



Planning, action and reflection at PIBID: a Lesson Study with Origami in light of the Didactic Suitability Criteria

Planejamento, ação e reflexão no PIBID: um Estudo de Aula com Origamis à luz dos Critérios de Adequação Didática

Rodrigo Sychocki da Silva¹

Abstract

This article aims to problematize the teaching practice actions developed within the scope of the Institutional Teaching Initiation Scholarship Program (PIBID) at a Brazilian public university. For this purpose, authors who discuss Class Studies and Teaching Suitability Criteria, as well as their connections with initial teacher training, are used as theoretical foundations. The research methodology used for this followed a qualitative approach, of an exploratory nature, with a documentary nature. It is inferred that a reflection produced jointly from a Class Study context, which involved scholarship teachers, supervising teachers and the coordination of the subproject, with a view to meeting the Didactic Suitability Criteria, made the educational process qualified and improved as the practices took place. This gave students on the Mathematics Degree course who participated in PIBID the opportunity to build a plural education, being capable of reflecting beyond the content to be taught.

Keywords: Didactic Suitability Criteria; Lesson Study; Initial teacher training.

Resumo

O presente artigo traz como tema problematizar sobre as ações de prática de ensino desenvolvidas no âmbito do Programa Institucional de Bolsa de Iniciação à Docência (PIBID) em uma universidade pública brasileira. Para tal intento, utilizam-se, como fundamentação teórica, autores que dialogam sobre Estudos de Aula e Critérios de Adequação Didática, bem como suas conexões com a formação inicial docente. A metodologia de pesquisa utilizada para tanto seguiu uma abordagem qualitativa, de natureza exploratória, com caráter documental. Infere-se que uma reflexão produzida em conjunto a partir de um contexto de Estudo de Aula, a qual envolveu professores-bolsistas, professores supervisores e a coordenação do subprojeto, com vistas ao atendimento dos Critérios de Adequação Didática, tornou o processo educacional qualificado e aperfeiçoado conforme as práticas aconteciam. Isso oportunizou aos acadêmicos do curso de Licenciatura em Matemática e que participaram do PIBID construir uma formação plural, sendo capazes de refletir para além do conteúdo a ensinar.

Palavras-chave: Critérios de Adequação Didática; Estudos de Aula; Formação Inicial de professores.

Introduction

This article materializes a journey that began in the first semester of 2020, more specifically from the research carried out by Hummes (2022). At that time, a partnership was established between two universities, one in Brazil (Federal University of Rio Grande do Sul) and the other in Spain (University of Barcelona), with the aim of developing, in a

Submetido em: 31/05/2024 – **Aceito em:** 15/08/2024 – **Publicado em:** 31/10/2024

¹ PhD in Informatics in Education from the Federal University of Rio Grande do Sul (UFRGS). Professor at the Federal University of Rio Grande do Sul, Brazil. E-mail: sychocki.rodrigo@gmail.com. ORCID: <https://orcid.org/0000-0002-7406-2517>

colaborative way, a training action with Basic Education teachers, which led to the researcher's doctoral thesis. At the time, professors from both universities and a group of teachers who worked in Basic Education schools participated in the research action. Hummes (2022) developed his doctoral thesis involving the following themes: (i) Lesson Study, (ii) Didactic Adequacy Criteria and (iii) Teacher Training.

In parallel, but not coincidentally, after an initial appropriation of the themes (i), (ii) and (iii) mentioned above, Franzen's (2022) academic research was carried out. His study (2022), structured on the same theoretical pillars addressed by Hummes (2022), took place in the context of Popular Education. At the end of the research, which culminated in an academic master's dissertation in Mathematics Teaching, the author reflects:

It is expected that teachers who participated in the course, especially those who completed it, will continue to promote Lesson Study in their groups and discuss this methodology with more professional colleagues, even though this increases the complexity of teaching by involving other teachers in the process. This would increase the reach of the planning, execution and reflection cycle technique in the teaching practice of an increasing number of teachers who teach Mathematics in Brazil (Franzen, 2022, p. 115).

From the experiences built through the research experiences mentioned above, in which the author of this article actively and engagedly participated, it was conjectured about the possibility of exploring such theoretical references in a prominent way in the field of initial teacher training. In this sense, it was understood that, in the implementation of the Institutional Teaching Initiation Scholarship Program (PIBID), funded by CAPES (Coordination for the Improvement of Higher Education Personnel), through notice number 23, of 2022, held at the Federal University of Rio Grande do Sul (UFRGS), there would be a fruitful opportunity to bring the group of students closer to the theoretical assumptions involving Classroom Study and Didactic Adequacy Criteria.

In the academic literature, it is possible to find studies such as Bezerra, Caetano and Peron (2022); Pinheiro and Soares (2023); and Bezerra, Caetano and Morelatti (2023), which present reflections involving an explicit approximation between the initial training of Mathematics teachers and Lesson Study. Similarly, we find in Carvalho and Pietropaolo (2015); Silva and Tinti (2021); and Frade and Tinti (2023) reflections that directly involve the initial training of Mathematics teachers and the Didactic Adequacy Criteria. These studies allow us to infer how pertinent it is, from the initial training of future Mathematics teachers, to reflect and relate the elements previously mentioned, in the tripod (i), (ii) and (iii).

In this sense, this article aims to analyze a cycle of teaching practice² that occurred in the PIBID – Mathematics subproject³ in 2023, in light of the work plan documents and

² It is understood, in this text, that the set of actions: meetings and referrals regarding planning, carrying out teaching practices with students in the field school and preparing, followed by recording, reflections in report documents, define the practice cycle.

³ Projects in specific areas of knowledge (Mathematics, Biology, Chemistry, Portuguese Language, English Language, Physics, among others) that are part of the PIBID Institutional Project.

reports produced by a group of “scholarship teachers”. This designation will be used in the text to name the university students who participated in PIBID actions and worked in field schools. The question that guides the work is: How is the Lesson Study process carried out by scholarship teachers influenced by the reflection on their practice in light of the Didactic Adequacy Criteria? The research methodology used for this purpose is qualitative (Silveira & Córdova, 2009), exploratory (Gil, 2002) and documental (Creswell, 2007).

The article is organized as follows: in the next section, there is the presentation, followed by a theoretical reflection on Lesson Study and Didactic Adequacy Criteria, with references that support the theoretical perspective of the study; next, the methodological aspects are outlined, with the materials and methods used; in the following section, an analysis of the empirical material is carried out, through the theoretical references already discussed. Finally, in the last section, the final considerations are presented, with the lessons learned, accompanied by a reflection on potential answers to the guiding question of this work.

Theoretical contributions: Lesson Study and Didactic Adequacy Criteria

The aim of this section is to bring theoretical ideas that: (I) will allow the reader to understand the methodological choices in the way activities are conducted in the PIBID – Mathematics subproject, from November 2022 to April 2024, at the Federal University of Rio Grande do Sul; and (II) will provide an opportunity to carry out and present an analysis of a practice cycle that occurred within the scope of the subproject. The section is based on ideas about Lesson Study and Didactic Adequacy Criteria, as well as their relationship with the teacher training area. Throughout the text, the works, researches and authors referred to in order to guide and articulate the thought of the section are specifically mentioned.

In this text, the understanding about the meaning of Lesson Study is in line with that outlined by Souza, Wrobel and Baldin (2018, p. 116), for whom this is:

[...] an action that started in Japanese schools and involves the training of teachers, in a shared and reflective way, aiming to meet the learning needs of their Mathematics students. This action comprises three main stages: planning, execution of the planned lesson and post-lesson reflection, all carried out jointly and aiming to create/expand knowledge about the students' learning processes and, simultaneously, promote the development of professional practice.

According to this premise, it is assumed that the tripod of planning, execution and reflection must permeate the context for carrying out a Lesson Study. In this sense, PIBID, since its genesis at the national context, in Brazil, in 2008, emphasizes that work in field schools must be developed collaboratively with the team of supervising teachers⁴. The focus of the actions must be on improving the processes related to the teaching profession, with special attention to the comprehensive development of school's students. In a Lesson Study

⁴ Supervisor: the teacher at the partner school who is part of the Institutional Project, and is responsible for monitoring and supervising the activities of the teaching initiation scholarship holders (Source: Portaria CAPES nº 90, de 25 de Março de 2024).

process, this is of utmost importance, since planning must enhance the learning processes and, at the same time, promote the development of the professional practice of those involved. Professor Catherine Carol Lewis, interviewed during the II International Seminar on Lesson Study in Mathematics Teaching (II SILSEM), held in 2023, argues that, in general, the aim of Lesson Study is to achieve:

A way to build knowledge about student thinking; an effective pedagogy; and an effective curriculum. These are goals often pursued by educational researchers outside Japan, but most commonly sought in Japan, through collaboration with school and university educators (Souza, Baldin & Lewis, 2023, p. 7).

As previously stated, several actions, including those already mentioned in the introduction, have been taking place around the world, and, when constructing practices, it is necessary to take into account the students, the curriculum, and the school community, without, however, hierarchizing them. Although Lesson Study is, in a number of texts found in the academic literature, a work methodology developed over a cycle, it was identified and typified, during the execution of the PIBID – Mathematics (2022-2024) subproject, as a spiral-shaped model, as assumed by Gaigher, Souza, and Wrobel (2017). From this model, it is possible to understand that Lesson Study, once initiated for a given topic related to teaching, becomes a complex object to be thought about and carried out. This interpretation of the model of understanding Lesson Study also appears in the research carried out by Franzen (2022).

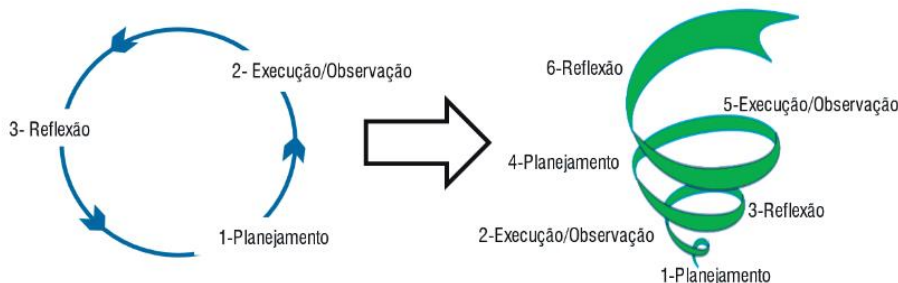


Figure 1 – From Cycle to Spiral in a Lesson Study.

Source: Gaigher et al. (2017, p. 55).

From this perspective, according to the authors, it is essential that school students take an “active stance towards their learning, sharing ideas, reasoning, and doubts with their colleagues and teachers” (Gaigher et al., 2017, p. 55). Therefore, an increase in the complexity of teaching throughout a Lesson Study is related to the active engagement of all participants in the process: students, teachers (in training or at the school), and the community. Lesson Study, as a methodological possibility, can be carried out during initial teacher training, configuring itself as an even mechanism that can group different types of knowledge, which are apparently isolated in the curriculum schedules provided by training institutions. In this regard, Ponte and Quaresma (2022, p. 341) argue that:

Lesson Study requires the mobilization of content knowledge, in terms of concepts, procedures, task-solving strategies and mathematical representations, and of didactic

knowledge, such as lesson planning, task selection and analysis of students' thought and reasoning processes, as well as communication in the classroom. Lesson study, which is developed around the preparation, execution and reflection of a lesson, offers a remarkable possibility of linking theory and practice. In addition, it allows the mobilization of different theoretical perspectives on initial teacher training.

Therefore, by mobilizing, involving and relating different types of knowledge in a Lesson Study, it is possible to re-signify teaching and, consequently, the teaching and learning processes. It is with this tone that the Didactic Adequacy Criteria assume an essential role in the teacher training process. They are part of a broader theoretical framework, called the Ontosemiotic Approach to Cognition and Mathematical Instruction, which can be found in Godino, Batanero and Font (2007). It is assumed, in this article, that Didactic Adequacy is related to teaching practices and the elements that circumscribe it (means, resources, methods, etc.), in such a way that, from them, it is possible to reflect and infer about the quality of learning achieved by students, through the interrelations among the elements available, by the teachers (Breda, Font & Pino-Fan, 2018).

In the academic literature, it is possible to find works, over time, such as Godino, Bencomo, Font and Wilhelmi (2007); Ramos and Font (2008); Breda (2016); Hummes, Seckel and da Silva (2023); and Breda, Hummes, Silva and Sánchez (2021), in which various explanations and applications of the Didactic Adequacy Criteria occur in the most diverse training spaces. To reach the aim of this article, however, it is enough to understand that the following six criteria are used, according to Godino (2013): epistemic, cognitive, ecological, affective, interactional and mediational. The author also argues that such criteria should not be considered independent or isolated, since there are interactions between them and their multiple components. Although this is not the focus of this text, it is understood that it is pertinent, at least, to mention it.

For each of the criteria, Godino (2013) presents components and indicators, which can be used to observe how much a criterion is being achieved within a context of didactic proposition. During the implementation of the PIBID subproject – Mathematics (2022-2024), the focus of this text, each of the criteria was explored by the scholarship teachers in a more comprehensive way, according to the characteristics presented in Chart 1, below.

Chart 1 – Didactic Adequacy Criteria explained in the PIBID subproject – Mathematics.

Criterion	Characteristic
Epistemic	Refers to the degree of representativeness of the institutional meanings implemented or intended, in relation to a reference meaning.
Cognitive	Expresses the degree in which the intended/implemented meanings are in the potential development area of the students, as well as the degree of proximity between the personal meanings achieved and the intended/implemented meanings.
Ecological	Degree in which the study process adjusts itself to the educational project, the school, society and the environment where it is developed.
Affective	Degree of student involvement in the teaching process. It is related to factors that depend on the institution, the student and their previous school history.
Interactional	It is related to the teaching and learning process. It focuses on the identification and resolution of semiotic conflicts that arise during the teaching process.

Mediational	Degree of availability and adequacy of the resources demanded for the development of the teaching and learning process.
-------------	---

Source: Braga and Santos-Wagner (2021).

Hummes, Breda and Font (2022, p. 59), in general terms, present the above criteria, being that, the first of which, epistemic, deals with the reflection on the quality of Mathematics taught to students. The cognitive criterion refers to the students' degree of proximity to the object of study that is intended to be taught and vice versa; the ecological aspect refers to a reflection on the conditions circumscribed to the social and professional context, with an assessment of the established educational project. The affective criterion aims to assess the interests and motivations of students when experiencing the teaching process; the interactional adequacy questions whether it is possible to overcome/bypass the obstacles experienced by students in the educational process; and, finally, the mediational aspect seeks to observe whether the resources, both material and temporal, used in the teaching process, were adequate.

Each of these six criteria was reflectively explored by the group of academics participating in the PIBID – Mathematics subproject (2022-2024) at a later stage of the practice, as highlighted in step (3) of figure 1. It is understood that these criteria are important elements for outlining the characteristics inherent to the teaching practice process, limited to contexts involving Lesson Study, as presented in the first part of this section. In a similar way to what is exposed by the excerpts from academic literature, work methodologies involving Lesson Study, accompanied by the Didactic Adequacy Criteria, can enhance the construction of collaborative and reflective teaching practices, as it is intended to be approached in the next sections.

Methodological contributions: foundations, materials and methods

The work began in the PIBID – Mathematics subproject, at the Federal University of Rio Grande do Sul, in November 2022, and completed its activities at the end of April 2024. During this period, a total of twenty-six students (scholarship teachers) participated in the program, working in three field schools – two state and one federal institution. In each of the field schools, there was a Mathematics teacher in the role of supervisor, being responsible, on average, for a group of eight students.

During this period, the whole group created plans, called “work plans.” These documents were the result of a collective creation, involving small groups of students, a supervising teacher, and an area coordinator⁵. There were minimum guidelines for organizing the plan, which was a constant element of dialogue between the team of students and teachers involved. While they were creating the plans, all the students experienced the reality of the classroom, immersed in the classes in which the plans would be put into practice. This

⁵ Area Coordinator: the university professor responsible for planning, organizing and guiding teaching initiation activities in his/her area of academic activity (Source: Portaria CAPES nº 90, de 25 de Março de 2024).

allowed the students, within a specific context, to learn about, in the form of bilateral belonging to that community, the characteristics of the classes of students in which the work was being developed. Chart 2 summarizes the elements that were part of the document model organized by the students.

Chart 2 – Elements and their characteristics in the work plan.

Element	Characteristics
Activity title	Name the proposed activity or sequence of activities.
Activity summary	Write a summary, with a maximum of 7 lines, about the proposed activity or sequence of activities.
General objective	Mention the general objective of the proposed activity or sequence of activities.
Specific objectives	Mention the specific objectives of the proposed activity or sequence of activities.
Mathematical concepts present in the activity	Mention which Mathematics content is included in the proposed activity or sequence of activities.
Cross-cutting or interdisciplinary concepts / Contemporary themes present in the activity	Mention which cross-cutting or interdisciplinary content is included in the proposed activity or sequence of activities. Or, the “Contemporary cross-cutting themes in the BNCC”.
Target audience	Mention the target audience that participated in the proposed activity or sequence of activities.
Why learn this content?	Write the reasons why you think it is appropriate for the students in the classes you teach at the field school to learn this content. This differs from a more formal justification, as you should present arguments of a personal and subjective nature.
Mobilized BNCC Skills	Cite/list the skills from the BNCC (National Common Curricular Base) that are being mobilized/used in the activities of this work plan.
Required resources or materials	Write down the materials used or resources needed to prepare/execute the proposed activity or sequence of activities.
Methodological design of classes/meetings	<p>This is the “core” of the work plan!</p> <p>Present an organization of your pedagogical work. It is the writing of your actions! To do so, you should organize it as follows:</p> <p>Lesson X, Y, ..., Z (as we discussed, the “lesson” can be defined based on every meeting you will have with the students).</p> <p>Estimated time:</p> <p>Activities</p> <p>Moment 1: (...)</p> <p>Moment 2: (...)</p> <p>Moment 3: (...)</p> <p>...</p> <p>Moment “n”: (...)</p> <p>You can link or mention information that is in the appendices.</p>
Assessment	Present a form of assessment for this activity or sequence of activities. The idea is to explain how students would be assessed, or which instruments can be used within a process of assessing students in the development of the class or sequence of activities.
Bibliography	Present the bibliography, with the list (according to ABNT formatting) of the materials used in the construction of your planning.

Source: the author.

Chart 2 might give one the idea that the work plan structure was had a complex organization, involving the elaboration of a pedagogical architecture of the meetings with students. All the information was constructed collectively and involved the subgroups of scholarship teachers, supervision and coordination of the subproject, that is, the planning encompassed at least three groups with different functions within the same context. The wealth of perspectives and ways in which the proposals were elaborated was consistent with the previous immersion in the performance of the scholarship teachers in the classes, in which the educational reality and circumscribed to the classroom was also considered a structuring element in the planning process.

After the set of classes/meetings contained in the work plan had taken place, the time for elaborating the reflections began. These reflections would be recorded in documents entitled “Reflections/report on the work plan” and “Reflections/analysis on student production”. Both documents aimed to record, through collective writing in the form of memories, the impressions, learning and other elements experienced in practice that could contribute to a possible reapplication of the work plan or in the elaboration of the subsequent materials.

Throughout the entire period of implementation of the PIBID – Mathematics subproject, thirty cycles with a view to teaching practices were outlined. It is understood that these cycles were composed of the following set of actions: meetings and referrals regarding planning; implementation of teaching practices with students in the field school; and preparation, followed by recording, of reflections in report documents. Over time, the subgroups of scholarship teachers were modified, for reasons specific to the interested parties or the need to adapt to the days and times of classes in the field schools. This provided an opportunity for the working groups to share experiences in a heterogeneous and fruitful manner.

With the conclusion of the PIBID – Mathematics subproject, it was possible to reflect and present to the academic community an example of a path taken, within the thirty planning possibilities that were constructed and are available⁶. Lesson Study, highlighted since the beginning of this text, besides being a theoretical emphasis, fully permeated the work developed in this last edition of the subproject carried out at the Federal University of Rio Grande do Sul. The Didactic Adequacy Criteria served as an epistemological compass, used to understand the relationships and interrelations in light of the facts that occurred during the practices. In view of this, it is important to recall the question presented in the introduction of this article and which guides this research: How is the Class Study process carried out by the scholarship teachers influenced by the reflection on their practice in light of the Didactic Adequacy Criteria? To explore this question, the research methodology is qualitative (Silveira & Córdova, 2009), exploratory (Gil, 2002) and documental (Creswell, 2007). Silveira et al. (2009, p. 34) define qualitative research as:

⁶ All materials related to teaching practices can be accessed freely and unrestrictedly at: <https://www.ufrgs.br/pibid-mat2022/> (Accessed in August 2024).

It attempts to understand the phenomena as a whole, rather than focusing on specific concepts; it has few preconceived ideas and emphasizes the importance of interpretations of events rather than the researcher's interpretation; it collects data without formal and structured instruments; it does not attempt to control the research context, but rather to capture the context in its entirety; it emphasizes the subjective as a means of understanding and interpreting experiences; it analyzes the narrated information in an organized but intuitive way.

Unlike a quantitative approach, it is understood that the one used is qualitative, as it respects the limits and context of the scopes in which the materials on the practice were designed and carried out. As empirical material, the documents “Reflections/report on the work plan” and “Reflections/analysis on student production”, part of the work plan entitled “Exploring the Art of Origami⁷: Geometry in Flat Figures”, were selected. These documents were selected for the following reasons: (i) these were reports of the last teaching practice that took place in one of the field schools and (ii) the group of scholarship teachers involved had started their participation in the program in November 2022, having participated in at least three complete cycles of teaching practice. The other two field schools began PIBID activities in June 2023, with the first cycles of planning, practice, and reflection ending around August/September of that year. It was thus seen that the documents produced in these other two schools acquired reflective notoriety only after at least one complete cycle of practice had been completed. With this, the set of documents to be brought and analyzed in this text was defined.

This study, which has a qualitative approach, is also exploratory, since its “main objective is to improve ideas or discover intuitions. Its planning is, therefore, quite flexible, so that it allows the consideration of the most varied aspects of the fact studied” (Gil, 2022, p. 41). In this sense, it is understood that its nature is documental, as it allows the researcher to “obtain the language and words of the participants; it can be accessed at a convenient time for the researcher; and it represents reflected data, to which the participants dedicated attention to compiling” (Creswell, 2007, p. 192). The materials presented in the next section, in which the analysis occurs, were constructed by a trio of scholarship teachers, who show, through their own language, the impressions and reflections constructed by the group after the moment of carrying out the teaching practices.

Highlighting two aspects is of pivotal importance: that the respective supervising teacher responsible for the group of scholarship teachers working at the field school also dialogued with them before/during/after the teaching practice sessions; this open and bilateral dialogue constituted a key element in the actions taken during the subproject. Also, that the scholarship teachers were free to reflect on any aspects of the practices that they considered worth addressing. There was no room for omissions, and all ideas and arguments were, as far as possible, welcomed recorded. With this in mind, the next section attempts to carry out the

⁷ It is a Japanese word composed of the verb to fold (折り=ori) and the noun paper (紙=kami). It literally means, “to fold paper”. Source: https://www2.ibb.unesp.br/Museu_Escola/Ensino_Fundamental/Origami/Documentos/indice_origami.htm (Accessed in August 2024).

analysis, based on the Didactic Adequacy Criteria framework, with a view to recognizing the potential of the work developed through the planning-action and didactic-reflection cycle, during the implementation of the PIBID – Mathematics subproject.

Results and discussion

This section aims to present an analysis of the information contained in the documents “Reflections/analysis on student production” and “Reflections/report on the work plan”, which are part of the teaching practice cycle carried out in the work plan entitled “Exploring the Art of Origami: Geometry in Flat Figures”. These materials were produced by a trio of scholarship teachers. The members of this group, individually, freely and consentingly expressed, through answers via e-mail, their agreement with the use of the productions for the preparation of this article, as well as their consent for this to be done.

Regarding the document “Reflections/analysis on student production”, the topics “Relevant data related to student performance”, “Difficulties presented”, “Revisiting the work plan” and “Other aspects considered relevant” are presented. These items, written by the trio of scholarship teachers, reveal, in a reflective tone, a set of elements that allow us to understand how valid it is, in a sequential manner, to pay attention to the stages corresponding to the Lesson Study spiral (Gaigher et al., 2017), depicted in figure 1. Chart 3, below, contains the elements, followed by an analysis. It is also worth noting that, in the document, there are other topics, of a technical nature, which revisit some information already included in the work plan.

Chart 3 – Elements and their characteristics in the work plan.

Element	Characteristics
Relevant data related to student performance	<p>(...)</p> <p>At the scholarship teacher PPP’s stand, most of the students showed interest in folding the box, probably because they were curious to know how it was possible to create a box from folding. During the visits, it was possible to work on some mathematical concepts, in a superficial way, for example, horizontal and vertical (in the direction of the folding), vertices and nomenclature of convex polygons, which were well perceived. Some results were very close to the expectations for the box, while other students realized that their boxes did not have some apparent properties of the model: edges perpendicular to the base, square base, “tabs” from the last folding stage meeting at the center of the base, among others.</p> <p>At the scholarship teacher NNN’s stand, all the students who passed by were very excited and interested in making Origami of a swan and of a fox. Some of the students even made both Origami in more than one color and in more than one size. As soon as the visits began, most of the students started at the scholarship teacher NNN’s stand, where they wanted to make the two Origami, a swan and a fox. In this way, it was possible to initially work on how to make a square from an A4 sheet of paper, since the basis for making an Origami is a square sheet. Thus, as the folds of each of the Origami were developed, it was possible to work on some mathematical notions, in a superficial way, for example, geometric</p>

	<p>figures and vertices. Most of the constructions resulted in a swan and a fox, but some students, when they finished, realized that they had not managed to get a swan, for example, a fact that occurred because the folding was done in reverse, or even, the folding of the right triangles was not done so that they met and formed a larger triangle.</p> <p>At the scholarship teacher AAA's stand, most of the students showed enthusiasm when they got involved in creating the Origami of the butterfly and the bat. This active participation can be attributed to the remarkable similarity between the resulting figures and the original animals. The experience allowed them to explore some mathematical concepts, since the Origami involved simple folding, enabling the presentation of notions such as proportion, related to the squares that were formed, as well as concepts of parallel and concurrent lines. In addition, it was possible to address the relationship between some internal angles, which were inserted in a brief manner throughout the development of the Origami.</p> <p>(...)</p>
Difficulties presented	<p>The students' main difficulties appeared when they were reproducing the folding procedures, especially regarding the notions of parallelism and angle between folds and/or edges of the square. It was noticed that some students constructed their initial squares without worrying about the need for coincidence between the sides that were being brought together by the formation of the folding crease, and others did not realize that the quadrilaterals formed throughout the process were not squares, much less congruent polygons. This led to the problems with the results in relation to the arts, already mentioned in the previous topic.</p> <p>The analysis of this kind of difficulty leads to the conjecture of the existence of a gap in the students' abstract thinking, since, when in contact with the Origami, they did not apply some knowledge that they had worked on previously, failing to observe incongruities or the consequences of incorrect folds, through the association of their own actions with what had already been constructed.</p> <p>Another difficulty observed, already expected, is related to the students' understanding of the printed diagrams available at the stands. This observation shows the importance of media as a form of representation for learning, so that a static image may sometimes not be able to play the role of fully representing a movement. Thus, simply observing the diagrams represented a challenge for the students.</p>
Revisiting the work plan	<p>In order to reconsider the work plan, it is understood as necessary to include one more topic to develop students' skills in the practice of Origami, focusing on the understanding of the concepts of parallelism, angles and congruence, as well as the effective interpretation of diagrams. We could, through the practices, reflect on and encourage students to observe their own actions and identify possible incongruencies and, in addition, integrate interactive activities that allow students to explore the diagrams in a dynamic way. This could involve the use of interactive applications, augmented reality, or even the creation of animated diagrams.</p> <p>Also, a specific moment could be created, in order to mitigate difficulties of interpreting the symbols contained in the diagrams, which is something that can present itself, including for us, teachers - demanding better prior preparation.</p>

	<p>Thus, by addressing the identified difficulties in a more proactive and engaging way, we would provide students with a more meaningful and enjoyable learning experience in the context of Origami, promoting not only the practice of folding, but also the development of abstract thinking and the ability to apply prior knowledge.</p>
Other aspects considered relevant	<p>It is rewarding to observe the success achieved in exploring the fascinating universe of paper art. The students demonstrated engagement, overcoming challenges and effectively constructing/revising the concepts covered. We can observe collective and individual achievements, such as the development of motor skills and the identification and overcoming of difficulties initially observed during the folding process. This demonstrates the students' ability to face challenges, as well as their ability to articulate mathematical concepts already constructed with their actions and in reflecting on their actions. The resulting attention to detail and error correction demonstrate the class' commitment to the quality of the final work.</p> <p>Besides, we noticed the students' remarkable interest in carrying out the constructions of the proposed Origami, as they wanted to understand and know how they would achieve the result using only a piece of square paper and several folds.</p> <p>In short, the success of this lesson plan lies not only in the Origami figures that were created, but in the experiences shared, the challenges overcome, and the skills developed by the students throughout this educational process. This is just the beginning of a journey that we hope will continue to inspire the pursuit of knowledge and creative expression.</p>

Source: research archive. Author's emphasis added.

Chart 3 shows, based on the passages emphasized, how latent the reflective exercise was for the trio of scholarship teachers, henceforth the elaboration of the document entitled “Reflections/analysis on student production”. From the perceptions of the three scholarship teachers, immersed in the context of this teaching practice involving folding and Origami, there is a sample of elements that precede and follow the practice. Considering a plan, prepared from a set of previous characteristics, it can be observed, in the chart above, that the group recovers in some detail the elements of the classes. From this, it can be problematized that there is a reflection, by the group, that guides a consensus with a view to improving the teaching and learning processes of the subjects registered/circumscribed to teaching practice, which is perfectly aligned with what Ponte et al. (2002) assert within the scope of initial teacher training, which is inserted in a context of Lesson Study, being a possibility of connection between theories and practice.

When reading the data emphasized in Chart 3, it is possible to perceive a narrative constructed with a reflective tone, thus converging with Ferreira de Souza et al. (2018, p. 116), for whom, during a Lesson Study, the stages of “planning, execution of the planned lesson and post-class reflection must occur, all carried out together and aiming to create/expand knowledge about the students’ learning processes”. This is expressed by the trio, among other passages, in chart 3 above, in the excerpt “This is just the beginning of a journey that we hope will continue to inspire the pursuit of knowledge and creative

expression”. This shows, to a certain extent, a type of formative and social commitment that PIBID, as a national program, has to provide, simultaneously, to university students involved, since the beginning of their initial training, to reflect on characteristics related to professional development of teachers. In this sense, it is understood that PIBID's social commitment lies in the tripod of planning/carrying out/reflecting on practices, which invite school students to redefine their relationship with scientific knowledge.

Regarding the document “Reflections/report on the work plan”, it is presented how the trio of scholarship teachers structured the reflections produced after the practical moment, a moment in which they proposed to relate the teaching practice with the Didactic Adequacy Criteria presented in the theoretical framework. Chart 4, below, brings excerpts from the document, organized according to the respective criterion. The passages emphasized aim to highlight the evidence observed regarding the respective criterion. In order to limit the length of the article, two excerpts related to each of the criteria are presented, making it possible to observe a reflection, by the trio of scholarship teachers, on how such criterion was put into practice during the teaching practice.

Chart 4 – Analysis of practice using the Didactic Adequacy Criteria.

Adequacy Criteria	Observation/Analysis
Epistemic	<p>(...) However, although not explored in detail by the students in the textual production activity, the development of mathematical concepts seemed to be present, since the students mentioned, for example, the articulation of (...) polygons, angles and parallelism. This shows that the set of activities, due to its rich epistemic nature, was able to highlight intersections between Art and Mathematics. (...)</p> <p>(...) Through the Origami workshop, we aimed that students could create different types of constructions and relate folding and geometric shapes to the content learned throughout the year. From this, we understand that both of these objectives were successfully achieved, since most students visited all the stands and managed to build each of the Origami that was being presented. (...)</p>
Cognitive	<p>(...) In this way, we understand that, through the Origami workshop, students were able to recall the concepts of Plane Geometry learned previously, reflect on the importance of making each of the folds correctly and analyzing whether they are congruent, because, if this is not done, the result will be different from what was expected. The activities were fully cohesive with the concepts worked on, so students did not have difficulties in highlighting the concepts worked on in the textual production activity or in the discussion of the investigative questions.</p> <p>In relation to the textual productions, specifically, it was possible to analyze that the students revisited concepts learned previously, expressed their opinions about the class and the workshop, and, in addition, demonstrated that they really liked the approach used, which allowed them to decide which Origami they wanted to make or not. The writing activity, therefore, strengthened the meaning of the whole moment, instigating the students to understand the applicability of the constructed concepts.</p>
Ecological	The contents related to the work plan analyzed are in accordance

	<p>with the curricular guidelines, and the activity was significant in the context of the interdisciplinarity between Art and Mathematics, so that the students could be in contact with a pedagogical resource with formative potential. The art of Origami, in the context of Brazilian education, can be a path to the development of sensitivity and respect for a distinct culture. The opening moment, with a talk about the history of Origami, was an attempt in this direction.</p> <p>Besides, the activity represented a moment of articulation of several mathematical concepts in the field of Geometry, among themselves, and with motor, visual and abstraction skills. We understand, with this, that a formative moment was offered to the students, through the perception that they can perform an interpretation of the world from the perspective of Mathematics. (...)</p>
Affective	<p>In the initial stage of the presentation, which addressed the fundamental concepts related to Origami, the students were highly engaged and participated significantly in the development process. In this context, many of their responses had a great contribution to the progression of the constructed reasoning and added pertinent elements to the discussion.</p> <p>The students were very excited about the Origami workshop, showing happiness, interest and motivation, as most of them completed the construction of all the Origami presented and, even if some difficulties arose during the folds, everyone was helping each other so that they could reach the final fold. (...)</p>
Interactional	<p>The activity proposal was based on a methodology that made it possible to easily provide room for interaction and development between scholarship teachers and students. The environment was built collectively, and the Origami activity was based on individual production, reinforcing the aspect of autonomy and socialization of strategies to achieve the intended result.</p> <p>Regarding the interaction between subject (students) and object (sheet of paper), the Origami workshop was essential, constantly demanding that the students act on the material and understand the consequences of this action, mathematically recognizing what was happening and comparing it with expectations (their own or those of the references from the scholarship teachers).</p>
Mediational	<p>The activity from the work plan developed involved a contextualization of the approach to mathematical concepts using the art of Origami, so that the contact and manipulation of the material represented a moment of significant learning. The material used, especially the paper, was available all the time for the application of the activity, allowing, consequently, the application of the activity in a significant manner.</p> <p>The other conditions for the development of the activity were sufficient, as well as the time available - since the students were able to circulate around all the stands to explore the different Origami and think calmly about the textual production - and the layout of the room - which did not allow incidents due to simultaneous activities.</p>

Source: research archive. Author's emphasis added.

The excerpts shown in Chart 4, above, may lead to the perception of a reflection guided and based on the actions carried out during the teaching practice. According to Godino (2013), such criteria are not limited to six; however, it is possible to perceive a plenty of details in the document produced by the trio under issue in the qualification of their teaching practice and establishment of relations with the aforementioned criteria. The organization of materials, time planning, synchronicity of actions, observation of the actions of the school students during the practice, among other characteristics, can be observed in the excerpts highlighted in the chart above.

In each passage of the document “Reflections/report on the work plan”, it is possible to observe that the trio of scholarship teachers is concerned to carry out their action with zeal and equity, demanding themselves to be fully commitment to develop an appropriate practice that could qualify the processes of teaching and learning Mathematics, taking into account the reality of which they are a part, as stated by Breda et al. (2018). It is worth highlighting that the use of criteria (Hummes et al., 2022) that can qualify pedagogical action is not limited to a single set of six (Godino, 2013); as highlighted in the theoretical framework, there may be other variables that circumscribe the process of teaching practices. In this sense, the analysis performed in this section, which used the Lesson Study and Didactic Adequacy Criteria references, is followed by the final considerations, which explore the resumption of the problem addressed within the scope of the research.

Final considerations

This last section can be opened bringing the feeling that a scenario has been created, a favorable environment to answering the question presented in the introduction to this article, namely: How is the Lesson Study process carried out by scholarship teachers influenced by reflection on their practice in light of the Didactic Adequacy Criteria? Even though this text has brought a cutout/example in a set of thirty practice cycles that took place at PIBID – Mathematics, from November 2022 to April 2024, it is possible to see that Lesson Study, as proposed by Gaigher et al. (2017), constituted, in the case under issue, a fruitful methodological resource for relating the stages of the tripod planning/carrying out/reflecting on teaching practices since initial training.

Furthermore, a joint reflection produced from a Lesson Study context, capable of involving scholarship teachers, supervising teachers and subproject coordination, guided by the observation of compliance with the Didactic Adequacy Criteria (Godino, 2013), made the educational process qualified and improved as the practices took place. This can be observed from a progressive improvement in the quality of the report documents prepared by the groups of scholarship teachers who carried out more than one complete practice cycle, making a comprehensive coverage from planning to reflection.

In short, the use of criteria, to some extent, required attention and observation, by the scholarship teachers, in the processes adjacent to the teaching and learning of those involved in the classroom environment. One lesson learned is that, by knowing and exploring the

Didactic Adequacy Criteria since the initial training of teachers, it is possible to give the academics the opportunity to build a plural and more comprehensive training, in observance of the aspects that go beyond the Mathematics content to be taught.

Acknowledgements:

To CAPES, for the financial resource made available through Notice No. 23/2022 for the execution of the PIBID – Mathematics subproject (2022-2024), within the scope of the Federal University of Rio Grande do Sul (UFRGS). To Professor Dr. Lúcia Rottava for successfully conducting the PIBID – UFRGS project contemplated in Notice No. 23/2022, in the role of Institutional Coordinator.

References

- Bezerra, R. C., Caetano, R. S., & Peron, L. D. C. (2022). Lesson Study na Formação Inicial de Professores: uma Experiência no Projeto Pibid/Matemática. *Jornal Internacional De Estudos Em Educação Matemática*, 15(2), 132–141. <https://doi.org/10.17921/2176-5634.2022v15n2p132-141>
- Bezerra, R. C., Caetano, R. S., & Morelatti, M. R. M. (2023). Un estudio de clase en el contexto de PIBID: desafíos y posibilidades para la Formación Inicial de profesores brasileños de Matemáticas. *Paradigma*, 44(2), 293-316. <https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p293-316.id1421>
- Braga, N. H., & Santos-Wagner, V. M. P. (2021). Utilização de ferramentas de idoneidade didática para análise do conceito de função em livros didáticos. *Revemop*, 3, e202127. <https://doi.org/10.33532/revemop.e202127>
- Breda, A., Font, V., & Pino-Fan, L. R. (2018). Criterios valorativos y normativos en la Didáctica de las Matemáticas: el caso del constructo idoneidad didáctica. *Bolema: Boletim De Educação Matemática*, 32(60), 255–278. <https://doi.org/10.1590/1980-4415v32n60a13>.
- Breda, A. (2016). *Melhorias no ensino de matemática na concepção de professores que realizam o mestrado profmat no Rio Grande do Sul: uma análise dos trabalhos de conclusão de curso*. Tese de Doutorado em Educação em Ciências e Matemática. Porto Alegre: Pontifícia Universidade Católica do Rio Grande do Sul.
- Carvalho, M. P., & Pietropaolo, R. C. (2015). Perspectivas de estudantes da licenciatura em matemática sobre o programa de bolsas de iniciação à docência. *Práxis Educacional*, 11(19), 171-190. <https://periodicos2.uesb.br/index.php/praxis/article/view/826>
- Creswell, J. W. (2007). *Projeto de Pesquisa: métodos qualitativo, quantitativo e misto*. Porto Alegre: Artmed.
- Gaigher, V. R., Souza, M. A. V. F. de, & Wrobel, J. S. (2017). Planejamentos colaborativos e reflexivos de aulas baseadas em resolução de problemas verbais de matemática. *VIDYA*, 37(1), 51–73. <https://periodicos.ufn.edu.br/index.php/VIDYA/article/view/1929>
- Gil, A. C. (2002). *Como elaborar projetos de pesquisa?* São Paulo: Atlas.
- Godino, J. D., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in

- mathematics education. *ZDM*, 39(1–2), 127–135.
- Godino, J. D., Bencomo, D., Font, V., & Wilhelmi, M. R. (2007). Análisis y valoración de la idoneidad didáctica de procesos de estudio de las matemáticas. *Paradigma*, 27(2), 221-252.
- Godino, J. D. (2013). Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas. *Cuadernos de Investigación y Formación en Educación Matemática*. 8(11), 111-132.
- Hummes, V., Breda, A., & Font, V. (2022). Critérios de adequação didática implícitos na reflexão de professores quando planejam, implementam e redesenham uma aula em uma experiência de Lesson Study. In A. Richit, J. P. Ponte & E. S. Gómez (Orgs.), *Estudos de aula na formação inicial e continuada de professores* (pp. 53-88). São Paulo: Livraria da Física.
- Breda, A., Hummes, V., Silva, R. S. da, & Sánchez, A. (2021). El Papel de la Fase de Observación de la Implementación en la Metodología Estudio De Clases. *Bolema: Boletim de Educação Matemática*, 35(69), 263–288. <https://doi.org/10.1590/1980-4415v35n69a13>
- Hummes, V., Seckel, M. J., & da Silva, R. S. (2023). Diseño de un curso de formación que articula los Criterios de Idoneidad Didáctica y el Estudio de Clases como herramienta para desarrollar la reflexión sobre la práctica de profesores de matemáticas. *Paradigma*, 44(4), 221-245. <https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p221-245.id1395>
- Frade, I. M. S. de A., & Tinti, D. da S. (2023). O Programa Residência Pedagógica e a mobilização do conhecimento metadidático: uma análise focalizando a adequação de meios. *Paradigma*, 44(4), 390-408. <https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p390-408.id1390>
- Franzen, T. (2022). *O Estudo de Aula no contexto da formação de professores na Educação Popular: uma análise a partir dos Critérios de Idoneidade Didática*. Dissertação de Mestrado em Ensino de Matemática. Porto Alegre: Universidade Federal do Rio Grande do Sul.
- Hummes, V. (2022). *Uso combinado del Lesson Study y de los Criterios de Idoneidad Didáctica para el desarrollo de la reflexión sobre la práctica en la formación de profesores de matemáticas*. Tese de Doutorado em Educação. Barcelona: Universidade de Barcelona.
- Pinheiro, J., & Soares, G. (2023). Uma Orquestração Instrumental para o ensino de porcentagem no 7º Ano do Ensino Fundamental Anos Finais: um relato de experiência no contexto do PIBID. *Revista Insignare Scientia - RIS*, 6(6), 319-335. <https://doi.org/10.36661/2595-4520.2023v6n6.13682>
- Ponte, J. P., & Quaresma, M. (2022). Estudos de aula na formação inicial de professores de Matemática In A. Richit, J. P. Ponte & E. S. Gómez (Orgs.), *Estudos de aula na formação inicial e continuada de professores* (pp. 337-362). São Paulo: Livraria da Física.
- Ramos, A. B., & Font, V. (2008). Criterios de idoneidad y valoración de cambios en el proceso de instrucción matemática. *Revista latinoamericana de investigación en*

matemática educativa, 11(2), 233-265.

Silva, J. F. da, & Tinti, D. da S. (2021). Planejamento de espaços formativos e a mobilização do Conhecimento Didático-Matemático: um olhar para o Programa Residência Pedagógica. *Revemop*, 3, e202136. <https://doi.org/10.33532/revemop.e202136>

Silveira, D. T., & Córdova, F. P. (2009). A Pesquisa Científica. In T. E. Gerhardt & D. T. Silveira (Orgs.), *Métodos de pesquisa* (pp. 31-42). Porto Alegre: Editora da UFRGS.

Souza, M. A. V. F. de, Baldin, Y. Y., & Lewis, C. C. (2023). From Japan to the United States: an interview with Catherine C. Lewis about Lesson Study. *Revista Paranaense De Educação Matemática*, 12(29), 5–16. <https://doi.org/10.33871/22385800.2023.12.29.5-16>

Souza, M. A. V. F. de, Wrobel, J. S., & Baldin, Y. Y. (2018). Lesson Study como Meio para a Formação Inicial e Continuada de Professores de Matemática - Entrevista com Yuriko Yamamoto Baldin. *Boletim GEPEN*, (73), 115–130. <https://doi.org/10.4322/gepem.2018.020>