UNIVERSIDADE FEDERAL DO RIO GRANDE DO SULFACULDADE DE CIÊNCIAS ECONÔMICAS
PROGRAMA DE PÓS-GRADUAÇÃO EM ECONOMIA

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DYNAMIC VINE COPULAS FOR TAIL RISK ASSESSMENT IN ENERGY COMMODITIES

Porto Alegre - RS

2022

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Dissertação apresentada à Universidade Federal do Rio Grande do Sul como parte das exigências para a aprovação do título de Mestre em Economia Aplicada.

Orientador: Flávio Augusto Ziegelmann

Porto Alegre - RS

2022

CIP - Catalogação na Publicação

Abreu, Ludyson Ramon
Dynamic Vine Copulas for Tail Risk Assessment in Energy Commodities / Ludyson Ramon Abreu. -- 2022.
26 f.
Orientador: Flávio Augusto Ziegelmann.

Dissertação (Mestrado) -- Universidade Federal do Rio Grande do Sul, Faculdade de Ciências Econômicas, Programa de Pós-Graduação em Economia, Porto Alegre, BR-RS, 2022.

1. Vine copulas. 2. Generalized autoregressive score. 3. Energy commodities. 4. Tail risk. I. Ziegelmann, Flávio Augusto, orient. II. Título.

Elaborada pelo Sistema de Geração Automática de Ficha Catalográfica da UFRGS com osdados fornecidos pelo(a) autor(a).

ABSTRACT

In this paper, we seek to measure and forecast tail risk for a energy commodities portfolio. To address this challenge, we propose a dynamic D-vine copulas. This model allow us to capture the complex dependence structures of energy commodity returns, while also accommodating their specific characteristics, such as asymmetries and heavy tails. We also use generalized autoregressive score models as an updating mechanism for the copula parameters, which allows us to incorporate time-varying dependence structures and use the information about the copula distribution to improve parameter estimation. Our results demonstrate that the dynamic D-vine copula approach accurately forecasts tail risk in energy commodity returns and outperforms other models in terms of average loss.

Key words: Vine copulas; Generalized autoregressive score; Energy commodities; Tail risk.

RESUMO

Nessa dissertação, tivemos como objetivo medir e prever risco na cauda para um portfolio de commodities de energia. Para esse fim, propomos D-vine copulas dinâmicas. Com esse modelo, podemos capturar características complexas da estrutura de dependência dos retornos de commodities de energia, e também incluir características individuais, tais como assimetria e caudas pesadas. Nós também utilizamos o modelo generalized autoregressive score como mecanismo de atualização dos parâmetros das cópulas, o que nos permite incluir dinâmica na estrutura de dependência e utiliza informação da distribuição da cópula para melhorar a estimação de parâmetros. Nossos resultados indicam que o modelo D-vine copulas dinâmicas mede corretamente risco na cauda para os retornos de commodities de energia e é o menor com menor perda média dentre os considerados.

Palavras-chave: Vine copulas; Generalized autoregressive score; Commodities de energia; risco caudal.

CONTENTS

1 INTRODUCTION

Energy commodities hold relevant importance in the global economy, as they account for over 30% of the world's energy consumption, in which oil and natural gas are among the most widely used energy sources in industries, transportation and agriculture (HAMILTON, 2008; ENERGY, 2020). As for financial assets, they are frequently of interest in both speculative and hedging strategies (BASHER; SADORSKY, 2016). Therefore, factors such as the phase of economic cycles, precautionary demand and geopolitical risk can become short-run drivers for their prices (and price returns), resulting in volatility clusters and sharp price changes (LAPORTA; MERLO; PETRELLA, 2018; AMARO et al., 2022). In this sense, understanding and accurately measuring risk is an important task for economic regulators and financial institutions.

These energy commodities price returns are characterized by their complex and intricate behavior, which can be reflected on their nonstandard marginal distributions. Commonly their marginal distributions present non-normal behavior, heavy tails and volatility. Also, the complex nature of production and consumption of these commodities gives rise to possible time changing nonlinear relationships among the distinct commodities (SERLETIS; TIMILSINA; VASETSKY, 2011). In this sense, accurately including these features in the returns joint distribution is crucial to correctly measure and forecast the risk of a portfolio formed by energy commodities returns.

One way of dealing with this complex joint behavior of the returns is via copulas, which provide a powerful approach for modeling complex patterns of dependence among variables. By using copulas, we can effectively separate the modeling of the dependence structure from that of the marginal distributions, enabling us to more flexibly model the joint distribution. This allows us to capture a wide range of dependencies among variables and better understand the underlying relationships among them.

Nevertheless, in a higher dimension problem, the dependence structure might become too complicate to characterize, which leads us to using vine copulas. Vine copulas present a flexible and efficient method for modeling the dependence structure among multiple variables. The pair-copula construction (PCC) introduced by Aas et al. (2009) employs pair-copulas as building blocks, which can then be organized into different levels to capture the whole dependence structure. By using this approach, it is possible to model the joint distribution of multiple variables and also build conditional distributions. This capability has been shown to be a powerful tool for modeling higher dimensional dependence structures and also forecasting risk (RIGHI; SCHLENDER; CERETTA, 2015; TRUCÍOS; TIWARI; ALQAHTANI, 2020; YU etal., 2018).

The time dynamic nature of dependence structures among financial assets has been commonly indicated in the literature. To address this feature, Tófoli et al. (2019) propose a dynamic vine structure, which is based on the copula dynamics introduced by Patton (2006). Also, Almeida, Czado e Manner (2016) propose to include a generalized autoregressive score

(GAS) model to update copula parameters. The important advantage of using GAS is that the parameters updating mechanism includes information of the most recent gradient of the copula density to improve the parameters updating process (similar a Newton-type optimization procedure), adjusting the copula dynamics according to the particular chosen copula.

Measuring risk in energy commodities is crucial for economic policy and financial institutions. A commonly used measure is the Value-at-risk (VaR), which is used to quantify the level of financial risk of an asset or portfolio at certain level α, calculated as the α-quantile of the marginal distribution of an asset or a portfolio (JORION, 2000). To accurately compute a portfolio returns VaR one needs to fully understand the joint behavior of the returns that compose the portfolio. Thus, the use of multivariate models that incorporate the dependence structure among the asset returns is crucial in order to effectively manage and mitigate risk of a portfolio.

The literature has explored tail risk modeling under both univariate and multivariate perspectives. Žikovic´ (2017), Amaro et al. (2022), Laporta, Merlo e Petrella (2018) present ARMA-GARCH models for univariate risk modelling. In general, one of the most common findings is that specifying marginal distributions that account for heavy tails and asymmetry tend to be effective in risk forecasting for energy commodities. However, fitting more sophisticated models for conditional variance not always produces the best forecasts (AMARO et al., 2022). As for multivariate modeling, one possible approach is to use copulas or Extreme Value Theory (EVT), as explored in the works by Hsu, Tseng e Wang (2008), Lu, Lai e Liang (2014), Ghorbel e Trabelsi (2014). These studies investigate the dependence structure among different oil price returns, such as between WTI and Brent oil, and between oil and natural gas. While these models provide insights into tail risk and portfolio management, they do not fully capture the complex dependence structures.

To address this limitation, González-Pedraz, Moreno e Peña (2014) propose an asymmet- ric DCC-GARCH model in the context of energy commodities. In addition to oil and natural gas, their model includes coal and electricity in the portfolio. The authors note that energy returns exhibit high volatility, dynamic correlation, and tail dependence, and that the inclusion of information about these characteristics can improve the accuracy of portfolio risk assess- ment and forecasting. Their empirical results show that the asymmetric DCC-GARCH model with generalized hyperbolic errors outperforms standard models such as multivariate normal orCCC-GARCH models.

While there is a considerable body of literature on risk assessment for energy commodities using univariate approaches, and some studies investigating the risk of energy commodity portfolios, to the best of our knowledge, no previous research has employed dynamic vine copulas to model the dependence structure of the asset returns which compose an energy commodities portfolio. Our study is the first to address this gap in the literature by employing this methodology. By utilizing dynamic vine copulas, we are able to capture the complex interdependence among the several energy commodities returns, which has important implications for risk management and hedging strategies. Hence, the purpose of this study is to measure and predict returns risk of a portfolio composed by energy commodities by using dynamic D-vine copulas. We analyze the most actively traded energy commodities futures, including West Texas Intermediate (WTI) oil , gasoline, heating oil, natural gas and Brent oil. We find that dynamic vine copulas accurately forecast risk and have the lowest average loss among the considered models.

Beyond this introduction, the dissertation is structured into three chapters. Chapter 2 describes all methodological aspects present in this work, including definitions, models and estimation procedures. Chapter 3 shows our comprehensive empirical analysis, exposing the results and discussions. Finally, Chapter 4 brings our final comments.

2 METHODOLOGY

2.1 MODELS FOR THE MARGINAL DISTRIBUTIONS

As documented in literature, energy commodities price returns are characterized by their complex behavior, usually possessing nonstandard marginal distributions with heavy tails and volatility. We use an ARMA (p,q) to capture the autocorrelation in mean and an EGARCH(m,n), as proposed by Nelson (1991), to capture the asymmetric behavior of volatility. We describe these models in (2.1).

$$
\begin{aligned}
x_t &= \phi_0 + \sum_{i=1}^{p} \phi_p x_{t-i} + \sum_{j=1}^{q} \theta_i \varepsilon_{t-1} + \varepsilon_t \\
\varepsilon_t &= \sigma_t . \eta_t, \; \eta_t \sim Skewed - t(v, \lambda) \\
ln(\sigma_t^2) &= \omega + \sum_{k=1}^{n} (\alpha_j \varepsilon_{t-k}^2 + \gamma_k(|\varepsilon_{t-j}| - E|\varepsilon_{t-j}|)) + \sum_{l=1}^{m} \beta_l ln(\sigma_{t-l}^2)
\end{aligned}
\tag{2.1}
$$

In the expressions above, $\delta \geq 0$ and $\sum_{k=1}^{n} \alpha_k + \sum_{l=1}^{m} \beta_l < 1$. The parameter γ indicates a negative asymmetry, which represents the leverage effect. As for the marginal distribution η_t, we use the skewed-t distribution proposed Fernández e Steel (1998) to model both asymmetry and heavy tails.

If the model is correctly specified, then we have that $F(\eta_t) = F(x_t|\mu_t, \sigma_t) = Skewed - t_{v,\lambda}((\frac{x_t - \mu_t}{\sigma_t}))$, in which μ_t as conditional mean of x_t, and σ_t^2 as conditional variance. Additionally, we have by the probability integral transform (PIT) that $u_t = F(x_t|\mu_t, \sigma_t) \sim U[0, 1]$. We use Kolmogorov-Smirnoff (KS) and Cramer-von Mises tests to check the specification of the marginal distributions.

2.2 VINE COPULAS

In this section, we present the vine copula models, which we use to characterize the dependence structure of the energy commodities. The advantages of using this model is that we can model the dependence structure in a flexible way, without making binding assumptionsabout the marginal distribution. This fits properly with the asymmetries and fat tails documentedin literature for the marginal behavior of these commodities, and also to the complex relations among them.

Let $X_t = (x_{1,t}, ..., x_{n,t})$ be a random vector and $F(X_t)$ its joint distribution function. The theorem of Sklar (1973) states that every multivariate joint function can be expressed as a copula $C(\cdot, \cdot)$ and the $F_i(x_i)$ as marginals distributions of $(X_1, \dots, X_n)$. Additionally, following the probability integral transform, a cumulative distribution function (cdf) is also a random variable with uniform distribution. As such, we have that each $F_i(x_i) \sim U[0, 1]$. The expression (2.2) indicates a n-variate case and the joint density function.

$$
\begin{aligned}
F(x_1, ..., x_n) &= C(F_1(x_1), ..., F_n(x_n)) \\
f(x_1, ..., x_n) &= c_{12...n}(F_1(x_1), ..., F_n(x_n)).f_1(x_1)....f_n(x_n).
\end{aligned} \tag{2.2}
$$

We can use copulas to model dependence structure between variables in a flexible way, separating the dependence structure of marginal distribution of the variables. Following Aas et al. (2009), we can factorise a joint pdf, such as (2.3).

$$
f(x_1, ..., x_n) = f(x_n).f(x_{n-1}|x_n). \dots .f(x_1|x_2, ..., x_n). \tag{2.3}
$$

Aas et al. (2009) points that we can generalize this result for a n-variate case. We have the expressions in (2.4) showing the $f(x|v)$ and $F(x|v)$, respectively. In this notation, v is a vector of dimension d, where v_j and v_{-j} are chosen arbitrarily. The multivariate density, as showed in (2.4), involves conditioned marginal distributions in the form presented by Joe (1996):

$$
\begin{aligned}
f(x|v) &= c_{xv_j|v_{-j}}(F(x|v_{-j}), F(v_j|v_{-j})).f(x|v_{-j}). \\
F(x|v) &= \frac{\partial C_{xv_j|v_{-j}}(F(x|v_{-j}), F(v_j|v_{-j}))}{\partial F(v_j|v_{-j})}.
\end{aligned} \tag{2.4}
$$

Since the decomposition is arbitrary, we can have many possibilities, and these increase with the number of variables. As a way to organize the relations between variables, Bedford e Cooke (2002) propose regular vines (R-vines). Vines are a graphical method which we build a sequence of nested trees, defining each edge as a pair-copula and each edge is an node in the following tree (JOE; KUROWICKA, 2011). For n variables, we have $n - 1$ number of trees, where each tree is denoted as T_j, $j = 1, 2, ..., n - 1$. However, as pointed by Aas et al. (2009), R-vines still are very general with many possible decompositions. Two special cases can be highlighted: canonical vines (C-vines) and drawable vines (D-vine).

In canonical vines, the first tree is built associating all variables to one central variable, which is useful when we know the key variable that governs the interactions. As for drawable vines, the only restriction is that no node is connected to more than one edge at any T_j tree and these trees has an unique node connected to $n - j$ edges (AAS et al., 2009).

To select the optimal tree structure, we use the algorithm proposed by Dissmann et al. (2013). In this procedure, we select the pairing of the variables that maximize the empirical Kendall's τ for each tree level, then select the best fitting copula each pair using Schwarz Information Criterion (SIC or SBC).

For some of these pair-copula, we may also have time-varying parameters. Almeida, Czado e Manner (2016) introduces dynamic D-vine copulas, where we have all of the pair-copulae as dynamic and also, in which one of the proposed updating mechanism for copula parameters is a generalized autoregressive score model, as proposed by Creal, Koopman e Lucas (2013). Tófoli et al. (2019) introduces a dynamic D-vine copula, where some of the copula presents dynamic behavior, following the dynamic proposed by Patton (2006). In both cases, the iterative algorithm proposed for building conditional distributions and pair-copulae remains the same, as long as we account for the parameter dynamics in (2.4). In this paper, we use a dynamic D-vine copula, using a GAS (1,1) to model the evolution of dependence parameters over time[1].

2.3 TIME-VARYING COPULAS

A common fact in the finance literature is the evolution of the dependency structure over time. We use generalized a autoregressive score model (GAS) to introduce dynamic in copula parameters. Following Manner e Reznikova (2012), the most used models to introduce dynamics in copula parameters are GAS models and the ARMA(1,m) dynamics, proposed by Patton (2006). The advantages of using GAS models is that we use information regarding the full conditional density, not only the first and second moment (CREAL; KOOPMAN; LUCAS, 2013).

As proposed by Creal, Koopman e Lucas (2013), the GAS (p,q) model has an updating mechanism for time-varying parameters that takes into account both the lagged parameter and score of the conditional density. Let θ_t be a time-varying parameter of interest and $\boldsymbol{u_t}$ our vector of marginal distributions. We assume that $\boldsymbol{u_t} \sim c(\boldsymbol{u_t}|\theta_t, \mathrm{F}_{t-1}, \omega)$, where F_{t-1} is the informationset up to time $t-1$ and ω are fixed parameters. Following Almeida, Czado e Manner (2016), for $t = 1, ..., T$, our copula parameters in $1 \leq \boldsymbol{i} \neq \boldsymbol{j} \leq d$. (2.5) describes the copula distribution and our time-varying parameter.

$$(u_{i,t}, u_{j,t}) \sim c(\cdot, \cdot, \theta_t^{i,j}). \tag{2.5}$$

Since the parameters of different copula families range through different intervals, we bound the parameters to a range of values according to the correspondent copula. In this sense, for the elliptical copulas, we use a logistic transformation to associate the unrestricted parameter $\lambda_t^{i,j}$ and the correlation parameter $\rho_t^{i,j}$. As for all the other copulas that the parameters had to assume a positive value, we use a restriction similar to the one indicated by Oh e Patton (2018), as indicated in Expression 2.6.

[1] We have a dynamic D-vine in which some of the selected pair-copulae are static.

$$\rho_t^{i,j} = 0.99 * \frac{1 - exp(-\lambda_t)}{1 + exp(-\lambda_t^{i,j})}$$
$$\delta_t^{i,j} = 0.001 + exp(\lambda_t^{i,j}) \tag{2.6}$$
$$\gamma_t^{i,j} = 1.001 + exp(\lambda_t^{i,j})$$

For the parameter $\lambda_t^{i,j}$, we have the following GAS(1,1) dynamics:

$$\lambda_t^{i,j} = \omega_{i,j} + \boldsymbol{A}_1 s_{t-1}^{i,j} + \boldsymbol{B}_1 \lambda_{t-1}^{i,j}$$
$$s_t^{i,j} = S_t^{i,j}.\nabla_t^{i,j} \tag{2.7}$$
$$\nabla_t^{i,j} = \frac{\partial c(u_{i,t}, u_{j,t}|\omega_{i,j}, \mathrm{F}_{t-1})}{\partial \theta_t^{i,j}}.$$

In (2.7), $\omega_{i,j}$ is a vector of constants, A_1 and B_1 are coefficient matrices with appropriated dimensions, $S_t^{i,j}$ is a scaling matrix. For the choice of scaling matrix, Creal, Koopman e Lucas (2013) indicate that some possibilities, such as $S_t = \mathrm{I}_{t|t-1}^{-d}$, $d = \{0, \frac{1}{2}, 1\}$, where $\mathrm{I}_{t|t-1} = \boldsymbol{E}_{t-1}[\nabla_t \nabla_t']$ is Fisher information matrix up to time $t-1$. If $d = 0$, we have $\boldsymbol{S}_t = \boldsymbol{I}$, but for $d = 1$, we have S_t as the variance of the parameter θ_t up to time $t-1$.

Following Almeida, Czado e Manner (2016) and Kielmann, Manner e Min (2022), we choose $d = 1/2$, which indicates the scaling matrix as the square-root of the inverse of the Fisher Information matrix. As such, the scaling matrix presents information referring to copula parameters added by new observations. This feature can capture dynamics in a more complete way than the updating mechanism proposed by Patton (2006), since the new information depends on copula function, not only by mean of the distance between the two marginals (as a measureof comonotonicity) (CREAL; KOOPMAN; LUCAS, 2013).

2.4 ESTIMATION OF COPULAS

For the joint distribution function of our five variables, we have (2.8) as log-likelihood function of a five-dimensional D-vine:

$$
\begin{aligned}
\ell(\boldsymbol{\alpha}, \boldsymbol{\gamma}; \boldsymbol{X}) = & \sum_{t=1}^{T} (\log f(x_{4,t}; \alpha_4) + \log f(x_{2,t}; \alpha_2) + \log f(x_{1,t}; \alpha_1) + \log f(x_{5,t}; \alpha_5) + \log f(x_{3,t}; \alpha_3)) + \\
& \sum_{t=1}^{T} (\log c_{42}(F_4(x_{4,t}; \alpha_4), F_2(x_{2,t}; \alpha_2); \gamma_{42}) + \log c_{21}(F_2(x_{2,t}; \alpha_2), F_1(x_{1,t}; \alpha_1); \gamma_{21}) + \\
& \log c_{15}(F_1(x_{1,t}; \alpha_1), F_5(x_{5,t}; \alpha_5); \gamma_{15}) + \log c_{53}(F_5(x_{5,t}; \alpha_5), F_3(x_{3,t}; \alpha_3); \gamma_{53})) + \\
& \sum_{t=1}^{T} (\log c_{41|2}(F_{4|2}(x_{4,t}|x_{2,t}; \alpha_4, \alpha_2, \gamma_{42}), F_{1|2}(x_{1,t}|x_{2,t}; \alpha_1, \alpha_2, \gamma_{21}); \gamma_{41|2}) + \\
& \log c_{25|1}(F_{2|1}(x_{2,t}|x_{1,t}; \alpha_2, \alpha_1, \gamma_{21}), F_{5|1}(x_{5,t}|x_{1,t}; \alpha_5, \alpha_1, \gamma_{15}); \gamma_{25|1}) + \\
& \log c_{31|5}(F_{3|5}(x_{3,t}|x_{5,t}; \alpha_3, \alpha_5, \gamma_{53}), F_{1|5}(x_{1,t}|x_{5,t}; \alpha_1, \alpha_5, \gamma_{15}); \gamma_{31|5})) + \\
& \sum_{t=1}^{T} (\log c_{45|12}(F_{4|12}(x_{4,t}|x_{1,t}, x_{2,t}; \alpha_4, \alpha_1, \alpha_2, \gamma_{42}, \gamma_{21}, \gamma_{41|2}), \\
& F_{5|12}(x_{5,t}|x_{1,t}, x_{2,t}; \alpha_5, \alpha_1, \alpha_2, \gamma_{15}, \gamma_{21}, \gamma_{25|1}; \gamma_{45|12}) + \\
& \log c_{23|15}(F_{2|15}(x_{2,t}|x_{1,t}, x_{5,t}; \alpha_2, \alpha_1, \alpha_5, \gamma_{21}, \gamma_{15}, \gamma_{25|1}), \\
& F_{3|15}(x_{3,t}|x_{1,t}, x_{5,t}; \alpha_3, \alpha_1, \alpha_5, \gamma_{53}, \gamma_{15}, \gamma_{31|5}; \gamma_{23|15})) + \\
& \sum_{t=1}^{T} (\log c_{43|125}(F_{4|125}(x_{4,t}|x_{1,t}, x_{2,t}, x_{5,t}; \alpha_4, \alpha_1, \alpha_2, \alpha_5, \gamma_{42}, \gamma_{21}, \gamma_{15}, \gamma_{41|2}, \gamma_{25|1}, \gamma_{41|12}), \\
& F_{3|125}(x_{3,t}|x_{1,t}, x_{2,t}, x_{5,t}; \alpha_3, \alpha_1, \alpha_2, \alpha_5, \gamma_{53}, \gamma_{15}, \gamma_{21}, \gamma_{25|1}, \gamma_{31|5}, \gamma_{23|15}); \gamma_{43|125}))
\end{aligned}
\tag{2.8}
$$

To estimate the parameters in an asymptotic efficient manner, we use sequential maximum likelihood estimation (CZADO, 2019). As such, we estimate each level of the vine and use the estimated parameters in the following level. The first step of the procedure is similar to "inference from margins" as proposed by Joe e Xu (1996) (IFM), since we use directly the estimated marginal distributions from our variables.

In the second step, we estimate the conditional distribution using the h-function, then we estimate the conditional pair-copulas. This procedure is followed for the other two steps, where we first generate the conditional distribution functions using the copulas from the previous levels, then we estimate the conditional copulas. Even including dynamics, the sequential procedure remains the same for computing conditional marginals using the proper h-function (ALMEIDA; CZADO; MANNER, 2016).

2.5 DCC-GARCH

As one of the dynamic models proposed, we use Dynamic Conditional Correlation GARCH (DCC-GARCH), as proposed by Engle (2002). In this model, we can model both covariance matrix and conditional variance of each variable in a dynamic form as separated processes. The advantages of this approach is considering first the conditional variance, then conditional correlation, accounting for each separately. Also, the conditional correlation is dynamic, encompassing changes in the relation between financial assets. As such, the model is commonly used as multivariate model in risk management (BRECHMANN; CZADO, 2013; ZHANG et al., 2014; MARCHESE et al., 2020). Expression (2.9) presents the DCC-GARCH model.

$$\begin{aligned} \boldsymbol{X_t} &= \boldsymbol{\mu}_t + \varepsilon_t \\ \boldsymbol{\varepsilon}_t &= H_t^{1/2} z_t,\ z_t \sim iid(0,1) \\ H_t &= D_t R_t D_t \end{aligned} \tag{2.9}$$

Similar to univariate case, we have an expression for conditional mean given by the first expression in (2.9). As for conditional variance, we decompose the covariance matrix in two elements: a diagonal matrix $D_t = diag(h_{1,t}^{1/2}, ..., h_{5,t}^{1/2})$ defined as the standard deviation matrix and each $h_{i,t}$ representing the univariate GARCH (p,q) of each variable that might have different orders; and R_t as the correlation matrix. Since H_t is a covariance matrix, it has to be positive definite. To ensure this requirement, Engle (2002) proposes to further decompose R_t in $R_t = P_t^{-1} Q_t P_t^{-1}$. Expressions in (2.10) define a DCC(1,1) model.

$$\begin{aligned} Q_t &= (1 - a - b)\overline{Q} + a\xi_{t-1}\xi_{t-1}' + bQ_{t-1} \\ \overline{Q} &= \frac{1}{T}\sum_{t=1}^{T} \xi_t \xi_t' \end{aligned} \tag{2.10}$$

We define $\xi_t = \boldsymbol{D}_t^{-1}\varepsilon_t \sim N(0, \boldsymbol{R}_t)$ as the standardized residuals, and both a and b are scalars. To ensure positive definite matrix, the following conditions are imposed: $a, b \geq 0$ and $a + b < 1$. The matrix P_t is a normalization matrix, composed by rescaled elements of Q_t.

2.6 VALUE-AT-RISK AND BACKTESTING

We define Value-at-Risk as the α-quantile of the joint distribution function (JORION, 2000). To calculate VaR forecasts, we use different approaches depending on the fitted model. For dynamic and all-static D-vines, we follow an approach proposed by Müller e Righi (2018), in which we have the following algorithm to compute $VaR(\alpha)$:

1. Simulate a sample $\{\hat{u}_{i,t}\}_{i=1}^{5}$ from the fitted D-vine copula. This procedure was done 1000 times;

2. From the simulated uniforms, we apply the inverse distribution to obtain the marginals, since $\hat{\eta}_{i,t} = F^{-1}(\hat{u}_{i,t}),\ i = 1, ..., 5;$

3. We estimate each return as $\hat{r}_{i,t} = \hat{\mu}_{i,t} + \hat{\sigma}_{i,t}\hat{\eta}_{i,t},\ i = 1, ..., 5;$

4. We compute a portfolio equally weighted $R_t = \sum_{i=1}^{5} 0.2 * (r_{i,t})$;
5. We calculate $VaR(\alpha) = -inf\{\boldsymbol{R_t} \in \mathbb{R} | \alpha \leq F(\boldsymbol{R_t})\}$ as the quantile of the portfolio $\boldsymbol{R_t}$ at time t. The significance levels we choose are 0.10, 0.05 and 0.01.

For DCC-GARCH, we have similar steps for conditional mean and variance, but $\hat{\eta}_{i,t}$ are the standardized residuals obtained in estimation. To compare the different risk forecasts, first we test for VaR violations using the unconditional coverage test proposed by Kupiec et al. (1995), the conditional coverage test proposed by Christoffersen (1998), and the dynamic quantile test proposed by Engle e Manganelli (2004).

In the dynamic quantile test (DQ), we denote $Hit_t(\alpha) = I(\alpha) - \alpha$, a demeaned process, in which I is an indicator function that assumes 1 when $r_t < VaR_t$. The idea of this test is to check whether the violation today has correlation with its lags. For this, we test the joint significance of the lags in a linear regression, in which $Hit_t(\alpha) = \beta_0 + \beta_1 Hit_{t-1} + e_t$, $e_t \sim iid$. Under the null hypothesis, the hits are uncorrelated.

Also, we use the loss function proposed by González-Rivera, Lee e Mishra (2004). The loss function is an altered version of the one proposed by Koenker e Jr (1978) to estimate parameters in quantile regression. As such, for a given α, (2.11) indicate the loss function Q.

$$Q = P^{-1} \sum_{t=R}^{T} (\alpha - d^{\alpha}_{t+1})(y_{t+1} - VaR^{\alpha}_{t+1}). \quad (2.11)$$

In (2.11), we have P as the prediction period, and $d^{\alpha}_{t+1} = 1(y_{t+1} < VaR^{\alpha}_{t+1})$ as an indicator function. In this loss function, we have a higher penalization of $(1 - \alpha)$ if $y_{t+1} - VaR^{\alpha}_{t+1} < 0$. Smaller values of Q indicate better fit of the proposed model. In order to compare model forecasts, we use the test proposed by Diebold e Mariano (1995). Here we use loss functions to test if there is significant statistical difference between them. Under null hypothesis, we have that there is no significant difference between forecasts.

3 EMPIRICAL ANALYSIS

3.1 DATA

The data we use in this paper consists in log-returns of the futures prices of selected energy commodities, such as West Texas Intermediate (WTI) oil, gasoline, heating oil, natural gas, and Brent oil. We choose these energy commodities since they are the most traded and also, the most influential to other energy commodities Ferreira et al. (2022). The full period of analysis is january-2011 to december-2022, totalling 3000 observations with a daily frequency. The variables represent the log returns of one month ahead future prices traded on New York Merchant Exchange (NYMEX). Chart 1 describes the variables as we refer to it in this paper.

Chart 1 - Variables used in the estimated models

Number in vine	**Notation**	**Description**	**Source**
1	$doil_t$	WTI future prices returns	NYMEX
2	$dgasol_t$	RBOB gasoline future prices returns	NYMEX
3	$dheat_t$	Heating oil futures prices returns	NYMEX
4	$dgas_t$	Natural gas future prices returns	NYMEX
5	$dbrent_t$	Brent oil future prices returns	NYMEX

Source:author elaboration.

To perform the estimation, we split the data in sub-periods: first, we have an in-sample of 1500 observation we use to select the tree format of the vine copulas, the copula distributions and whether the copulas are dynamic of static. After this first estimation, we proceed to re-estimate using a rolling window with the size of 1500. To conditional mean and variance, we use the rolling window to forecast one-step ahead. As for the parameters in GAS dynamics for the copulas, we re-estimated at each 500 observations. This is due to the dependence structure not changing so drastically over time, as compared to conditional variance.

Table 1: Descriptive statistics and unit root test for in-sample period

Variable	Mean	SD	Skewness	Kurtosis	PP	KPSS
$doil_t$	0	0.02	0.16	2.95	-1628.60**	0.11
$dgasol_t$	0	0.02	0.17	9.55	-1553.30**	0.10
$dheat_t$	0	0.02	-0.64	10.26	-1654.80**	0.16
$dgas_t$	0	0.03	0.24	1.71	-1585.70**	0.09
$dbrent_t$	0	0.02	0.11	3.79	-1680.60**	0.16

Source:author elaboration. Asterisks indicate the rejection of null hypothesis.

Table 1 presents the descriptive statistics and unit root tests for the log-returns of the selected energy commodities for the in-sample period. According to presented results, there is evidence that log returns of gasoline and heating oil present heavy tails, while all other variables present kurtosis closer to 3. All log returns of energy commodities present positive asymmetry (with exception of heating oil). Additionally, both unit root tests present evidence of stationarity for all variables.

3.2 ESTIMATION OF DYNAMIC D-VINE COPULAS

In this section, we present the results for the estimation of dynamic D-vine copulas. First, we test whether the marginal distributions are correctly specified, using the Kolmogorov-Smirnoff and the Cramer-von Mises tests. Then, if the marginal are correctly specified, we proceed to select the best fitting pair-copulae among the presented families. To estimate the marginal distributions, we use an ARMA-eGARCH model and we select the lags with Schwarz Information Criterion. The results are presented in Table 2.

Table 2: Results for the diagnosis of the estimated marginal distributions for energy commodities

Coefficient	$doil_t$	$dgasol_t$	$dheat_t$	$dgas_t$	$dbrent_t$
$Q(20)$	0.9998	0.9775	0.8827	0.8456	0.8165
$Q^2(20)$	0.0716	0.6267	0.6097	0.1598	0.0718
$K-S$	0.2487	0.5387	0.2467	0.1096	0.8357
CvM	0.9813	0.4365	0.9240	0.8244	0.2132

Source: author elaboration. P-value in parenthesis.

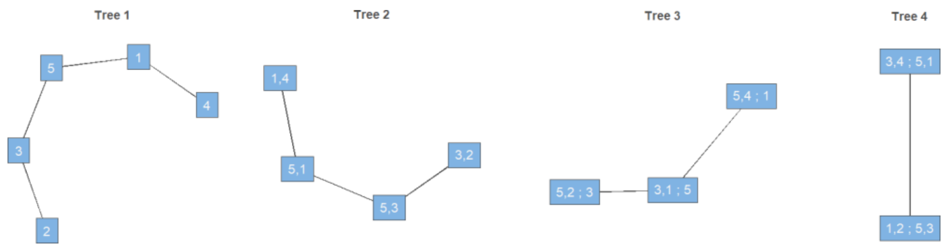
By the results in Table 2, the $Q(20)$ and $Q^2(20)$ tests indicate no evidence of autocorrelation or ARCH-type residuals up to lag 20. Also, we do not reject the null hypothesis for both Kolmogorov-Smirnoff and Cramer-von Mises, which indicate that we have evidence that the estimated marginal distributions are $U[0, 1]$. As such, we can proceed to copula estimation.

Now we proceed to the dynamic vine copulas. The first step is to organize the dependence structure, and for such, we use the spanning tree algorithm proposed by Dissmann et al. (2013). After selecting the appropriate vine structure, we proceed to select the best fitting copula for each pair-copulae. Figure 1 present the selected tree format.

As indicated in Figure 1, we select a D-vine copula. In this pairing, we have that the first pair-copula is gasoline and heating oil returns, then the second is heating oil and Brent oil, the third Brent oil and WTI oil, and the last, WTI oil and natural gas. The upper levels are conditioned to the first.

Next, we select the best fitting copula for each pair. For that, we first select between five copula families among their static and dynamic version: Gaussian, t, BB7, Rotated Gumbel and

Figure 1 – Selected tree format for dynamic vine copulas



Source: author elaboration.

Gumbel. Both Gaussian and t model a linear correlation, but t copula also present symmetric tail dependence, while Gaussian does not present tail dependence. The Rotated Gumbel and Gumbel copulas present asymmetric tail dependence, being upper and lower tail dependence, respectively. We also include BB7, that allow us to model both lower and upper tail dependence in a separate manner. Table 3 presents the selected families and their SIC for the static and dynamic versions.

Table 3: Results for family selection for vine copula

Copula	Family	SIC
Tree 1		
$dgas_t, dheat_t$	static Gaussian	−0.0251
$dheat_t, dbrent_t$	dynamic t	−2.1813
$dbrent_t, doil_t$	dynamic t	−3.008
$doil_t, dgasol_t$	dynamic t	−3.9181
Tree 2		
$dgas_t, dbrent_t \vert dheat_t$	static Gaussian	−0.0010
$doil_t, dheat_t \vert dbrent_t$	dynamic t static	−0.5445
$dgasol_t, dbrent_t \vert doil_t$	t	−0.2676
Tree 3		
$doil_t, dgas_t \vert dheat_t, dbrent_t$	static Gaussian	0.0052
$dgasol_t, dheat_t \vert doil_t, dbrent_t$	static t	−0.2080
Tree 4		
$dgasol, dgas \vert doil_t, dheat_t, dbrent_t$	static Gaussian	0.0051

Source:author elaboration. Asterisks indicate the best fitting model by SIC criteria.

The results in Table 3 indicate that, for all pair-copulae, we choose symmetric copulas, where t copula is the most common. According to Righi, Schlender e Ceretta (2015), t copulas are one of the most used in dependence structures of financial assets. These findings are in line with part of the literature which use copulas for energy commodities returns modelling (HSU; TSENG; WANG, 2008; LU; LAI; LIANG, 2014).

Out of the ten estimated copulas, we select dynamic copulas for four of them, being present in the first and second trees of the vine. These copulas indicate evidence of significant change of these dependence structures over time. However, most of the second, and all of the pair-copula in third and fourth trees, present a better fit with static copulas. As such, we have evidence of constant relation over time. In Table 4, we present the estimated parameters of the dynamic D-Vine copulas.

Table 4: Estimates from dynamic copulas models for energy commodities

Copula	ω_1	ω_2	α	β
$dgas_t$, $dheat_t$	0.1226 (0.0253)	–	–	–
$dheat_t$, $dbrent_t$	0.6345 (0.4548)	4.0112 (0.0501)	0.3060 (0.0760)	0.7634 (0.1684)
$dbrent_t$, $doil_t$	0.0352 (0.0038)	2.4789 (0.0369)	0.0998 (0.0417)	0.9847 (0.000)
$doil_t$, $dgasol_t$	0.1529 (0.000)	8.3752 (0.0152)	0.1261 (0.000)	0.9124 (0.000)
$dgas_t$, $dbrent_t \| dheat_t$	−0.0095 (0.0244)	–	–	–
$doil_t$, $dheat_t \| dbrent_t$	0.2045 (0.0358)	8.9847 (0.0150)	0.2131 (0.0226)	0.5272 (0.0913)
$dgasol_t$, $dbrent_t \| doil_t$	0.4669 (0.0201)	9.5969 (0.0211)	–	–
$doil_t$, $dgas_t \| dheat_t$, $dbrent_t$	0.0638 (0.0258)	–	–	–
$dgasol_t$, $dheat_t \| doil_t$, $dbrent_t$	0.2488 (0.0252)	13.1406 (0.0224)	–	–
$dgasol$, $dgas \| doil_t$, $dheat_t$, $dbrent_t$	0.018 (0.0163)	–	–	–

Source: author elaboration. Standard error in parenthesis.

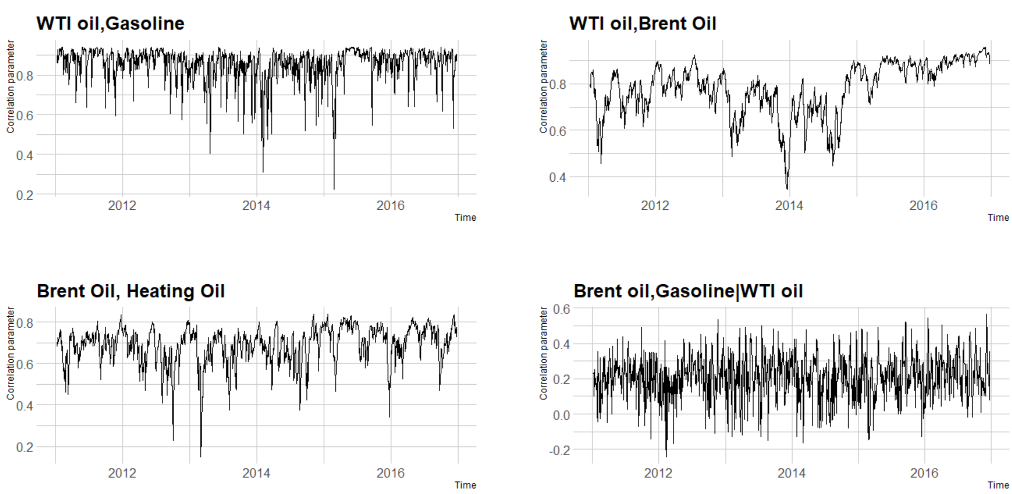
For first tree, we have that all time-invariant copula parameters are statistically significant. Also, we have evidence of symmetric tail dependence and a time-invariant linear correlation. However, in the second and third trees, we have lower correlation between energy commodities, and even a non-significant time-invariant parameter for $c_{4,5|3}$and $c_{2,4|1,3,5}$, which might indicate no correlation. All t copulas present significant symmetric tail dependence.

Understanding some of the correlations between energy commodities might be complex due to nature of production and consumption, but some are quite direct, as for correlation between WTI and Brent oil. We can see a reflection of oil markets integration through the high correlation between both oil prices, in which Brent is a benchmark for european oil and WTI (West Texas Intermediate) is an american benchmark. In some extension, the price differences might indicate the effect of local markets, but the most predominant factor are global oil markets drivers, such as global supply and demand (HAMILTON, 2009; KRUSE; WEGENER, 2020).

Other energy commodities, such as gasoline and heating oil, are linked with oil since they are oil derivatives, which implicates price spillovers between these markets (FERREIRA et al., 2022; ALBULESCU; TIWARI; JI, 2020; MARCHESE et al., 2020). A result that was not expected is the low time-invariant correlation between gasoline and WTI oil prices (the estimated parameter was 0.1529), but high dynamic correlation. Additionally, in contrast to oil markets, the price of these energy commodities are dominated by factors associated with local markets, specially on demand side (ENERGY, 2020).

Regarding to the parameters associated with GAS dynamics, an important finding is that for all dynamic copulas, the coefficient referring to the scaling matrix is significant, whichgives us evidence that information about copula family affects dynamic parameters. Specifically, since our scaling matrix S_t is the square-root of the inverse of the Fisher Information matrix, wehave information specifically about the asymptotic standard deviation of our copula estimated parameters [1]. The autoregressive parameter is also significant for all copulas, which also means a degree of dependence of the parameter in time *t* to its first lag. Figure 2 indicates the trajectory of the estimated dynamic parameters in-sample.

Figure 2 – Estimated correlation parameter from time-varying copulas



Source: author elaboration.

3.3 VAR ESTIMATION

Our results are based in the following models: dynamic D-Vine copula, static D-vine copula, and DCC-GARCH. We analyze the results of the unconditional coverage test proposed by Kupiec et al. (1995), in which it is evaluated if the number of effective exceedances is equal to the nominal α. Also, the independence test proposed by Christoffersen (1998), which tests whether the breaches are independent, as well as Dynamic Quantile test, which tests unconditional coverage and independence at the same time. The results are presented in Table 5.

[1] For MLE estimators, the inverse of the Fisher information matrix is equal to asymptotic variance of the estimators.

Table 5: Results for backtesting procedures of VaR

Test	10%	5%	1%
	Dynamic D-vine		
Kupiec	0.0359	0.6209	1.8405
	(0.8497)	(0.1054)	(0.1748)
Christoffersen	0.0444	2.8222	6.8094**
	(0.9780)	(0.2438)	(0.0332)
DQ	1.3999	4.4287	19.1693**
	(0.8442)	(0.3510)	(0.0007)
	Static D-vine		
Kupiec	12.1964**	5.1701**	0.2249
	(0.0004)	(0.0229)	(0.6353)
Christoffersen	12.2108**	5.7631**	3.0793
	(0.0022)	(0.0560)	(0.2144)
DQ	11.9404**	6.7937	10.8723**
	(0.0177)	(0.1471)	(0.0280)
	DCC-GARCH		
Kupiec	0.1222	3.4305	7.5516**
	(0.7266)	(0.0639)	(0.0059)
Christoffersen	1.1058	3.7436	10.5761**
	(0.5752)	(0.1538)	(0.0050)
DQ	1.7367	5.5886	22.1801**
	(0.7840)	(0.2320)	(0.0001)

Source: author elaboration. The asterisks indicate the rejection of null hypothesis.

The results of the unconditional coverage test indicate that VaR measures based on dynamic D-vine present the expected exceedances percentages for all levels of significance. However, we reject the hypothesis of independence of violations for 1% significance level. This is corroborated by DQ test, which evaluates the percentage of exceedances and independence at the same time. These results are similar to the ones of the ones from DCC-GARCH model. This might be due to both models presenting dynamic modelling of correlation structure, and also because most of the selected pair-copulae present linear dependence and symmetric tail dependence.

The VaR based on static D-Vine presents opposite results: the tests indicate evidence of the percentage of correct exceendances and independence only for 1% significance level, even though the DQ test indicates correct exceedances and independence for 5%. As such, we have evidence that dynamic D-vine and DCC-GARCH perform better than static D-Vine for measuring tail risk for 10% and 5% significance levels.

To compare the forecasts, we use the average of the loss function proposed by González-Rivera, Lee e Mishra (2004) and the Diebold-Mariano test. For Diebold-Mariano test, we use dynamic D-Vine as benchmark model. We present the results in Table 1.

Table 1 – Results for Average Loss and Diebold-Mariano tests

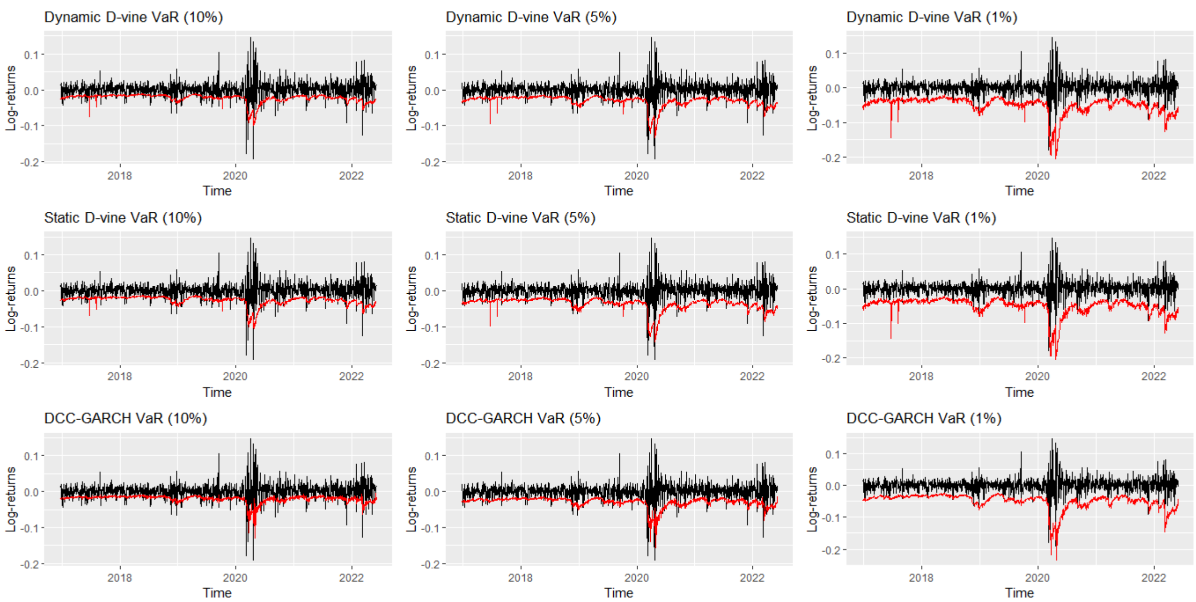
Model	10%	5%	1%
Dynamic D-vine	0.00389	0.00252	0.00078
Static D-vine	0.00394	0.00252	0.00078
	(0.0000)**	(0.0000)**	(0.0235)**
DCC-GARCH	0.00388	0.00250	0.00078
	(0.9026)	(0.4149)	(0.9100)

Source: author elaboration. P-value of Diebold-Mariano in parenthesis.

Following the results, we have that the average loss of DCC-GARCH and D-vine models are roughly equivalent at all significance levels, leading to no significant statistical difference between their average losses (as pointed by DM test). For all levels of significance, static D-vine presents similar loss to dynamic D-vine, however, the DM test indicates difference between its forecasts and the ones from dynamic D-vine. In this sense, we have evidence that dynamic D-vine and DCC-GARCH present the best forecast capacity in the collection of models we evaluate for this data set.

Figure 3 presents the time evolution of the portfolio returns of the energy commodities and the estimated tail risk measure VaR for all used models.

Figure 3 – One-step ahead VaR forecasts



Source: author elaboration.

Overall, the results from backtesting indicate that dynamic models present better for VaR forecast, since they present conditional coverage for two different quantiles, while the static D-vine presents conditional coverage only for one quantile. The concordance between the dynamic D-vine and DCC-GARCH for these commodities might be due to the observed linear relations between them. In this sense, the joint distribution function presented a similar format than the one indicated by the DCC-GARCH. This result goes in a similar direction to that in González-Pedraz, Moreno e Peña (2014), where the authors also conclude that models which incorporate dynamic correlation, asymmetry and heavy tails tend to perform better.

4 CONCLUSION

In this paper, we seek to measure and forecast tail risk for energy commodities. For such end, we propose the use of dynamic vine copulas. The advantages of the vine copulas arethat we can model the dependence structure in a separate manner from the joint distribution, and with it, capture the complex features of the energy commodities individually and also their possible nonlinear relations. Also, the vine copulas allow the decomposition of high dimensional dependence structure in simple building blocks, the pair-copulae. With such, we can estimate the parameters in a flexible and efficient manner. For the marginal distributions, we use ARMA-eGARCH models with a skewed-t distribution for the innovations, since the literature indicate the presence of asymmetries in conditional volatility, and also, heavy tails for the returns of the energy commodities.

The main results of the paper is that dynamic vine copulas present a good fit to measure risk for energy markets, since for backtesting, we only fail to present the correct exceendances for 1%. Also, the model presented a lower average loss than the static alternative, indicating that accounting for the dynamic behavior of the relations between energy commodities improve the risk measure. Due to the linear correlations found between energy commodities, the dynamic vine copulas presented a similar fit to the DCC-GARCH models, which also models dynamic correlation and includes information about heavy tails in the multivariate t distribution. For further studies, we indicate the possibility of including different energy commodities portfolios.

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