$\boldsymbol{\alpha_t} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda_j & \sin\lambda_j \\ 0 & -\sin\lambda_j & \cos\lambda_j \end{pmatrix} \boldsymbol{\alpha_{t-1}} + \begin{pmatrix} 0 & \sigma_\eta & 0 & 0 \\ 0 & 0 & \sigma_\omega & 0 \\ 0 & 0 & 0 & \sigma_\omega \end{pmatrix} \boldsymbol{u_t}$$

where $\boldsymbol{\alpha_t} = (\mu_t \quad \gamma_{j,t} \quad \gamma^*_{j,t})'$ and $\boldsymbol{u_t} = (\varepsilon_t \quad \eta_t \quad \omega_t \quad \omega^*_t)'$.

In other words, our model basically resembles a reduced-form new Keynesian Phillips curve, with inflation expectations term and output gap as explanatory variables. Nevertheless, it also captures, to some extent, past inflation behaviour through the decomposed trend and seasonality terms, in an attempt to mitigate an empirical deficiency that is commonly referred to in the literature.[15]

4.2.3 Data and econometric approach

Table 1 shows the series used to estimate equations (4.11) through (4.15) as well as the multivariate analysis in section 4.3. Monthly GDP series at current prices, from April 2000 to May 2011 (source: http://www4.bcb.gov.br/?SERIESTEMP[16]) was decomposed, following equations (4.7) through (4.10), using the Kalman filter algorithm from the OxMetrics 5 package (STAMP module). The trend component obtained from the estimation is a good approximation for the potential output in the period. The difference between the observed series and its seasonally adjusted trend is the output gap. In this case, disregarding the error term, as its variance was very close to zero, we can easily assume that the cyclical component corresponds to the output gap. Similar reasoning was used to extract the output gap from the IBC-Br series.

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[15] Fuhrer and Moore (1995) were the first to argue that standard new Keynesian price adjustment models could not explain persistence in the empirical process of inflation.

[16] The BCB interpolates IBGE's official quarterly series to obtain a monthly series.

Table 1: Data series

Monthly data series – April/2000 through May/2011

Variable	*Proxy*	*Source*
Inflation	IPCA	IBGE
Output	log(PIB-BC)	BCB
	IBC-Br	BCB
Inflation expectations	Median of daily expectations	FOCUS-BCB
Marginal cost in PC	PIB-BC and IBC-Br output gaps	UC-model
	Industrial capacity utilization rates (ICU)	CNI

Notes: The IBC-Br series is only available after January 2003. IPCA: Broad consumer price index; IBGE: Brazilian Institute of Geography and Statistics; BCB: Brazilian Central Bank; PIB-BC: Monthly output series published by the BCB; IBC-Br: BCB´s Index of Economic Activity; CNI: Brazil's National Confederation of Industry.

Figure 1 shows the comparison of the series obtained from the PIB-BC output gap, from the IBC-Br output gap and from relative deviations from the ICU, all of which are expressed as percentage. Note that the output gap, as specified, varies between positive and negative percentage values.[17] The ICU series was collected from the IPEADATA database, and the percentage difference between the real series and its average for the period (calculated as 80.87%) was used to construct the deviations. Some clear patterns among all variables match the stylized facts in the Brazilian economy: first, a continuous economic activity growth period started in early 2006 and lasted until the first half of 2008; thereafter the subprime crisis caused a dramatic drop in activity. Second, a new period of economy growth apparently brought the observed output again above potential output and then more or less stabilized it over 2010. Particularly industrial capacity seems to be quite above its natural rate since the beginning of 2010.

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[17] This occurs because the series trend, by definition, crosses the real series in different moments. Hence, positive output gap figures correspond to moments of heated economic activity, which result in inflationary pressures.

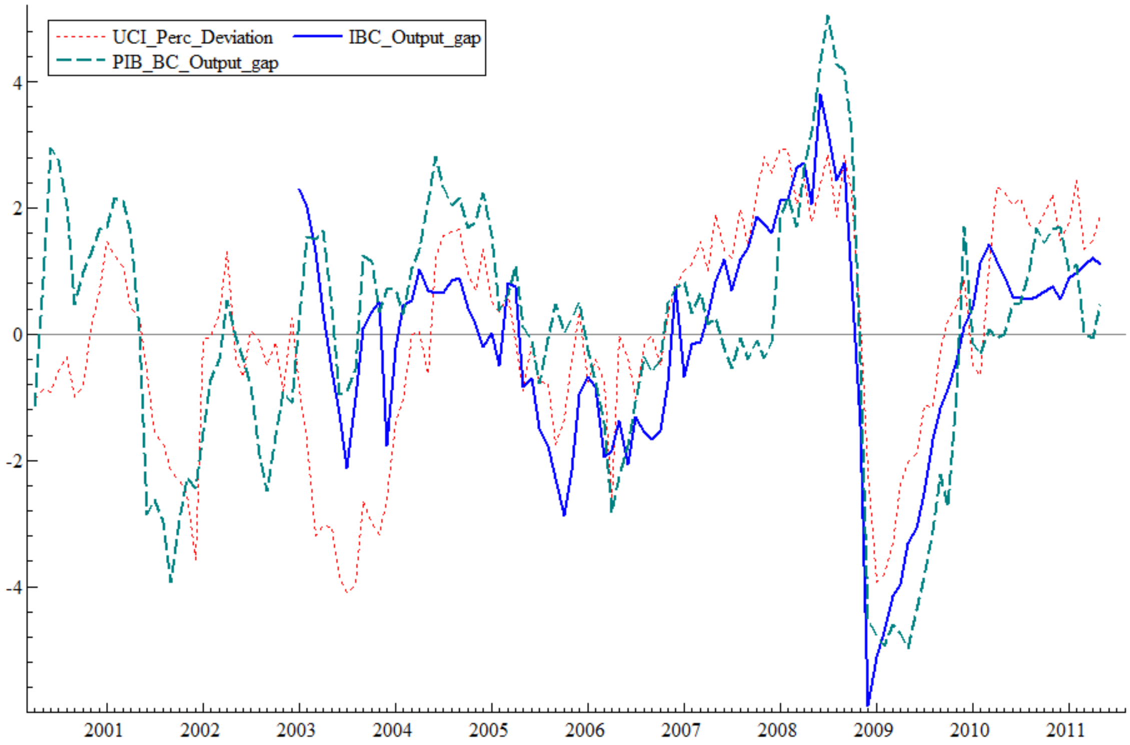

Figure 1: Comparison of economic activity series

Inflation expectations were obtained from the Central Bank's FOCUS survey. In the present study, we used the median of daily expectations within each month relative to the forward month.

With data from output gap, industrial capacity utilization rates and survey expectations for the period at hand, we carried out Phillips curve estimations, analytically shown in section 4.2.2.

First, we tested a model that is similar to the one used in Harvey (2011), which consists of equation (4.11) and is identified in Table 2 as model I. Then, to highlight the importance of introducing inflation expectations in the Phillips curve model with unobserved components, we use equation (4.12) (Model II). In both alternatives, the measure of marginal cost used is the output gap calculated from the monthly GDP provided by the BCB (PIB-BC). The third model concerns a Phillips curve that is identical to (4.12), but with ICU data instead of output gap. Finally, model IV consists of the same equation (4.12), with the difference that the output gap series was calculated using IBC-Br series. Due to some unusual inflation movements, especially between 2002 and 2003, the inclusion of interventions inevitably leads to a better fit, which leads us to include them in the model.

Model evaluation followed some fitting and residuals diagnostic statistics. With respect to fitting, the chief indicators contemplated in the estimation of the output gap and of the Phillips curve were the following: algorithm convergence, prediction error variance (PEV), and log-likelihood. According to Koopman et al (2007), a good convergence is key to showing that the model was properly formulated and that, in general, it does not have fitting problems. Prediction error variance is the basic measure of goodness-of-fit which, in steady state, corresponds to the variance of the one-step-ahead forecast errors. Other diagnostic statistics analysed include Box-Ljung's Q statistics, for the assessment of residuals autocorrelation, and normality (N) and heteroskedasticity (H) results.

4.2.4 Results

In the output gap estimation, a "very strong" convergence and a relatively small prediction error variance were obtained. The recent global financial crisis and the resulting sharp decrease in all economic activity measures in the last quarter of 2008 and subsequent recovery are noteworthy.

Table 2 shows key results from the different models described in Section 4.2.2. In all cases, convergence was again "very strong," satisfying the main modeling criterion proposed by Koopman et al (2007). As expected, the prediction error variance decreased from I to IV, indicating superior fit of the models (II through IV) that include inflation expectations. Log-likelihood indicators underscore this conclusion, as they increased from I to III. In the case of model IV, the reduction is more a result of sample size than of the goodness of fit, given that log-likelihood is an absolute and cumulative indicator.

The traditional coefficient of determination undergoes a slight change in case of seasonal data, R_s^2, and measures the relative performance of the specified model in relation to a simple random walk with drift and fixed seasonality. Again, the result is better for models II and III. According to Box-Ljung's Q statistics, serial correlation of residuals is absent in all models and significance is lower than 0.1%.

With values lower than one, heteroskedasticity (H) tests indicate that the variance of residuals decreases slightly over time. Unequivocally, this results from the improvement of the inflation targeting regime in Brazil, with an increasingly larger convergence of the inflation rate towards the targets.

Table 2: Phillips curve estimation results

	PEV	$Loglik$	R_s^2	Q	N	H	δ_f	ϕ
Model I (Harvey) $\pi_t = \mu_t + \gamma_t + \phi h_t + \sum_{k=1}^{l} d_k \theta_{k,t} + \varepsilon_t$	0.052	143.43	0.54	29.31	0.04	0.56	-	0.017 [0.41][#]
Model II $\pi_t = \mu_t + \gamma_t + \phi h_t + \delta_f E_t(\pi_{t+1}) + \sum_{k=1}^{l} d_k \theta_{k,t} + \varepsilon_t$	0.047	148.96	0.59	27.52	0.77	0.63	0.999 [3×10^{-5}]	0.015 [0.38]
Model III $\pi_t = \mu_t + \gamma_t + \phi ICU_t + \delta_f E_t(\pi_{t+1}) + \sum_{k=1}^{l} d_k \theta_{k,t} + \varepsilon_t$	0.047	148.87	0.59	30.32	0.90	0.63	0.995 [3×10^{-5}]	0.009 [0.66]
Model IV $\pi_t = \mu_t + \gamma_t + \phi IBCh_t + \delta_f E_t(\pi_{t+1}) + \sum_{k=1}^{l} d_k \theta_{k,t} + \varepsilon_t$	0.029	117.97	0.45	26.16	3.46	0.58	1.08 [0]	0.034 [0.001]

Source: Data obtained by the authors

Notes: [#]Values in square brackets: *p*-value.

The interventions, in order of importance, and respective *p*-values for models I through III were as follows:

- Outlier in 2002/11. Model I: 1.35 [0]; model II: 1.26 [0]; model III: 1.27 [0]
- Level break in 2003/6: Model I: -0.74 [0.001]; model II: -1.01 [1×10^{-5}]; model III: -1.02 [0]
- Outlier in 2003/9: Model I: 0.66 [2×10^{-4}]; model II: 0.65 [1×10^{-4}]; model III: 0.66 [1×10^{-4}]
- Outlier in 2000/8: Model I: 0.94 [1×10^{-5}]; model II: 0.81 [1×10^{-4}]; model III: 0.82 [1×10^{-4}]
- Outlier in 2000/7: Model I: 0.83 [1×10^{-4}]; model II: 0.63 [0.003]; model III: 0.64 [0.002]

In model IV, the resulting interventions were:

- Level break in 2003/3: -0.75 [0]
- Outlier in 2003/6: -1.25 [0]
- Outlier in 2006/6: -0.36 [0.06].

Even in model IV, with a more recent sample, the pattern signals at gradually lower variances. As to normality (N), the models clearly succeeded on the test, based on Doornik-Hansen's statistic whose critical value at a 5% significance level is 5.99.

The slopes (ϕ) of the different Phillips curve specifications – which are the coefficients for output gap and ICU deviation at the end of the sample – were positive in all cases, as theoretically expected, though not statistically significant in cases I to III. On the other hand, the output gap measure calculated based on the IBC-Br series was positively correlated with the inflation rate, with a high level of significance, although the amount of available data is smaller. The comment made by Tombini and Alves (2006), that smaller coefficients than most of those described in the literature are due to the monthly frequency of data, applies here. In regard to the coefficients of inflation expectations, the values showed high statistical significance and are close to one, suggesting alignment of inflation expectations with the observed rates.

Test statistics particularly indicate a fitting improvement as we move from an approach without inflation expectations, as in Harvey (2011), to an approach that includes them. Among output gap measures and the ICU deviations, there is no clear superiority, when it comes to fitting.

Figure 2 shows the decomposition obtained in model II, which performed clearly better than the model adapted from Harvey (2011). The first chart compares observed inflation values (black line) with the decomposition of trend, regression and intervention effects (red line). The middle chart shows seasonal effects. The values correspond to the absolute contribution due to seasonality. For instance, a value of 0.2 indicates that in that month 0.2% of price fluctuation is exclusively due to the seasonal effect of the month. Note also that the variance of this effect decreases in more recent years, with a sharp increase in the effect in February and decrease in the effect in June over the last two years.

Finally, the lower graph shows the irregular component. Considering that the period between late 2002 and mid-2003 had the three largest discrepant observations regarded as outliers, this chart also depicts some gradual reduction in the variability of disturbances.

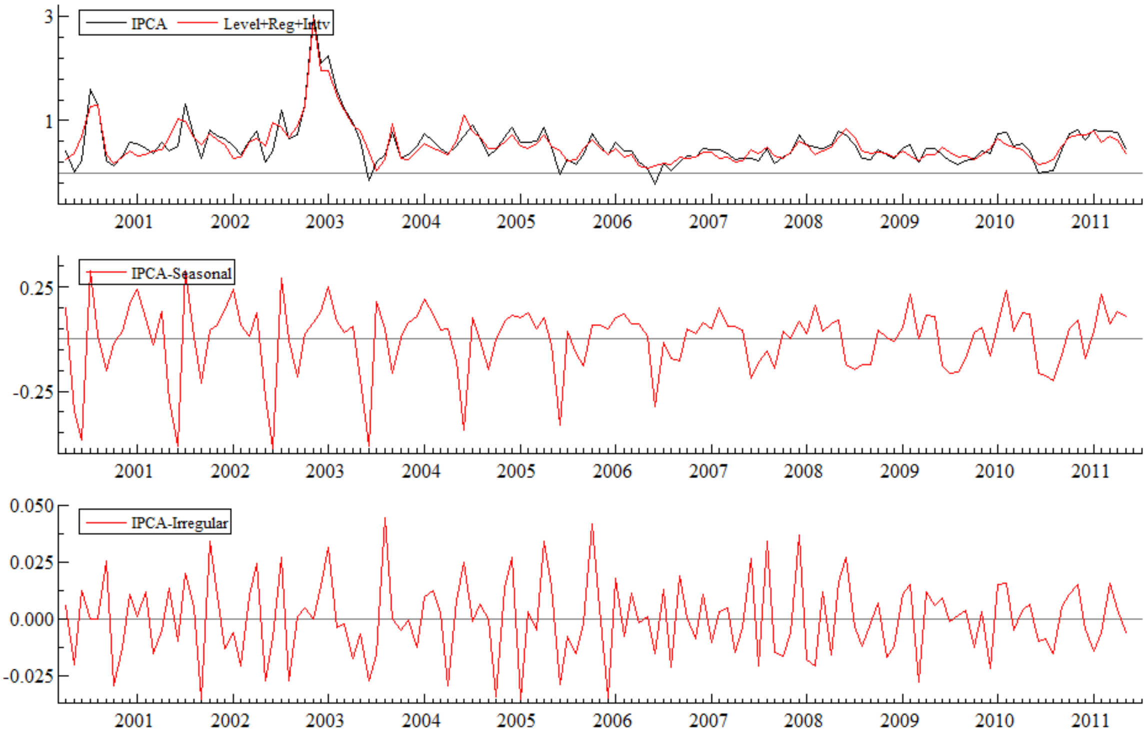

Figure 2: Inflation decomposition - Model II

4.3 Multivariate estimation

To complement the previous analysis, we fitted a bivariate Phillips Curve, in which inflation and output are jointly decomposed into unobserved components. This specification has the advantage of averting the exogenous estimation of the output gap, as it is implicitly present in the output equation. In this case, we have a vector of observations that now relies on two seemingly unrelated time series equations (SUTSE).

The joint specification of inflation and output differs from Harvey (2011) because of the introduction of the seasonal component and, especially, of the expectations term. Our main model is now:

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} \mu_t^\pi \\ \mu_t^y \end{bmatrix} + \begin{bmatrix} \gamma_t^\pi \\ \gamma_t^y \end{bmatrix} + \begin{bmatrix} \psi_t^\pi \\ \psi_t^y \end{bmatrix} + \delta_f \begin{bmatrix} E_t(\pi_{t+1}) \\ 0 \end{bmatrix} + \begin{bmatrix} \sum_{k=1}^{l} d_k \theta_{k,t}^\pi \\ \sum_{k=1}^{m} f_k \theta_{k,t}^y \end{bmatrix} + \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^y \end{bmatrix} \tag{4.16}$$

where $\theta_{k,t}^\pi$ and $\theta_{k,t}^y$ represent the sets of outliers considered for the inflation and output series respectively.

The link between the series in the SUTSE approach is generally established by the correlations of errors of one or more components. Following Harvey (2011), we assume the cycles have the same autocorrelation function and spectrum. In other words, inflation and output cycles are modeled as "similar cycles". In algebraic terms, supposing $\boldsymbol{\psi_t} = (\psi_t^{\pi}, \psi_t^{y})'$,

$$\begin{bmatrix} \boldsymbol{\psi_t} \\ \boldsymbol{\psi_t^*} \end{bmatrix} = \left[\rho \begin{pmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{pmatrix} \otimes \boldsymbol{I_2}\right] + \begin{bmatrix} \boldsymbol{\psi_{t-1}} \\ \boldsymbol{\psi_{t-1}^*} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\kappa_t} \\ \boldsymbol{\kappa_t^*} \end{bmatrix}, \tag{4.17}$$

$$t = 1, \dots, T$$

where $\boldsymbol{\kappa_t}$ and $\boldsymbol{\kappa_t^*}$ are 2×1 error vectors, such that $E(\boldsymbol{\kappa_t}\boldsymbol{\kappa_t}) = \boldsymbol{E}(\boldsymbol{\kappa_t^*}\boldsymbol{\kappa_t^*}') = \boldsymbol{\Sigma_\kappa}$, and $\boldsymbol{\Sigma_\kappa}$ is a 2×2 covariance matrix and $E(\boldsymbol{\kappa_t}\boldsymbol{\kappa_t^*}') = \boldsymbol{0}$.

The series can also be expressed in state-space form, with each component now being a vector. Like the univariate estimation, the inflation trend component follows a local level model, as in (4.13), and the output component conforms to a smooth trend model, as in (4.8) and (4.9). Seasonality here is also allowed to be stochastic, in order to check its variability in the inflation series.

The cyclical component of inflation can be broken down into two independent parts, as follows:

$$\psi_t{}^{\pi} = \beta\psi_t{}^{y} + \psi_t{}^{\pi^*} \tag{4.18}$$

where $\beta = \frac{Cov(\psi_t{}^{\pi}, \psi_t{}^{y})}{Var(\psi_t{}^{y})} = \frac{Cov(\kappa_t{}^{\pi}, \kappa_t{}^{y})}{Var(\kappa_t{}^{y})}$ and $\psi_t{}^{\pi^*}$ is a cyclical component specific to inflation.

Thus, the inflation equation may be rewritten as:

$$\pi_t = \mu_t{}^{\pi} + \gamma_t + \beta\psi_t{}^{y} + \psi_t{}^{\pi^*} + \varepsilon_t{}^{\pi} \tag{4.19}$$

$$\varepsilon_t{}^{\pi} \sim N(0, \sigma_\varepsilon{}^2)$$

Considering that the cycle of the output equation gives a good notion about the output gap, as occurred in Section 4.2, and that disturbances $\kappa_t{}^{\pi}$ and $\kappa_t{}^{y}$ are perfectly correlated, it is possible to conclude that coefficient β corresponds to parameter ϕ of the univariate Phillips curve, or its slope. Therefore, from the correlation matrix of cycles, one obtains $\phi = \beta$.

In the bivariate case, three basic specifications are tested. Again, a similar approach to that of Harvey (2011) is compared with the model built above, in which one includes the future inflation expectations term, as shown in equation (4.16). They are represented as models V and VI. Finally, (4.16) through (4.19) are also employed considering the IBC-Br as proxy for y_t (Model VII).

The relevant goodness-of-fit criterion in this case is a correlation matrix for the prediction error variance and the log-likelihood. Test statistics already used in the univariate model are also reproduced in Table 3. We show diagnostics referring only to the inflation equation in (4.16), since our main interest here is on Phillips curve estimations.

4.3.1 Results

Multivariate estimates again reveal that the introduction of expectations clearly improves the model's fit and determination, as one can easily check from log-likelihood and R_s^2 outcomes. It is also important to highlight that model V had a "weak" algorithm convergence compared to a "very strong" convergence in model VI. The comparison with model VII is hindered simply because the samples are different, but a similar pattern can be detected in the coefficients for inflation expectations and output gap. When the IBC-Br is used, its parameter shows a greater sensitivity of inflation than when PIB-BC is employed as a measure of the output gap. This result is probably due to the nature of both series. Intuitively, a properly measured index of economic activity, such as the IBC-Br is expected to have greater impact on inflation than a series built by mere interpolation of quarterly series.

Table 3: Estimation results - bivariate case

	Loglik	R_s^2	Q	N	$H^{\dagger}$	δ_f	ϕ
Model V (Harvey): $\begin{bmatrix}\pi_t\\ y_t\end{bmatrix}=\begin{bmatrix}\mu_t^{\pi}\\ \mu_t^{y}\end{bmatrix}+\begin{bmatrix}\gamma_t^{\pi}\\ \gamma_t^{y}\end{bmatrix}+\begin{bmatrix}\psi_t^{\pi}\\ \psi_t^{y}\end{bmatrix}+\begin{bmatrix}\sum_{k=1}^{l} d_k\theta_{k,t}^{\pi}\\ \sum_{k=1}^{m} f_k\theta_{k,t}^{y}\end{bmatrix}+\begin{bmatrix}\varepsilon_t^{\pi}\\ \varepsilon_t^{y}\end{bmatrix}$	604.39	0.57	19.39	2.95	0.43	-	0.028*
Model VI (PIB-BC) $\begin{bmatrix}\pi_t\\ y_t\end{bmatrix}=\begin{bmatrix}\mu_t^{\pi}\\ \mu_t^{y}\end{bmatrix}+\begin{bmatrix}\gamma_t^{\pi}\\ \gamma_t^{y}\end{bmatrix}+\begin{bmatrix}\psi_t^{\pi}\\ \psi_t^{y}\end{bmatrix}+\delta\begin{bmatrix}E_t(\pi_{t+1})\\ 0\end{bmatrix}+\begin{bmatrix}\sum_{k=1}^{l} d_k\theta_{k,t}^{\pi}\\ \sum_{k=1}^{m} f_k\theta_{k,t}^{y}\end{bmatrix}+\begin{bmatrix}\varepsilon_t^{\pi}\\ \varepsilon_t^{y}\end{bmatrix}$	609.52	0.61	23.32	2.50	0.55	0.93 $[4\times10^{-5}]^{\#}$	0.024*
Model VII (IBC-Br) $\begin{bmatrix}\pi_t\\ y_t\end{bmatrix}=\begin{bmatrix}\mu_t^{\pi}\\ \mu_t^{y}\end{bmatrix}+\begin{bmatrix}\gamma_t^{\pi}\\ \gamma_t^{y}\end{bmatrix}+\begin{bmatrix}\psi_t^{\pi}\\ \psi_t^{y}\end{bmatrix}+\delta\begin{bmatrix}E_t(\pi_{t+1})\\ 0\end{bmatrix}+\begin{bmatrix}\sum_{k=1}^{l} d_k\theta_{k,t}^{\pi}\\ \sum_{k=1}^{m} f_k\theta_{k,t}^{y}\end{bmatrix}+\begin{bmatrix}\varepsilon_t^{\pi}\\ \varepsilon_t^{y}\end{bmatrix}$	87.59	0.39	24.00	3.30	0.57	1.05 $[0]^{\#}$	0.044*

Source: Data obtained by the authors

Notes: *The significance of parameter ϕ is not available as this parameter could only be indirectly estimated, as explained in (4.18) and (4.19).

[#]: Values in square brackets: *p*-value.

[†]: Statistics R_s^2, Q, *N* and *H* refer only to the inflation equation.

The interventions considered in the inflation equation in models V and VI, in order of importance, and the respective *p*-values were:
- Outlier in 2002/11. Model V: 1.41 [0] ; Model VI: 1.29 [0]
- Level break in 2003/6. Modelo V: -0.66 [6×10^{-4}] ; Model VI: -0.87 [0]
- Outlier in 2003/9. Model V: 0.71 [3×10^{-4}] ; Model VI: 0.67 [3×10^{-4}]
- Outlier in 2000/8. Model V: 0.61 [0.005] ; Model VI: 0.56 [0.008]

In the output equation, level break in 2008/12. Model V: -0.09 [0]; Model VI: -0.09 [0].

In model VII, the resulting interventions were:
- Outlier in 2003/6: -1.16 [0]
- Level break in 2003/2: -0.74 [9×10^{-4}]
- Outlier in 2003/9: 0.55 [0.005].

Figure 3 shows the resulting components of model VII, where we used both inflation and IBC-Br data in different equations. As opposed to Figure 2, recent dynamics of inflation in Brazil was rather less unstable. Particularly the seasonal effects proved to be relatively constant, although a reminder about the smaller size of the sample should be made. Taking a closer look at the period between 2006 and early 2008, there is clearly less variability in the components of inflation than in the rest of the sample. This illustrates the relatively calm pre-crisis setting in terms of monetary policy tensions, which was also in place in Brazil.

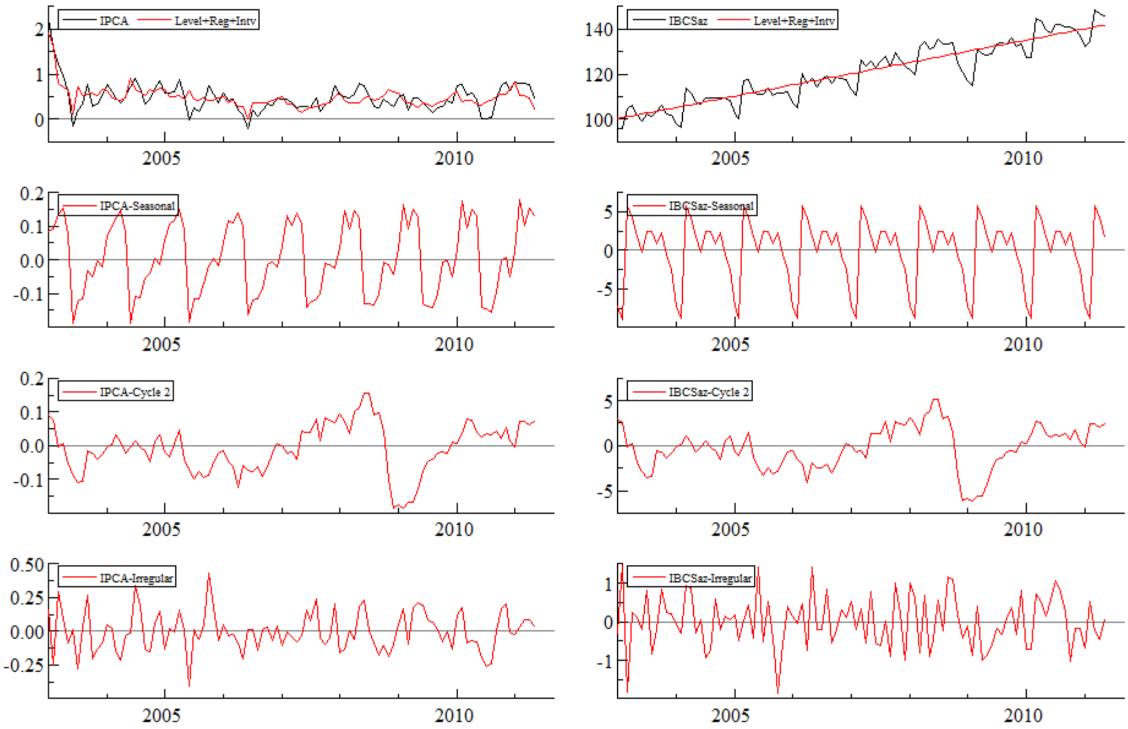

Figure 3: Inflation and output (IBC-Br) – Bivariate model VII

Note: The IBC-Br is constructed based on the value of 100 in 2002. Inflation is expressed in monthly rates.

As to GDP, seasonal effects were reasonably constant in the sample. On the other hand, the cyclical component, which gives some notion about the output gap, showed a more erratic behaviour, with a sharpened drop at the end of 2008,[18] due to the impact of the U.S. subprime crisis. Also, note how the modeling of similar cycles allowed for a contemporaneous pattern in both series coinciding with the crisis episode.

4.4 Extensions

Some analyses were added to the basic models in order to better understand the dynamics of Phillips curve components in the Brazilian case. The first one concerns the flattening of the Phillips curve, observed in studies for some developed countries. As shown by Kuttner and Robinson (2010), the parameter ϕ of equation (4.12), which represents the response of the observed inflation to the output gap, has decreased in empirical analyses of

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[18] The behaviour of IBC-Br in late 2008 would also suggest a level break in trend, which was not feasible in practice due to restrictions on the algorithm and to the relatively small amount of observations.

the United States and Australia. A possible explanation is that, as inflation expectations become better anchored, the inflation response to supply shocks tends to be accommodated. An alternative source of this observed phenomenon points to the fact that, the frequency of price-setting may depend on the average inflation rate, hence monetary policy could have indirectly influenced the slope of the Phillips curve, by lowering the inflation trend, as Kuttner and Robinson (2010) argue. To investigate whether the same occurs in Brazil, a variant of models I to IV was tested, in which the output gap coefficient was allowed to vary over time, i.e., we now have ϕ_t, t = (Jan/2003,...,May/2011). In this case, a smoothing spline was used, in which the slope of the Phillips curve varies according to:

$$(\phi_t - \phi_{t-1}) = (\phi_{t-1} - \phi_{t-2}) + u_t \qquad (4.20)$$

$$u_t \sim N(0, {\sigma_u}^2)$$

The estimation of this new model is carried out with equations (4.12) through (4.15) plus (4.20), which is an additional state equation.

The prediction error variances of this estimation were generally slightly lower than that of the models in which ϕ was fixed, pointing to an intuitively better fit. Results indicate that the flattening of the Phillips curve has recently been underway in Brazil too, as is shown from the time evolution of coefficient ϕ_t in Figure 4. With data starting in 2003, model IV generated a general decline in the Phillips curve slope only until 2009, when it started to rise again. However, the general trend is still downwards. This result confirms the importance to consider time-varying parameters in Phillips curve estimations, as underscored by Sachsida, Ribeiro and Santos (2009). In addition, there is an important implication that the potential cost of disinflation in terms of lost output may have increased. On the other hand, increases in economic activity may have been accompanied by gradually smaller inflationary pressures. Tombini and Alves (2006) highlight that the mere uncertainty caused by the 2002 electoral crisis would have been strong enough to change the parameters of the reduced-form Phillips curve, leading to higher costs of disinflation. The authors also find evidence of reduction in parameter ϕ_t.

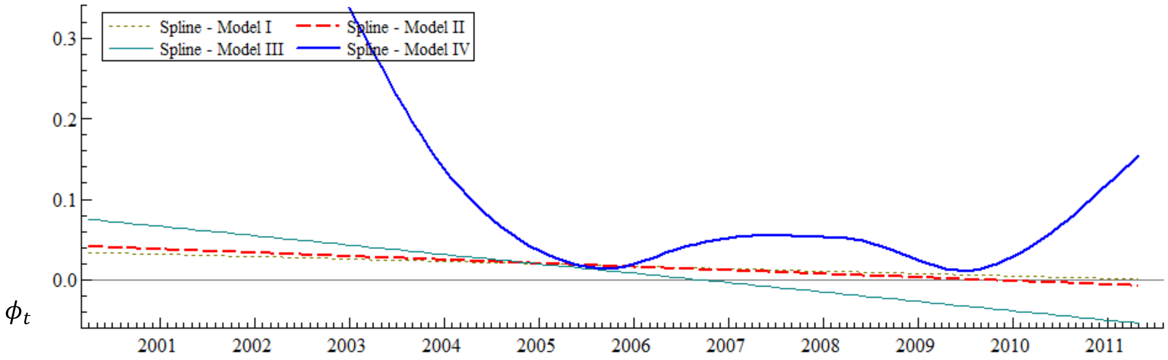

Figure 4: Dynamics of output gap coefficients in the Phillips Curve

The proposed model additionally allows assessing the forecasting power of a Phillips curve model by comparing the observed inflation with the one calculated through the models built by the Kalman filter, based on the minimization of one-step-ahead forecast errors. Stock and Watson (2008) reviewed works that dealt with forecasting inflation based on some form of Phillips curve and observed that these types of forecast are advantageous in some cases. However, Atkeson and Ohanian (2001) advocate that these forecasts tend to be worse than those which are based on simple univariate models. Notwithstanding, the widespread use of the NKPC in the literature and in actual policy with such forecasts require that their forecasting power be evaluated[19].

In the present study, the last 12 observations were excluded and the one-step-ahead inflation forecast was estimated for the period between April 2010 and March 2011. The mean squared error values for each model are shown in Table 4.

Note that the models without inflation expectations had a lower forecasting power than the model with expectations, again corroborating the main argument of the present study. This occurred both in the univariate and bivariate cases. In the univariate specification, output gap extracted from IBC-Br was not very successful, but in the bivariate case, it yielded the lowest mean squared error among all estimations, despite its smaller number of observations.

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[19] Araujo and Guillen (2008) test the forecasting power of different Phillips curves based on output gap specifications and conclude that the best performance was obtained by the output gap extracted by a multivariate method of unobserved components.

Table 4: Mean squared forecast error – different specifications

Model	I	II	III	IV	V	VI	VII	Naive
Mean squared error:	0.0258	0.0235	0.0215	0.0269	0.0222	0.0208	0.0184	0.0296

Source: Data collected by the authors

The forecasting power clearly increases in all cases when a multivariate specification is used. Finally, the mean squared error of a naive inflation model was calculated. In such a model, expected inflation value is forecasted by its current value, i.e., $E_{t-1}\pi_t = \hat{\pi}_t = \pi_{t-1}$. All analysed cases of Phillips curve outperformed this specification.

4.5 Conclusions

Given the clear-cut empirical difficulties surrounding the Phillips Curve and in order to fill a gap in its investigation in Brazil, the present study assessed inflation dynamics using an estimation with unobserved components for the Brazilian economy. By modifying Harvey's (2011) approach, introducing an inflation expectation term in the Phillips curve, the model manages to parsimoniously reproduce stylized facts about the relation between inflation and output gap. With the additional advantage of the graphical result, which allows a more direct economic interpretation of the components, we highlight the variability of the seasonal component of inflation, even with a sample of relatively few years. The relative reduction in this variability in the past years suggests that the inflation targeting system has contributed to reducing not only the inflation rates, but also their volatility within each year, at least until the subprime crisis effects started to hit the Brazilian economy, by 2008.

Output gap obtained in the trend-cycle models from the PIB-BC series and the ICU deviation series did not yield good statistical results for the analysed Phillips curve, even though positive coefficients were always found. In the case of an output gap extracted from the IBC-Br series, the result was clearly better, showing that this index, yet not used in academic works, may be of greater value in monitoring Brazilian monetary policy. Such success indicates that the output gap can also be representative in the Brazilian inflation dynamics, depending on the index used. Previous studies have normally used quarterly GDP series or the BCB's monthly series, as the one used here, in model II. In the former case, the number of observations is too small and, in the latter, the extrapolation of quarterly figures to monthly ones is unlikely to allow capturing the output dynamics. Thus, output gap results of the IBC-Br series may again strengthen the crucial relation of the Phillips curve for the Brazilian case. Bivariate estimation clearly produced more attractive results and strengthened our results from the univariate estimation.

The analysis of the Phillips curve slope, represented by parameter ϕ_t, is another important aspect, indicating a flattening Phillips curve in Brazil, as in Tombini and Alves (2006) and similarly to what is observed in developed countries, as reported by Kuttner and Robinson (2010). An important policy implication is that if this movement is neglected, the real activity parameter is overestimated at the end of the sample period, and monetary policy may, in disinflationary periods, be looser than would otherwise be required. In fact, the observed reduction in the impact of deviations of output from its real level on inflation means, *ceteris paribus*, that increases in economic activity, would not produce so much inflationary pressure as they used to. On the other hand, the costs of disinflation, in terms of lost output, would tend to increase in this scenario.

Finally, the forecasting power of different models was tested against a simple forecasting model. Whereas all models could outperform it in terms of squared forecast errors for the last 12 months of the sample, models considering inflation expectations performed better than Harvey's formulation.

Some issues, which were not dealt with here and that could be subject of investigation of future research, include the following: comparison of the performance of the output gap with that of other measures, such as unit labour cost or deviation from the natural rate of unemployment; another approach that considers different dynamics of free and administered prices, and even possible distinctions between tradable and nontradable goods, which concern the exchange rate influence; and finally, similar estimations for other countries. Also, instead of the trend-seasonal decomposition that produced the underlying level of inflation, we could re-estimate the model using the ideas of Chapter 3 in order to capture inflation persistence based on intrinsic and expectation sources.

As more data become available, it is likely that quarterly samples will have a higher forecasting power and will be more successfully used in similar estimations. In the meantime, the study also showed that IBC-Br can be an important tool for the formulation of monetary policy in Brazil.

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5 CONSIDERAÇÕES FINAIS

Na presente tese foram desenvolvidos três ensaios sobre os tópicos inter-relacionados de política monetária e dinâmica da inflação. O pano de fundo consiste de modelos novo-Keynesianos, em que uma curva de Phillips e uma equação IS representam respectivamente oferta e demanda agregada.

No primeiro ensaio, foi estudada a resposta ótima de política monetária a flutuações nos preços de ativos, com um ferramental de aprendizagem adaptativa.

Nele, as principais novidades apresentadas em relação à literatura foram: considerou-se heterogeneidade dos agentes em relação a suas crenças; procedeu-se a análise de política monetária ótima; e em relação a Carlstrom e Fuerst (2007) foi considerada uma regra de juros reagindo a valores futuros das variáveis.

Os principais resultados foram: a introdução de preços de ativos numa regra instrumental de taxa de juros em geral não é benéfica do ponto de vista de determinação e E-Estabilidade do equilíbrio; com heterogeneidade, a presença de agentes que formam expectativas levando em conta o passado leva a maior possibilidade de equilíbrios não-fundamentais; quanto à política monetária ótima, mostramos que o princípio de Taylor permanece importante na presença de desequilíbrios de preços de ativos.

Tais resultados indicam, como solução alternativa para o enfrentamento de desequilíbrios nos preços de ativos, uma maior regulação macroprudencial, como sugeriu, por exemplo, um trabalho recente do Fundo Monetário Internacional (IMF, 2011).

No tocante ao capítulo 3, o trabalho se concentrou em medir a persistência inflacionária no Brasil, usando uma abordagem multivariada com componentes não-observados e análise Bayesiana para identificar o grande número de variáveis não-observadas. Ao contrário das medidas tradicionais, baseadas em modelos univariados, que utilizam apenas dados de inflação, aqui diferentes fontes de persistência inflacionária puderam ser distinguidas e então quantificadas.

Seguindo em parte o trabalho - fruto de extensa pesquisa - realizado no âmbito da “Rede de Persistência Inflacionária” do Banco Central Europeu, o ensaio contribui para a literatura empírica brasileira no assunto, primordialmente baseada em modelos univariados.

Destacam-se os seguintes resultados: a persistência baseada em expectativas e a chamada persistência extrínseca tiveram grande importância em explicar a persistência inflacionária de forma geral; em especial, a persistência baseada em expectativas foi alta e permaneceu relativamente constante no período recente, enquanto que a persistência intrínseca decaiu.

Tais resultados não são inócuos para a política monetária: em ambientes mais instáveis, em que mudanças nas metas inflacionárias são mais comuns, a persistência inflacionária acaba sendo bastante influenciada pelos desvios nas percepções dos agentes com relação à verdadeira meta. Como conseqüência, os esforços de desinflação são maiores, devido ao lento ajuste das expectativas. Um resultado paralelo das estimações mostrou que as taxas de juros observadas não têm se distanciado demasiadamente dos seus valores naturais, contrariando argumentos de que o Banco Central do Brasil teria exercido política monetária muito conservadora nos últimos anos.

Por fim, o último ensaio se dedicou a estimar diversas curvas de Phillips Novo-Keynesianas com uma decomposição em componentes de tendência, ciclo e sazonalidade, em que foram empregados dados de inflação, PIB, utilização de capacidade industrial e uma série mais nova de produto, IBC-Br.

Tal decomposição, além do emprego da série elaborada pelo Banco Central do Brasil, IBC-Br, refletem uma contribuição para a literatura empírica brasileira. Em relação a Harvey (2011), a principal mudança foi a introdução de expectativas de inflação que aqui foram obtidas de dados coletados pela Pesquisa Focus do BCB.

Os resultados desse ensaio foram: em primeiro lugar, pôde-se analisar a dinâmica recente da inflação: seus efeitos sazonais diminuíram até o período anterior à recente crise, e depois subiram levemente; na estimação multivariada, foi possível identificar ciclos comuns entre inflação e produto, o que reforça a relação da curva de Phillips. Em segundo lugar, o IBC-Br mostrou-se uma série bastante promissora em refletir tempestivamente o comportamento do produto no Brasil e apresentou boa conexão com as taxas de inflação, como se esperava do modelo teórico de curva de Phillips. Por fim, destaca-se a diminuição do parâmetro da inclinação da curva de Phillips no período recente no Brasil, assim como foi visto em Tombini e Alves (2006) e em alguns trabalhos no exterior.

Como implicação desses resultados, concluímos que a relação presente na curva de Phillips entre inflação e hiato não pode ser subestimada no Brasil. Isso é verdadeiro, sobretudo se levado em conta o IBC-Br, que surge como opção significante, em se tratando de pesquisas mensais, e considerando-se que a relação varia no tempo e em especial tem decaído na última década.

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