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SEASONAL WEIGHTED LAG ADAPTIVE LASSO

Porto Alegre
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Dissertação submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Mestre em Economia, com ênfase em Economia Aplicada.

Orientador: Prof. Dr. Flavio A. Ziegelmann

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RESUMO

O objetivo principal desse artigo é propor um novo tipo de penalidade LASSO com intuito de melhorar a performance de previsão fora da amostra para séries temporais em cenários de grande dimensionalidade. Também apresentamos resultados relativos a performance da estimação de parâmetros. Nossa abordagem leva ao que chamamos SWLadaLASSO (seasonal weighted lag adaptive LASSO) que atribui maiores penalidades para coeficientes de variáveis com maiores defasagens – baseado na ideia do WLadaLASSO de Konzen e Ziegelmann (2016) – com exceção daqueles associados a defasagens sazonais da variável a ser estimada. Pode ser considerado uma generalização do WLadaLASSO. Nas nossas simulações de Monte Carlo, SWLadaLASSO é superior em termos de previsão, estimação de parâmetros e também de seleção de variáveis na maioria dos casos quando comparado a outros modelos de penalidade LASSO. Uma aplicação empírica é conduzida para avaliar a capacidade da abordagem proposta na previsão do crescimento do PIB brasileiro. Adicionalmente, são implementadas algumas formas de combinação de previsões visando obter maior acurácia nas previsões.

Palavras-chave: séries temporais; LASSO; previsão; sazonalidade; combinação de previsões.
Classificação JEL: C51, C53, C55.

ABSTRACT

The main purpose of this paper is to propose a new LASSO type penalty aiming to improve out-of-sample forecasting performance for seasonal time series in high dimensionality scenarios. We also present results concerning parameter estimation performance. Our approach leads to what we call SWLadaLASSO (seasonal weighted lag adaptive LASSO) which assigns larger penalties for higher-lagged covariate coefficients - based on the idea of WLadaLASSO by Konzen e Ziegelmann (2016) - but those associated to the seasonal lags of the variable being estimated. It can be considered a generalization of the WLadaLASSO. In our Monte Carlo studies, the SWLadaLASSO is superior in terms of forecasting, parameter estimation and also covariate selection in most of the cases when compared to other LASSO-type penalty models. An empirical application is conducted to evaluate the capability of the proposed approach to forecast Brazilian GDP growth. Additionally, a set of forecast combinations is implemented in search of forecast accuracy improvement.

Keywords: time series; LASSO; forecasting; seasonality; forecast combination.
JEL Codes: C51, C53, C55.

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1 INTRODUCTION

Forecasting emerging market macroeconomic variables has proven to be specially difficult, given the great instability in these economies (CHAUVET; LIMA; VASQUEZ, 2015). While obtaining data could be a hard task in the past, nowadays almost everything is registered somewhere. The hardship now is to filter the relevant pieces among all the available information.

Expanding the set of predictors increases parameter estimation uncertainty, which translates into additional uncertainty in the resulting forecasts. However, when the number of predictors is restricted, it becomes crucial to select the most informative variables for consideration (ÇAKMAKLI; DIJK, 2016).

Variable selection is particularly important when the true underlying model has sparse representation and identifying significant predictors will enhance the prediction performance of the fitted model (ZOU, 2006). Therefore, the pursuit for models capable of handling this kind of data becomes crucial in scenarios with a great amount of available data and variables.

Usually, the number of observations, n, is much larger than the number of variables or parameters, p. Yet, when the dimensionality of p is large compared to the sample size n, traditional methods face some challenges related to model interpretability, estimation and efficiency (KONZEN; ZIEGELMANN, 2016).

Traditionally, analysts use best-subset selection to choose significant variables. Nevertheless this procedure has two fundamental limitations: firstly when the number of predictors is large, it is computationally infeasible to perform subset selection and secondly the procedure is extremely variable due to its inherent discreteness (ZOU, 2006).

Another way to challenge the problems caused by high dimensionality is through the sparsity assumption on the parameter vector, forcing that many of its components are exactly equal to zero. Although it generally produces biased estimators, the sparsity assumption helps to identify the important covariates, then obtaining a more parsimonious model and reducing its complexity as well as the computational cost of estimation (KONZEN; ZIEGELMANN, 2016).

According to Tibshirani (1996), there are two reasons why data analysts are often not satisfied with the OLS (ordinary least squares) estimates: prediction accuracy, since the OLS estimates often have low bias but large variance, and interpretation. It is well known that the fewer the coefficients, the lower the prediction variance is, but at the cost of increasing bias (BREIMAN, 1995). This is the traditional trade-off between variance and bias.

Therefore, Tibshirani (1996) proposed a method of estimation in linear models called LASSO (least absolute shrinkage and selection operator), which is capable of producing exactly zero coefficients by minimizing the residual sum of squares subject to the sum of absolute values of the coefficients being less than a constant, leading to estimation and variable selection simultaneously. LASSO continuously shrinks the coefficients toward 0 as λ (penalty parameter) increases, and some coefficients are shrunk to exact 0 if λ is sufficiently large (ZOU, 2006).

In his article, Tibshirani (1996) found evidences that when there is a small to moderate number of moderate effects, LASSO performs better than ridge regression and subset selection in terms of prediction accuracy. Furthermore, as reported by Zou (2006), continuous shrinkage often improves prediction accuracy due to the bias-variance tradeoff. But LASSO tends to arbitrarily select only one variable among a group of predictors with high pairwise correlations, and this may result in some unimportant predictors that are highly correlated with the important ones being selected by LASSO while important predictors are missed (MA; FILDES; HUANG, 2016).

Zou (2006) proposed a modification in Tibshirani's model targeting to obtain consistent variable selection. It was named adaLASSO (adaptive LASSO) and uses adaptive weights for penalizing different coefficients in the penalty term. It is essentially a convex optimization problem used for an ℓ_1 constraint and therefore, the adaptive LASSO can be solved by the same efficient algorithm for solving the LASSO (ZOU, 2006).

Although LASSO variable selection can be inconsistent in some scenarios, the adaptive LASSO enjoys the oracle properties by using the adaptive weighted ℓ_1 penalty (ZOU, 2006). Medeiros e Mendes (2016) showed that adaLASSO can be applied in a very general time-series framework, allowing the errors and the regressors to be non-Gaussian and conditionally heteroskedastic, which is important when financial or macroeconomic data are considered.

Konzen e Ziegelmann (2016) proposed a variation called WLadaLASSO (weighted lag adaptive LASSO), a method which assigns different weights to each coefficient and also further penalizes coefficients of higher-lagged covariates. Their results show the superiority of WLadaLASSO when it is compared to LASSO and adaLASSO, essentially for a higher linear dependence between predictors and a higher number of candidate lags (KONZEN; ZIEGELMANN, 2016). In their empirical application, Konzen e Ziegelmann (2016) found good forecasting results for the series of risk premium and US inflation.

In this study we propose an adaptation on the WLadaLASSO pursuing better out-of-sample forecasting performance for seasonal time series in high dimensionality scenarios, called SWLadaLASSO (seasonal weighted lag adaptive LASSO). It assigns larger penalization on higher-lagged covariates coefficients but those which are seasonal lags of the variable being estimated. A more precise explanation will be brought in the next sections.

In this paper, Monte Carlo studies as well as an empirical application are conducted. For the empirical analysis, we use Brazilian monthly GDP. Some measures of accuracy will be compared for SWLadaLASSO, WLadaLASSO, adaLASSO, LASSO as well as six non LASSO-type penalty models.

The rest of this paper is as follows. The next chapter will describe LASSO, adaLASSO, WLadaLASSO and the proposed SWLadaLASSO models. Chapter 3 shows the results obtained via Monte Carlo simulation comparing SWLadaLASSO with the other LASSO-type penalty models. Chapter 4 brings the empirical application to Brazilian monthly GDP growth rates and, finally, chapter 5 announces the conclusions.

2 LASSO-TYPE PENALTIES

Suppose a data set containing $(\boldsymbol{x}, \boldsymbol{y})$, where $\boldsymbol{x}$ is a $n \times p$ matrix of p covariates and n observations and $\boldsymbol{y}$ a $n \times 1$ vector containing the response variable. The main interest of this article is to obtain good predictions of $\boldsymbol{y}$ when it is a seasonal time series.

In this context, SARIMA (seasonal autoregressive integrated moving average) models, among others, have been very popular in the literature. However, they usually face a limitation concerning the number of predictors. Having in mind a scenario with a large set of covariates, the LASSO-type penalty models become attractive options.

The motivation for LASSO comes from Breiman's non-negative garotte, which has consistently lower prediction error than subset selection and is competitive with ridge regression except when the true model has small non-zero coefficients (TIBSHIRANI, 1996).

Let $\widehat{\beta}_k$ be the original OLS estimates. Solving the problem below for c_k under constraints $c_k \geq 0$, $\sum_k c_k \leq s$ we have that $\tilde{\beta}_k(s) = c_k \widehat{\beta}_k$ are the new coefficients (BREIMAN, 1995).

$$\min \quad \sum_k \left(y_n - \sum_k c_k \widehat{\beta}_k x_{kn} \right)^2 \tag{2.1}$$

The solution depends on both the sign and the magnitude of the OLS estimates (TIBSHIRANI, 1996). When s decreases, more of the coefficients c_k become zero and the remaining nonzero $\tilde{\beta}_k(s)$ are shrunk (BREIMAN, 1995). According to Tibshirani (1996), in overfit or high correlation settings where OLS estimates behave poorly, garotte may suffer as a result; LASSO, on the other hand, avoids the explicit use of the OLS estimates.

2.1 LASSO

Proposed by Tibshirani (1996), it is a shrinkage method like the ridge, with subtle but important differences (HASTIE; TIBSHIRANI; FRIEDMAN, 2009). The LASSO estimates $(\widehat{\alpha}, \widehat{\boldsymbol{\beta}})$ are defined as

$$\left(\widehat{\alpha}, \widehat{\boldsymbol{\beta}}\right) = \arg\min_{\alpha, \boldsymbol{\beta}} \left\{ \sum_{i=1}^{N} \left(y_i - \alpha - \sum_j \beta_j x_{ij} \right)^2 \right\}, \qquad \text{subject to } \sum_{j=1}^{p} |\beta_j| \leq t, \tag{2.2}$$

where $\hat{\boldsymbol{\beta}} = \left(\hat{\beta}_1, ..., \hat{\beta}_p\right)'$ and t is a tuning parameter controlling the amount of shrinkage that is applied to the estimates and $\forall t \ \widehat{\alpha} = \bar{y}$ (TIBSHIRANI, 1996). It is assumed that x_{ij} are standardized, meaning $\bar{x}_i = 0$ and $s^2_{x_i} = 1$. Considering $\bar{y} = 0$, we can omit α.

Another way to express the solution for $\boldsymbol{\beta}$ (considering $\bar{y} = 0$) is presented below:

$$\widehat{\boldsymbol{\beta}}_{LASSO} = \arg\min_{\boldsymbol{\beta}} \left\{ \left\| \boldsymbol{y} - \sum_{j=1}^{p} \boldsymbol{x}_j \beta_j \right\|^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}. \tag{2.2'}$$

The second term in (2.2') is referred to as ℓ_1 penalty, which is crucial for the success of LASSO (ZOU, 2006). LASSO uses an ℓ_1 penalty to achieve a sparse solution. Its constraints make the solutions nonlinear in y_i, and there is no closed form expression as in ridge regression (HASTIE; TIBSHIRANI; FRIEDMAN, 2009).

LASSO continuously shrinks the coefficient toward 0 as λ increases, shrinking some of them exactely to 0 if λ if sufficiently large (ZOU, 2006). t, or λ, should be adaptively chosen to minimize an estimate of expected prediction error (HASTIE; TIBSHIRANI; FRIEDMAN, 2009).

2.2 ADALASSO

Zou (2006) showed that LASSO cannot be an oracle procedure and, in order to correct this problem, he assigned different weights to different coefficients, which he called adaptive LASSO. It is a new version of the LASSO in which adaptive weights are used for penalizing different coefficients in the ℓ_1 penalty and enjoys the oracle properties.

Thus, his definition of weighted LASSO is given by

$$\begin{aligned} \widehat{\boldsymbol{\beta}}_{adaLASSO} &= \arg\min_{\boldsymbol{\beta}} \left\{ \left\| \boldsymbol{y} - \sum_{j=1}^{p} \boldsymbol{x}_j \beta_j \right\|^2 + \lambda \sum_{j=1}^{p} \widehat{w}_j |\beta_j| \right\}, \\ \widehat{\boldsymbol{w}} &= \left| \widehat{\boldsymbol{\beta}} \right|^{-\tau}, \qquad \gamma > 0 \end{aligned} \tag{2.3}$$

where $\widehat{\boldsymbol{w}}$ stands for the estimator of the vector of weights and $\widehat{\boldsymbol{\beta}}$ is an n-consistent estimator of $\boldsymbol{\beta}$, for example $\widehat{\boldsymbol{\beta}}_{OLS}$ (ZOU, 2006). Another possibility can be $\widehat{\boldsymbol{\beta}}_{ridge}$.

2.3 WLADALASSO

In a time series context, Konzen e Ziegelmann (2016) borrowed Prak and Sakaori's idea of penalizing lags differently proposing the WLadaLASSO (weighted lag adaptive LASSO), leading to the following estimator:

$$\begin{aligned} \widehat{\boldsymbol{\beta}}_{WLadaLASSO} &= \arg\min_{\boldsymbol{\beta}} \left\{ \left\| \boldsymbol{y} - \beta_0 - \sum_{j=1}^{p} \boldsymbol{x}_j \beta_j \right\|^2 + \lambda \sum_{j=1}^{p} \widehat{w}_j |\beta_j| \right\}, \\ \widehat{w}_j &= \left(\left| \widehat{\beta}_j^{ridge} \right| e^{-\alpha l} \right)^{-\tau}, \qquad \tau > 0, \quad \alpha > 0, \end{aligned} \tag{2.4}$$

where l is the lag order of the variable multiplied by β_j (KONZEN; ZIEGELMANN, 2016). β_0 can be omitted if $\bar{y} = 0$.

2.4 SWLADALASSO

Forecasting economics behavior is an important challenge in modern econometric's literature, contributing to reduce economics agents' uncertainty towards decision making (ZUANAZZI; ZIEGELMANN, 2014). Therefore, consumers, governments and companies aim to have as accurate forecasts as possible. Many time series like GDP, inflation, sales and consumption are measured monthly or quarterly and they are influenced by seasonality.

In this article we propose an adaptation in Konzen e Ziegelmann (2016)'s model, WLadaLASSO, in order to improve forecasts in seasonality scenarios. Assuming a monthly time series, for the first seasonal lag of the response variable we have $\widehat{w}_j = \left(\left|\widehat{\beta}_j^{ridge}\right| e^{-\alpha\times 12}\right)^{-\tau}$ as weight when using WLadaLASSO, nevertheless it is relatively big. Our intention is to reduce this weight for the coefficients of the covariates which are seasonal lags of the response variable, then reducing the penalty for those seasonal lags.

Thus, the model representation is pretty similar to the others presented previously,

$$\widehat{\boldsymbol{\beta}}_{SWLadaLASSO} = \arg\min_{\boldsymbol{\beta}} \left\{ \left\| \boldsymbol{y} - \beta_0 - \sum_{j=1}^{p} \boldsymbol{x}_j \beta_j \right\|^2 + \lambda \sum_{j=1}^{p} \widehat{w}_j |\beta_j| \right\},$$
$$\widehat{w}_j = \begin{cases} \left(\left|\widehat{\beta}_j^{ridge}\right| e^{-\alpha\times l\times\delta}\right)^{-\tau} & \text{if seasonal lag of the response variable} \\ \left(\left|\widehat{\beta}_j^{ridge}\right| e^{-\alpha\times l}\right)^{-\tau} & \text{otherwise} \end{cases}, \tag{2.5}$$
$$\tau > 0, \quad \alpha > 0, \quad 0 < \delta \leq 1.$$

When $\delta \in (0,1)$, the inclusion of the multiplicative term δ reduces the weights tied with seasonal variables, once the inequality $e^{-\alpha\times l\times\delta} < e^{-\alpha\times l}$ is always true following the proposed restrictions, and therefore $\widehat{w}_{j,SWLadaLASSO} \leq \widehat{w}_{j,WLadaLASSO}$. Consequently, its given them less penalty, and they are more likely of having nonzero coefficients.

Another advantage is that it allows $\delta = 1$, which brings us back to WLadaLasso. Hence, SWLadaLasso adapts well to both cases, series with seasonal component and series without seasonal component.

In our implementations we follow Konzen e Ziegelmann (2016) and define $\tau = 1$. δ and α are automatically chosen form a set of predefined values according to minimum squared errors.

3 SIMULATION

In order to verify the performance of the model, in this section we compare the simulation results for all the LASSO-type penalties presented before. Implementations were conducted with software *R* using the *glmnet* package for optimization.

There are several ways to choose the tuning parameter λ. If users are data rich, they can set aside some fraction of their data for this purpose, evaluate the prediction performance at each value of λ, and pick the model with the best performance (FRIEDMAN; HASTIE; TIBSHIRANI, 2010). Alternatively, one can use K-fold cross-validation, where the training data is used both for training and testing in an unbiased way (FRIEDMAN; HASTIE; TIBSHIRANI, 2010). Often a "one-standard error" rule is used with cross-validation, in which we choose the most parsimonious model whose error is no more than one standard error above the error of the best model (HASTIE; TIBSHIRANI; FRIEDMAN, 2009).

In all of our Monte Carlo simulation replicates we choose λ via cross-validation using the *cv.glmnet* function. Following Hastie, Tibshirani e Friedman (2009), we decide for $lambda.1se$, which is the largest value of λ such that estimation error is within 1 standard error of the minimum mean squared error (MSE). It allows more coefficients to be set exactly to zero when compared to the λ that gives the smallest MSE; in other words it may lead to the choice of fewer covariates. Our option for using cross-validation was based on Konzen e Ziegelmann (2016) and is justified by the fact that common AIC and BIC criteria are not applied to LASSO-type models.

We set a combination of δ and α, choosing λ (as previously explained) for all of the combinations of δ and α to be compared. Then, maintaining δ fixed, we choose α which provides the smallest MSE considering the optimal λs already obtained. Finally, we determine δ, also by the smallest MSE among the chosen λ and α.

A total of 1000 Monte Carlo simulation replicates were conducted considering the data generating process mimicking a monthly time series

$$\begin{aligned} y_t = &-0.5x_{1,t-1} + 0.5x_{2,t-1} - 0.5x_{3,t-1} + 0.5x_{4,t-1} \\ &- 0.4x_{1,t-2} + 0.4x_{2,t-2} - 0.4x_{3,t-2} + \\ &+ 0.2y_{t-1} + 0.3y_{t-12} + 0.2y_{t-24} + \varepsilon_t, \qquad t = 1, ..., T, \end{aligned} \tag{3.1}$$

where $x_{i,t} = \phi x_{i,t-1} + u_{i,t}$, $i = 1, 2, ..., 10$, are independent AR(1) processes and $u_{i,t} \sim \mathcal{N}(0, 1)$. Despite using only x_1, x_2, x_3 and x_4 in the simulation, we have generated ten independent variables in order to see how the method would respond to different quantities of irrelevant covariates. The disturbance term ε_t has two specifications: $\mathcal{N}(0, 1)$ and $t(5)$.

We estimate an expanded model (3.1) considering lags of exogenous variables as well as lags of the response variable. For x_i, $i = 1, 2, ..., 10$, and y_t the option was for using $L = 3$, 6 and 9 lags. In terms of seasonality the choice was for three lags, y_{t-12}, y_{t-24} and y_{t-36}, in all of the scenarios. Therefore we have $L \times 10 + L \times 1 + 3$ candidate covariates. The methods used were LASSO, adaLASSO, WLadaLASSO and SWLadaLASSO.

We also use three values of ϕ, (0.3, 0.5 and 0.7) and four sample sizes ($n = 120$, 180 and 240) considering 10, 15 and 20 years. To evaluate the forecasts we consider one and twelve-step-ahead forecasts for each of the 1000 replications.

Table 1 shows the measures used to compare the models. Seven of these measures refer precisely to the coefficient's estimation, two analyse the accuracy of one-step-ahead forecasts and another two relate to the 12 months horizon. Our primary interests are on the forecast's precision. Tables 14 to 25, in Appendix A and B, show the results for all combinations of ϕ, n, L and error distributions, resulting in $3 \times 3 \times 3 \times 2 = 54$ different scenarios.

As seen in these tables, all models showed good results for most of the measures, but the results of SWLadaLASSO were more precise, as we expected. We consider sample sizes 120 and 180 as the most relevant ones due to data availability. Sample sizes of 240 may be hard to obtain for some time series, once reliable data sets were not still widespread 20 years ago, therefore this scenario will probably be less common in empirical applications.

Table 1 – Performance measures used for comparison

Estimation	
frvi	Average fraction of relevant variables included
fvci	Average fraction of variables correctly identified: relevant as relevant and irrelevant as irrelevant
niv	Average number of included variables
five	Fraction of irrelevant variables excluded
tmi	Fraction where the true model is included: all the relevant variables are included
mae	MAE: Mean absolute error
rmse	RMSE: Root mean squared error
Forecast	
maef1	Forecast: MAE one-step-ahead
maefs	Forecast: MAE twelve-step-ahead
rmsef1	Forecast: RMSE one-step-ahead
rmsefs	Forecast: RMSE twelve-step-ahead

Under the 27 scenarios following normal disturbances, SWLadaLASSO performed well in terms of *fraction of irrelevant variables excluded* and *average number of included variables*, winning in 18 and 20 possibilities, respectively. When we disregard size 240, the number of wins increase from 66.67% to 77.78% and from 74.07% to 83.33%, each. It means that the model achieved the best results in variables exclusion and also selected the closest number of variables most of the times. Moreover, under the sample sizes we consider most important, results were even better. The measure *average fraction of variables correctly identified: relevant as relevant and irrelevant as irrelevant* had similar behavior. Under t disturbances the simulation as a whole was even more favourable to SWLadaLASSO for these three measures.

On the other hand, SWLadaLASSO performed poorly regarding *average fraction of relevant variables included* and *fraction where the true model is included: all the relevant variables are included* for both normal and t disturbances. Under normal distribution SWLadaLASSO had the largest value for *average fraction of relevant variables included* only six times, while adaLASSO performed best three times and LASSO nineteen times. It sums more than 27 due to a tie between LASSO and adaLASSO in the scenario of $\phi = 0.7$, $n = 180$ and $L = 6$, as can be seen in Table 18. For *fraction where the true model is included: all the relevant variables are included* these numbers were 4, 2 and 21.

These two measures are related. *Average fraction of relevant variables included* shows the average fraction of relevant variables included in the model. If all ten variables from 3.1 are included in all the replications, *average fraction of relevant variables included* $= 1$ and *fraction where the true model is included: all the relevant variables are included* $=$ 1 as well. However, if one variable is missed in all replications, *average fraction of relevant variables included* $= 0.9$ and *fraction where the true model is included: all the relevant are included* $= 0$. In other words, *fraction where the true model is included: all the relevant variables are included* is a very sensible measure.

Additionally, looking to the numbers for SWLadaLasso in the appendix, we see that it is not an issue we should worry about, since *average fraction of relevant variables included* stood between 0.9370 and 0.9977 considering all scenarios under Gaussian disturbances and between 0.8595 and 0.9735 under Student disturbances. For *fraction where the true model is included: all the relevant variables are included* the results laid between 0.5670 and 0.9770 and between 0.3060 and 0.8120, respectively, for normal and t. Even the 0.5670 result should not be considered a big problem when we analyse the overall results, because, once the other measures were good, it may only reflect that a small number of variables was not included in many repetitions. One explanation is the small number of variables chosen by SWLadaLASSO, which was smaller and closer to the true in the underlying data generating model.

In terms of *MAE* and *MSE* the outcomes were good for all of the four models, the worst results under Gaussian errors were 0.1042 and 0.1456 respectively, both for WLadaLasso. Under Student errors these numbers were 0.1098 and 0.1641, also for WLadaLasso. Under these two criteria, for some scenarios SWLadaLASSO performed best, for others adaLASSO.

As mentioned before, our greatest interest is related to forecasts. When we consider the one-step-ahead forecast, SWLadaLASSO performs best in all of the scenarios for both *MAE* and RMSE under Gaussian errors. For twelve-step-ahead forecast it was not the best only once in terms of *RMSE* and twice in terms of *MAE*. These cases are for adaLASSO when $\phi = 0.5$ and $n = 240$.

Under t disturbances, all of the 27 scenarios were favourable to SWLadaLASSO considering a forecast horizon of 12 months both in terms of *MAE* and *RMSE*. Yet, for one-month horizon it performed best 25 times in terms of *MAE* and 23 in terms of *RMSE*. Five out of the six losses were under sample size 240. The other one was under the scenario $\phi = 0.7$, $lag = 6$ and sample size 180.

Table 2 summarizes the results brought up in the twelve tables from Appendix A and B. Considering parameter estimation, SWLadaLASSO had the best results in about half of the scenarios, being even better under Student error disturbances.

For one-step-ahead forecasts under normal error distribution, SWLadaLASSO had the best results for all replicates in terms of *RMSE*. Under $t(5)$, it decreased to 88.89%, while for LASSO it increased to 11.11%. However, excluding $n = 240$ under t, SWLadaLASSO improves considerably to 97.22%. For twelve-step-ahead forecasts, the simulation results are pretty similar. Consequently, observing Table 2, the proposed model outperformed the other LASSO-type penalty models in terms of estimation and, specially, in terms of forecasts, which is our main interest.

Table 2 – Simulation: summary results, proportion of best results

		LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
Normal	Estimation	0.2116	0.3122	0.0000	0.4815
	1 Step Forecast	0.0000	0.0000	0.0000	1.0000
	12 Steps Forecast	0.0000	0.0556	0.0000	0.9444
t	Estimation	0.2381	0.1958	0.0000	0.5661
	1 Step Forecast	0.0000	0.1111	0.0000	0.8889
	12 Steps Forecast	0.0000	0.0000	0.0000	1.0000

Note: The first line adds more than one because of a tie between LASSO and adaLASSO

4 EMPIRICAL ANALYSIS

We chose the series of monthly Brazilian GDP growth rates from January 1998 to December 2016, totalizing 228 observations. The last 48 observations were left out-of-sample to evaluate out-of-sample forecasts. Besides the lags of the response variable, we also used 151 exogenous variables, detailed in Tables 26, 27 and 28 from the Appendix C. All data was obtained freely from the websites IPEADATA and Federal Reserve Bank of St. Louis.

Data was previously treated for non-stationarity and missing values. For missing value imputation we used the *auto.arima* function. We compare the forecasts obtained from the following ten methods: LASSO, adaLASSO, WLadaLASSO, SWLadaLASSO, three options of least squares with principal component as exogenous variables, SARIMA$(1,0,0)(1,0,0)_{12}$, SARIMA$(1,0,0)(1,0,0)_{12}$ with 6 covariates and SETAR of order 1 with one threshold. The same variables were considered in all of the models, except for SARIMA and SETAR which only used lags of the response variable.

The LASSO-type penalty options used as covariates the three first lags of the independent variables, six first lags and two first seasonal lags (12 and 24 months) of the response variable. Consequently, in the regressors matrix we had 461 variables. Since estimation sample size is 180, ordinary least squares it self was not able to estimate coefficients, and because of this we decided to use principal component analysis to reduce the dimensionality of exogenous variables when estimating by least squares.

The same lags used for the response variable in the LASSO-type penalty options were also used in the OLS model. The difference lays in in the exogenous variables. Three cases were tested for principal component: factors explaining 70%, 80% and 90% of the variance in the matrix of lagged exogenous variables.

Six covariates were used as exogenous variables for the SARIMA model: first difference of purchasing power parity rate; first difference of savings (% p.m.); number of business days; capacity utilization (%); first difference of 6-month treasury constant maturity rate and first difference of inflation - IPC (FGV) - (% p.m.). These variables were chosen due to the fact that they were the ones selected more often considering the four LASSO-type penalty models.

4.1 FORECASTS RESULTS

In Table 3 we show the RMSE values for forecast horizons of 1, 2, 3, 4, 5, 6 and 12 months. We can see that in terms of RMSE SWLadaLASSO had the best performance for all horizons excluding 4 months.

Table 3 – RMSE for GPD forecast

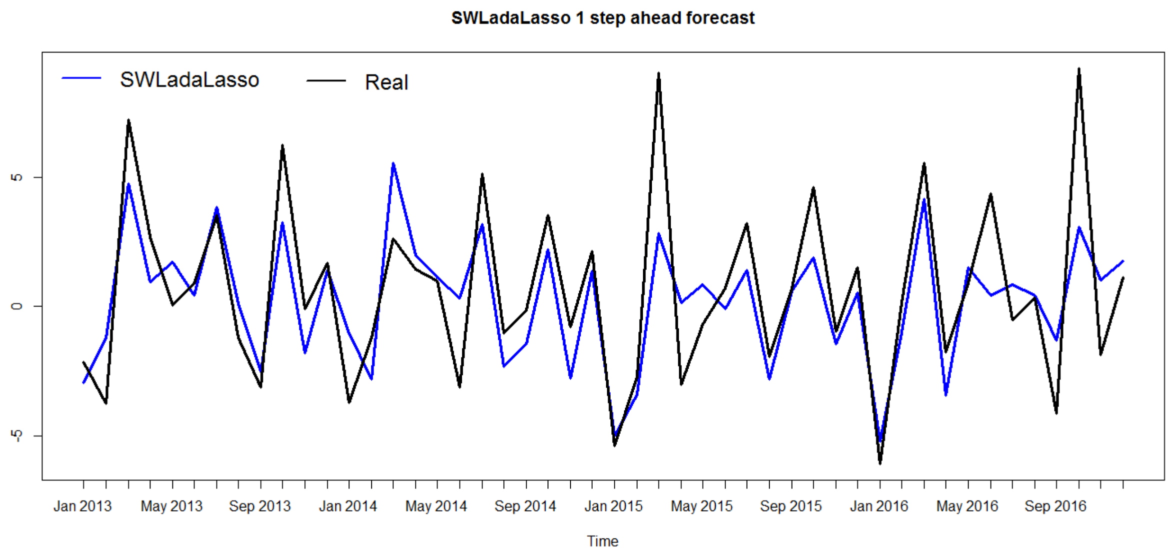
Horizon	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	f70	f80	f90	SARIMA	SARIMA X	Setar
1	2.2701	2.1825	2.4586	2.1477	3.0425	3.2752	3.2252	2.2719	2.1735	3.5867
2	2.2647	2.2003	2.7217	2.1535	3.0122	3.2575	3.2252	2.5355	2.2304	3.6382
3	2.2402	2.1690	2.4311	2.0636	2.9565	3.1816	3.2754	2.4968	2.1967	3.4394
4	2.2003	2.1330	2.4930	2.1855	3.0059	3.2035	3.2574	2.5044	2.2144	3.4246
5	2.2333	2.1611	2.4839	2.1517	2.9822	3.1876	3.2299	2.4255	2.2157	3.5047
6	2.1803	2.1358	2.5214	2.1014	2.8965	3.1272	3.1920	2.3888	2.2201	3.5446
12	2.2796	2.2101	2.7655	2.1938	3.0834	3.1594	3.2008	2.5292	2.2766	3.7283

Source: Own construction

4.1.1 Forecast Horizon 1

Table 3 gives us an idea of the importance of SWLadaLASSO, which is corroborated by Figure 1, where we can roughly realize how close to the observations the forecasts are.

Figure 1 – One-step-ahead forecast of GDP growth rate



Source: Own construction

According with the Modifed Diebold Mariano Test, in Table 4, we can say that SWLadaLASSO was better than OLS with principal component as well as SETAR. However, despite having lower RMSE, it can not be considered superior than LASSO, adaLASSO, WLadaLASSO, SARIMA and SARIMA with covariates in statistical terms.

Table 4 – Modified Diebold Mariano Comparison With SWLadaLASSO and Forecast Horizon 1

Model	p-value
LASSO	0.1563
adaLASSO	0.2231
WLadaLASSO	0.1200
f70	0.0025
f80	0.0003
f90	0.0005
SARIMA	0.2583
SARIMA X	0.4407
SETAR	0.0000

Note: Under H_1 SWLadaLASSO is greater than the other model

Source: Own construction

Seeking accuracy we also performed the Model Confidence Set (MCS) procedure, considering the squared error loss. The chosen models were SWLadaLASSO, SARIMA X and adaLASSO, as shown in Table 5 with their respective losses measures.

Table 5 – Forecast Horizon 1 - Superior Model Set

Method	Loss
SWLadaLASSO	4.6127
SARIMA X	4.7241
adaLASSO	4.7633

Source: Own construction

Therefore, we concluded that SWLadaLASSO brought good contribution in forecasting monthly Brazilian GDP growth rate considering the one month horizon. It had the best results in terms of RMSE and it was selected in the best set of models by MCS.

Furthermore, we wanted to see if any benefits emerge by combinating forecasts. So, we broke the 48 periods left out-of-sample into 24 in-sample and 24 out-of-sample forecasts to perform forecasts combinations. Thereafter, we used the models chosen by MCS, shown in Table 5, and combined them in six ways provided by *ForecastCombinations* package in R, as explained by Raviv (2015):

- Simple: simple average between forecasts;
- OLS: OLS using the realization as response variable and the forecasts as covariates;
- Robust: the same as OLS, but minimizes a different loss function, which is less sensitive to outliers;

- Variance based: constrained least squares that minimizes the sum of squared errors under the restriction that the weights sum up to 1, and that the forecasts themselves are unbiased;
- CLS: the variance-based method computes the MSE and weigh the forecasts according to their accuracy, giving relatively more weight to accurate forecasts;
- Best: all the weight is given to the best model regarding MSE.

Table 6 shows the weights given for each forecasts in the six types of combinations. It can be seen that in the first 24 observations the best model was SARIMA with covariates, despite the fact that considering the 48 periods of forecast the best one was SWLadaLASSO. It is a clear example in favor of forecast combination, once it assumes that even if one model is better than the others it does not mean that others may not offer useful information.

Table 6 – One-step-ahead forecast combinations weights

	adaLASSO	SWLadaLASSO	SARIMA X
Simple	0.3333	0.3333	0.3333
OLS	0.6085	-0.1254	0.5630
Robust	-1.1916	1.1072	1.0531
Variance based	0.3335	0.3178	0.3487
CLS	0.4448	0.0000	0.5552
Best	0.0000	0.0000	1.0000

Source: Own construction

From that point on, when analysing one-step-ahead forecast, we work with our 10 initial options adding the 5 new ones provided by the forecast combinations. We did not take into consideration the combination given by Best because it is exactly the SARIMA X forecast.

Table 7 – One-step-ahead: RMSE for the last 24 observations

LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	f70
2.6724	2.5402	2.7373	2.4494	3.1111
f80	f90	SARIMA	SARIMA X	Setar
3.5008	3.5530	2.4796	2.5510	3.8764
Simple	OLS	Robust	Variance Based	CLS
2.4487	2.4590	2.5879	2.4489	2.4728

Source: Own construction

In Table 7 we observe the RMSE for the one-step-ahead forecast of the last twenty four observations provided by our 15 options. Not considering any combination, the best model was SWLadaLASSO and a little gain was obtained by Simple and Variance Based combinations. Once again we proceed with MCS selecting three options: SWLadaLASSO, and 2 combinations - Simple and Variance Based -, as one can see in Table 8.

Table 8 – Forecast Horizon 1, Last 24 oservations - Superior Set Model

Model	Loss
Simple	5.9959
Variance Based	5.9971
SWLadaLASSO	5.9997

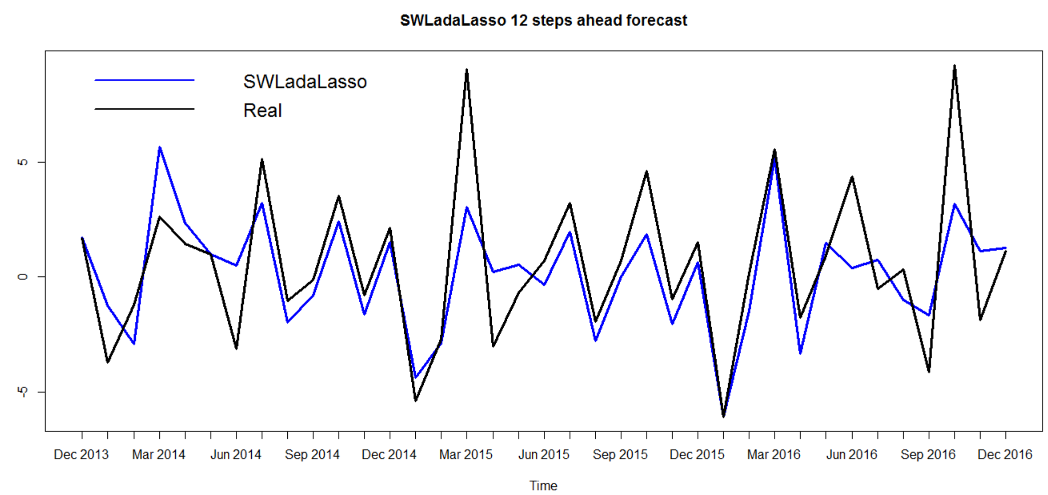
Source: Own construction

It is interesting to notice that SWLadaLASSO is used in all of the 3 chosen methods. Therefore, when considering the last 24 realizations of one-step-ahead forecasts, SWLadaLASSO also performed better than the other options in debate: it had the smallest RMSE and it was the only one included in the MCS, meaning it really brought contributions in forecasting one-step-ahead Brazilian GDP growth rate. Additionally, in this case the Simple combination produced the best result, reinforcing the belief that combinations, even the simpler ones, may contribute to forecasts accuracy.

4.1.2 Forecast Horizon 12

For twelve-step-ahead forecast we lose eleven out-of-sample observations, reducing from forty eight to thirty seven. Once again, SWLadaLASSO had the best performance in terms of RMSE, as showed in Table 3. Visually, observing Figure 2, the forecasts seem also to be good, following the dynamics pattern of the observations.

Figure 2 – Twelve-step-ahead forecast GDP Estimation



Source: Own construction

At a 5% significance level, using the Modified Diebold Mariano test in Table 9 we conclude that SWLadaLASSO produces superior forecasts when compared pair-wisely with WLadaLASSO, least squares with principal component, SARIMA and SETAR.

Table 9 – Modified Diebold Mariano Comparison With SWLadaLASSO and Forecast Horizon 12

Model	pvalue
LASSO	0.2098
adaLASSO	0.4165
WLadaLASSO	0.0177
f70	0.0052
f80	0.0030
f90	0.0030
SARIMA	0.0218
SARIMA X	0.3404
SETAR	0.0006

Note: Under H_1 SWLadaLASSO produces superior forecasts than the other model

Source: Own construction

Considering the squared error loss, MCS procedure is in accordance with Diebold-Mariano results and selects SWLadaLASSO, adaLASSO, SARIMA X and LASSO as the best set of forecasting methods, as one can see in Table 10. It can also be observed that SWLadaLASSO has the lower squared error loss among our options.

Table 10 – Forecast Horizon 12 - Superior Model Set

Method	Loss
SWLadaLASSO	4.8129
adaLASSO	4.8844
SARIMA X	5.1830
LASSO	5.1964

Source: Own construction

In the following, the out-of-sample observations were split in a new group of 13 in-sample and 24 out-of-sample observations to see if there was a gain from combining forecasts. In Table 11 we observe that the best model forecasting the first 13 twelve-step-ahead observations was LASSO.

Table 11 – Twelve-step-ahead forecast combinations weights

	LASSO	adaLASSO	SWLadaLASSO	SARIMA X
Simple	0.2500	0.2500	0.2500	0.2500
OLS	1.1128	-3.1552	2.7079	0.3659
Robust	0.9452	-2.4229	2.6125	-0.1575
Variance Based	0.3110	0.2207	0.2253	0.2429
CLS	0.9296	0.0000	0.0000	0.0704
Best	1.0000	0.0000	0.0000	0.0000

Source: Own construction

As can be seen in Table 12, SWLadaLASSO had the best performance under RMSE for the last 24 observations considering forecasting horizon of 12. It is also worth mentioning that the Simple combination achieved the smallest RMSE compared to all the methods, but SWLadaLASSO. Once again it is corroborating the idea of benefits coming from combinations, even th elementary ones.

Table 12 – Twelve-step-ahead: RMSE for the last 13 observations

LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	f70
2.6113	2.4187	3.0628	2.4030	3.0540
f80	f90	SARIMA	SARIMA X	Setar
3.1886	3.3960	2.8064	2.5425	4.2209
Simple	OLS	Robust	Variance Based	CLS
2.4100	3.1036	2.8981	2.4195	2.5878

Source: Own construction

Finally, via the MCS procedure we select four methods: SWLadaLASSO, adaLASSO and the 2 combinations - Simple and Variance Based - as the best set of forecasting models for the last 24 observations. Therefore, SWLadaLASSO is included in 3 out of 4 selected forecasting methods, reinforcing its remarkable performance.

Table 13 – Forecast Horizon 12, Last 24 observations - Superior Set Model

Model	Loss
SWLadaLASSO	5.774526
Simple	5.807945
adaLASSO	5.850203
Variance based	5.854131

Source: Own construction

5 CONCLUSION

We propose a new LASSO-type penalty suitable for seasonal time series. From an applied point of view, this new approach is particularly interesting for macroeconomic time series because many of them are monthly or quarterly registered, and would in general inherit seasonality.

SWLadaLASSO proposes a generalization of WLadaLASSO suggested by Konzen e Ziegelmann (2016). In our Monte Carlo simulations SWLadaLASSO performed very well concerning parameter estimation. The main interest, though, was related to out-of-sample forecast improvement, where SWLadaLASSO had an even better performance.

In terms of our empirical analysis of monthly Brazilian GDP growth, SWLadaLASSO performed really well. In general, we can say that its performance was the best among all the single methods. Forecast combinations were also implemented, adding contributions in terms of forecast accuracy. Nevertheless, SWLadaLasso was often included with large weight in the best performing forecast combinations.

BIBLIOGRAPHY

BREIMAN, L. Better subset regression using the nonnegative garrote. *Technometrics*, Taylor & Francis Group, v. 37, n. 4, p. 373–384, 1995.

ÇAKMAKLI, C.; DIJK, D. van. Getting the most out of macroeconomic information for predicting excess stock returns. *International Journal of Forecasting*, Elsevier, v. 32, n. 3, p. 650–668, 2016.

CHAUVET, M.; LIMA, E. C.; VASQUEZ, B. Forecasting brazilian output in real time in the presence of breaks: a comparison of linear and nonlinear models. *Discussion paper 118*, Instituto de Pesquisa Econômica Aplicada (Ipea), 2015.

FRIEDMAN, J.; HASTIE, T.; TIBSHIRANI, R. Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, v. 33, n. 1, p. 1–22, 2010.

HASTIE, T.; TIBSHIRANI, R.; FRIEDMAN, J. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. 2. ed. [S.l.]: Springer, 2009. (Springer Series in Statistics).

KONZEN, E.; ZIEGELMANN, F. A. Lasso-type penalties for covariate selection and forecasting in time series. *Journal of Forecasting*, Wiley Online Library, v. 35, n. 7, p. 592–612, 2016.

MA, S.; FILDES, R.; HUANG, T. Demand forecasting with high dimensional data: The case of SKU retail sales forecasting with intra-and inter-category promotional information. *European Journal of Operational Research*, Elsevier, v. 249, n. 1, p. 245–257, 2016.

MEDEIROS, M. C.; MENDES, E. F. $\ell 1$-regularization of high-dimensional time-series models with non-gaussian and heteroskedastic errors. *Journal of Econometrics*, Elsevier, v. 191, n. 1, p. 255–271, 2016.

RAVIV, E. *ForecastCombinations: Forecast combinations in R*. 2015. Accesso em: 22 ago. 2017. Disponível em: <https://CRAN.R-project.org/package=ForecastCombinations>.

TIBSHIRANI, R. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, JSTOR, v. 58, n. 1, p. 267–288, 1996.

ZOU, H. The adaptive lasso and its oracle properties. *Journal of the American statistical association*, Taylor & Francis, v. 101, n. 476, p. 1418–1429, 2006.

ZUANAZZI, P. T.; ZIEGELMANN, F. A. Previsões para o crescimento do pib trimestral brasileiro com séries financeiras e econômicas mensais: uma aplicação de midas. *Economia Aplicada*, SciELO Brasil, v. 18, n. 2, p. 295–318, 2014.

APPENDIX A – **SIMULATION RESULTS: ESTIMATION**

Table 14 – Estimation results for $\phi = 0.3$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	five	0.8470	0.9075	0.5531	0.9209	0.8828	0.9074	0.6984	0.9257
3	180	five	0.8915	0.9577	0.1482	0.9625	0.9158	0.9588	0.3861	0.9656
3	240	five	0.9157	0.9799	0.0577	0.9797	0.9413	0.9813	0.1725	0.9847
6	120	five	0.8951	0.8963	0.7971	0.9226	0.9219	0.8981	0.8353	0.9308
6	180	five	0.9209	0.9520	0.6845	0.9667	0.9392	0.9537	0.7811	0.9728
6	240	five	0.9365	0.9737	0.3524	0.9832	0.9559	0.9768	0.6386	0.9892
9	120	five	0.9175	0.8881	0.8449	0.9246	0.9383	0.8926	0.8688	0.9298
9	180	five	0.9355	0.9441	0.7865	0.9637	0.9517	0.9449	0.8320	0.9704
9	240	five	0.9501	0.9696	0.6861	0.9807	0.9642	0.9715	0.7889	0.9880
3	120	frvi	0.9656	0.9398	0.8534	0.9370	0.8726	0.8559	0.7781	0.8595
3	180	frvi	0.9963	0.9863	0.9387	0.9830	0.9668	0.9363	0.8734	0.9278
3	240	frvi	0.9998	0.9962	0.9703	0.9956	0.9891	0.9668	0.9179	0.9554
6	120	frvi	0.9394	0.9368	0.8540	0.9465	0.8141	0.8495	0.7804	0.8708
6	180	frvi	0.9941	0.9877	0.8780	0.9860	0.9570	0.9425	0.8605	0.9381
6	240	frvi	0.9998	0.9971	0.9183	0.9962	0.9854	0.9725	0.8736	0.9635
9	120	frvi	0.9192	0.9330	0.8478	0.9507	0.7804	0.8365	0.7727	0.8740
9	180	frvi	0.9916	0.9887	0.8936	0.9881	0.9465	0.9471	0.8664	0.9430
9	240	frvi	0.9996	0.9978	0.8908	0.9969	0.9837	0.9765	0.8731	0.9679
3	120	niv	13.6350	11.8020	20.1540	11.4260	11.7730	10.9660	15.6220	10.5270
3	180	niv	12.7830	10.9620	31.5330	10.8040	11.8570	10.4350	24.6960	10.1720
3	240	niv	12.1890	10.4840	34.2020	10.4830	11.4160	10.1530	30.6940	9.9520
6	120	niv	15.5850	15.4850	20.5130	14.0290	12.7460	14.5050	17.5210	12.7890
6	180	niv	14.6090	12.7080	27.3930	11.8220	13.1590	12.1550	21.5220	10.9860
6	240	niv	13.7450	11.5200	47.3920	10.9530	12.4550	11.0910	30.0560	10.2740
9	120	niv	16.7820	19.6240	22.7440	16.4420	13.4770	18.2430	19.7940	15.1960
9	180	niv	15.8460	15.0270	28.5790	13.2240	13.9090	14.5440	24.1230	12.1550
9	240	niv	14.5860	12.7750	37.7830	11.7430	13.1310	12.3830	28.1520	10.7810
3	120	tmi	0.7420	0.5560	0.2300	0.5670	0.4420	0.3070	0.1070	0.3060
3	180	tmi	0.9660	0.8710	0.5180	0.8510	0.7710	0.5560	0.2200	0.5360
3	240	tmi	0.9980	0.9630	0.7130	0.9590	0.9200	0.7330	0.3410	0.6900
6	120	tmi	0.6160	0.5650	0.0820	0.6130	0.3140	0.3050	0.0350	0.3870
6	180	tmi	0.9480	0.8860	0.2080	0.8750	0.7240	0.6070	0.0810	0.6020
6	240	tmi	0.9980	0.9720	0.4610	0.9650	0.8910	0.7740	0.1980	0.7420
9	120	tmi	0.5330	0.5500	0.0600	0.6470	0.2690	0.2860	0.0350	0.3910
9	180	tmi	0.9270	0.8990	0.1760	0.8930	0.6680	0.6410	0.0790	0.6350
9	240	tmi	0.9960	0.9780	0.2250	0.9720	0.8780	0.8070	0.1070	0.7610

Source: Own construction

Table 15 – Estimation results for $\phi = 0.3$ RMSE and MAE

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	mae	0.0486	0.0427	0.0816	0.0410	0.0615	0.0568	0.0844	0.0537
3	180	mae	0.0353	0.0283	0.0741	0.0290	0.0467	0.0397	0.0819	0.0398
3	240	mae	0.0297	0.0224	0.0609	0.0236	0.0398	0.0325	0.0754	0.0338
6	120	mae	0.0304	0.0283	0.0422	0.0247	0.0372	0.0368	0.0472	0.0315
6	180	mae	0.0211	0.0163	0.0426	0.0155	0.0273	0.0225	0.0412	0.0210
6	240	mae	0.0174	0.0121	0.0543	0.0124	0.0227	0.0174	0.0464	0.0173
9	120	mae	0.0230	0.0240	0.0309	0.0191	0.0273	0.0300	0.0353	0.0240
9	180	mae	0.0155	0.0125	0.0284	0.0111	0.0197	0.0171	0.0297	0.0148
9	240	mae	0.0125	0.0087	0.0327	0.0084	0.0162	0.0123	0.0296	0.0117
3	120	rmse	0.0995	0.0907	0.1288	0.0892	0.1278	0.1192	0.1445	0.1155
3	180	rmse	0.0755	0.0640	0.0989	0.0665	0.1012	0.0892	0.1207	0.0911
3	240	rmse	0.0640	0.0516	0.0783	0.0553	0.0876	0.0746	0.1017	0.0794
6	120	rmse	0.0820	0.0734	0.0909	0.0675	0.1026	0.0950	0.1069	0.0867
6	180	rmse	0.0598	0.0478	0.0807	0.0473	0.0791	0.0661	0.0867	0.0649
6	240	rmse	0.0500	0.0372	0.0818	0.0393	0.0674	0.0537	0.0844	0.0561
9	120	rmse	0.0730	0.0673	0.0776	0.0586	0.0892	0.0852	0.0921	0.0745
9	180	rmse	0.0522	0.0411	0.0642	0.0392	0.0682	0.0565	0.0725	0.0534
9	240	rmse	0.0430	0.0310	0.0645	0.0316	0.0576	0.0444	0.0672	0.0451

Source: Own construction

Table 16 – Estimation results for $\phi = 0.5$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	five	0.8515	0.9095	0.4483	0.9150	0.8776	0.9086	0.6203	0.9235
3	180	five	0.8865	0.9586	0.0862	0.9552	0.9184	0.9585	0.2754	0.9639
3	240	five	0.9125	0.9802	0.0491	0.9757	0.9415	0.9805	0.1093	0.9810
6	120	five	0.8911	0.8964	0.7702	0.9181	0.9129	0.8968	0.8140	0.9242
6	180	five	0.9198	0.9542	0.5807	0.9602	0.9398	0.9554	0.7300	0.9677
6	240	five	0.9388	0.9766	0.2107	0.9801	0.9572	0.9756	0.5057	0.9854
9	120	five	0.9099	0.8875	0.8259	0.9165	0.9310	0.8903	0.8516	0.9211
9	180	five	0.9353	0.9460	0.7523	0.9583	0.9505	0.9481	0.8050	0.9633
9	240	five	0.9498	0.9697	0.6480	0.9764	0.9664	0.9754	0.7429	0.9843
3	120	frvi	0.9727	0.9539	0.8675	0.9507	0.8954	0.8723	0.8002	0.8763
3	180	frvi	0.9963	0.9885	0.9613	0.9847	0.9759	0.9504	0.8941	0.9407
3	240	frvi	0.9998	0.9973	0.9847	0.9953	0.9905	0.9722	0.9349	0.9575
6	120	frvi	0.9501	0.9479	0.8512	0.9509	0.8464	0.8659	0.7874	0.8785
6	180	frvi	0.9933	0.9898	0.8767	0.9872	0.9657	0.9566	0.8512	0.9474
6	240	frvi	0.9995	0.9984	0.9464	0.9962	0.9886	0.9794	0.8728	0.9644
9	120	frvi	0.9326	0.9405	0.8416	0.9558	0.8202	0.8563	0.7898	0.8914
9	180	frvi	0.9906	0.9908	0.8868	0.9924	0.9582	0.9567	0.8632	0.9522
9	240	frvi	0.9995	0.9987	0.8903	0.9974	0.9860	0.9821	0.8643	0.9683
3	120	niv	13.5890	11.8910	23.0200	11.7160	12.1360	11.0990	17.8730	10.7510
3	180	niv	12.9140	10.9610	33.3730	11.0120	11.8800	10.5840	27.7800	10.3460
3	240	niv	12.2740	10.4870	34.5710	10.5840	11.4250	10.2280	32.5080	10.0690
6	120	niv	15.9270	15.5900	22.0730	14.3420	13.6010	14.7460	18.8500	13.2570
6	180	niv	14.6630	12.6010	33.5040	12.2220	13.2060	12.1970	24.4440	11.3770
6	240	niv	13.6060	11.3640	56.0350	11.1340	12.4130	11.2310	37.8910	10.5080
9	120	niv	17.6190	19.7510	24.4290	17.2380	14.5460	18.6570	21.5480	16.1710
9	180	niv	15.8550	14.8720	31.6590	13.7640	14.1330	14.3400	26.5720	12.8980
9	240	niv	14.6160	12.7790	41.2910	12.1430	12.9500	12.0840	32.2960	11.1230
3	120	tmi	0.7840	0.6280	0.3240	0.6420	0.4640	0.3180	0.1510	0.3520
3	180	tmi	0.9640	0.8940	0.6750	0.8690	0.8080	0.6300	0.3260	0.5950
3	240	tmi	0.9980	0.9730	0.8470	0.9550	0.9230	0.7660	0.4430	0.7050
6	120	tmi	0.6610	0.6160	0.0940	0.6550	0.3460	0.3210	0.0420	0.3870
6	180	tmi	0.9390	0.9000	0.3100	0.8910	0.7450	0.6800	0.1200	0.6440
6	240	tmi	0.9950	0.9840	0.6570	0.9670	0.9080	0.8280	0.2960	0.7320
9	120	tmi	0.5880	0.5730	0.0470	0.6890	0.2760	0.2970	0.0300	0.4230
9	180	tmi	0.9170	0.9120	0.2340	0.9350	0.6960	0.6820	0.0870	0.6740
9	240	tmi	0.9950	0.9870	0.2940	0.9760	0.8850	0.8450	0.1400	0.7680

Source: Own construction

Table 17 – Estimation results for $\phi = 0.5$ RMSE and MAE

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	mae	0.0466	0.0415	0.0896	0.0399	0.0587	0.0547	0.0910	0.0512
3	180	mae	0.0339	0.0274	0.0776	0.0286	0.0439	0.0381	0.0880	0.0384
3	240	mae	0.0285	0.0220	0.0612	0.0238	0.0380	0.0314	0.0790	0.0333
6	120	mae	0.0291	0.0272	0.0458	0.0242	0.0357	0.0355	0.0490	0.0310
6	180	mae	0.0200	0.0155	0.0533	0.0155	0.0254	0.0211	0.0476	0.0204
6	240	mae	0.0165	0.0116	0.0632	0.0125	0.0215	0.0165	0.0577	0.0172
9	120	mae	0.0221	0.0230	0.0332	0.0188	0.0262	0.0292	0.0364	0.0240
9	180	mae	0.0146	0.0117	0.0325	0.0110	0.0184	0.0160	0.0329	0.0146
9	240	mae	0.0119	0.0084	0.0370	0.0085	0.0152	0.0115	0.0348	0.0115
3	120	rmse	0.0963	0.0890	0.1341	0.0871	0.1223	0.1157	0.1483	0.1108
3	180	rmse	0.0722	0.0621	0.1005	0.0654	0.0956	0.0862	0.1234	0.0881
3	240	rmse	0.0615	0.0509	0.0782	0.0555	0.0839	0.0725	0.1039	0.0785
6	120	rmse	0.0783	0.0713	0.0950	0.0666	0.0978	0.0919	0.1079	0.0846
6	180	rmse	0.0568	0.0456	0.0915	0.0473	0.0740	0.0627	0.0937	0.0630
6	240	rmse	0.0479	0.0361	0.0882	0.0398	0.0640	0.0513	0.0951	0.0552
9	120	rmse	0.0695	0.0653	0.0807	0.0571	0.0849	0.0824	0.0921	0.0728
9	180	rmse	0.0492	0.0390	0.0694	0.0386	0.0638	0.0536	0.0763	0.0518
9	240	rmse	0.0410	0.0300	0.0694	0.0316	0.0546	0.0422	0.0736	0.0440

Source: Own construction

Table 18 – Estimation results for $\phi = 0.7$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	five	0.8352	0.8930	0.2785	0.9073	0.8622	0.8996	0.4582	0.9109
3	180	five	0.8780	0.9523	0.0547	0.9520	0.9072	0.9543	0.1525	0.9550
3	240	five	0.9050	0.9777	0.0431	0.9737	0.9336	0.9758	0.0730	0.9733
6	120	five	0.8818	0.8957	0.7085	0.9108	0.9032	0.8988	0.7666	0.9198
6	180	five	0.9121	0.9524	0.3966	0.9430	0.9345	0.9542	0.5902	0.9591
6	240	five	0.9311	0.9748	0.0673	0.9618	0.9533	0.9759	0.2719	0.9772
9	120	five	0.9022	0.8963	0.7645	0.9102	0.9186	0.8929	0.8075	0.9168
9	180	five	0.9291	0.9476	0.6573	0.9454	0.9449	0.9486	0.7405	0.9541
9	240	five	0.9423	0.9727	0.4775	0.9645	0.9622	0.9716	0.6412	0.9746
3	120	frvi	0.9716	0.9600	0.8975	0.9491	0.9111	0.8921	0.8327	0.8875
3	180	frvi	0.9962	0.9923	0.9850	0.9851	0.9736	0.9589	0.9280	0.9390
3	240	frvi	0.9996	0.9978	0.9946	0.9953	0.9896	0.9803	0.9584	0.9608
6	120	frvi	0.9571	0.9503	0.8341	0.9433	0.8824	0.8901	0.7871	0.8918
6	180	frvi	0.9942	0.9942	0.9049	0.9849	0.9677	0.9638	0.8584	0.9417
6	240	frvi	0.9990	0.9989	0.9817	0.9959	0.9877	0.9845	0.9107	0.9631
9	120	frvi	0.9429	0.9375	0.8227	0.9529	0.8606	0.8742	0.7885	0.9016
9	180	frvi	0.9914	0.9945	0.8774	0.9900	0.9625	0.9671	0.8592	0.9551
9	240	frvi	0.9985	0.9989	0.8980	0.9977	0.9860	0.9866	0.8670	0.9735
3	120	niv	14.0000	12.3830	27.7340	11.9010	12.6930	11.5310	22.4150	11.1910
3	180	niv	13.1330	11.1620	34.4290	11.0990	12.1490	10.7770	31.3150	10.5600
3	240	niv	12.4670	10.5570	34.8250	10.6380	11.6220	10.4320	33.6870	10.3020
6	120	niv	16.5420	15.6590	25.5410	14.6940	14.5380	14.8710	21.6410	13.6520
6	180	niv	15.1300	12.7480	44.6520	13.2120	13.5410	12.3380	32.7610	11.8290
6	240	niv	14.0570	11.4780	64.8450	12.2100	12.6330	11.2690	52.0630	10.9780
9	120	niv	18.4310	18.9110	29.8950	17.7880	16.0990	18.5980	25.5910	16.6660
9	180	niv	16.4390	14.7680	40.3030	14.9220	14.6970	14.3990	32.4650	13.7700
9	240	niv	15.2890	12.4970	57.0460	13.2420	13.3370	12.4830	41.6760	12.0680
3	120	tmi	0.7750	0.6920	0.5170	0.6440	0.4900	0.4000	0.2530	0.3860
3	180	tmi	0.9630	0.9280	0.8650	0.8740	0.8040	0.6810	0.5230	0.6140
3	240	tmi	0.9960	0.9780	0.9460	0.9570	0.9100	0.8370	0.6410	0.7360
6	120	tmi	0.6940	0.6500	0.1280	0.6450	0.4080	0.3990	0.0480	0.4490
6	180	tmi	0.9460	0.9450	0.5280	0.8860	0.7650	0.7270	0.2510	0.6430
6	240	tmi	0.9900	0.9890	0.8660	0.9620	0.8950	0.8740	0.5360	0.7470
9	120	tmi	0.6100	0.5900	0.0610	0.6890	0.3430	0.3550	0.0360	0.4740
9	180	tmi	0.9220	0.9480	0.2940	0.9140	0.7400	0.7540	0.1540	0.7230
9	240	tmi	0.9870	0.9890	0.4770	0.9770	0.8820	0.8890	0.2720	0.8120

Source: Own construction

Table 19 – Estimation results for $\phi = 0.7$ RMSE and MAE

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	mae	0.0447	0.0415	0.1042	0.0403	0.0555	0.0530	0.1098	0.0503
3	180	mae	0.0327	0.0276	0.0776	0.0292	0.0425	0.0379	0.0966	0.0386
3	240	mae	0.0276	0.0225	0.0620	0.0243	0.0367	0.0308	0.0801	0.0334
6	120	mae	0.0277	0.0266	0.0585	0.0243	0.0336	0.0333	0.0582	0.0296
6	180	mae	0.0193	0.0152	0.0704	0.0163	0.0242	0.0203	0.0642	0.0208
6	240	mae	0.0159	0.0116	0.0724	0.0131	0.0205	0.0158	0.0774	0.0175
9	120	mae	0.0211	0.0220	0.0469	0.0188	0.0251	0.0278	0.0457	0.0230
9	180	mae	0.0142	0.0112	0.0457	0.0113	0.0174	0.0149	0.0421	0.0146
9	240	mae	0.0115	0.0081	0.0525	0.0087	0.0145	0.0110	0.0462	0.0116
3	120	rmse	0.0915	0.0881	0.1456	0.0878	0.1148	0.1123	0.1641	0.1085
3	180	rmse	0.0694	0.0624	0.0996	0.0673	0.0916	0.0854	0.1291	0.0882
3	240	rmse	0.0594	0.0519	0.0796	0.0568	0.0801	0.0709	0.1042	0.0782
6	120	rmse	0.0736	0.0701	0.1103	0.0667	0.0910	0.0875	0.1183	0.0817
6	180	rmse	0.0544	0.0452	0.1065	0.0483	0.0701	0.0607	0.1097	0.0633
6	240	rmse	0.0458	0.0360	0.0944	0.0402	0.0609	0.0493	0.1112	0.0555
9	120	rmse	0.0653	0.0639	0.0986	0.0570	0.0794	0.0797	0.1038	0.0703
9	180	rmse	0.0473	0.0378	0.0843	0.0386	0.0599	0.0509	0.0868	0.0509
9	240	rmse	0.0394	0.0295	0.0856	0.0315	0.0517	0.0401	0.0856	0.0435

Source: Own construction

APPENDIX B – SIMULATION RESULTS: FORECASTS

Table 20 – Forecast results one-step-ahead $\phi = 0.3$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	maep1	0.9382	0.9362	1.1072	0.9249	1.2970	1.2864	1.4214	1.2487
3	180	maep1	0.8870	0.8709	0.9663	0.8642	1.1537	1.1336	1.2416	1.1216
3	240	maep1	0.8955	0.8829	0.9109	0.8766	1.0865	1.0654	1.1298	1.0610
6	120	maep1	0.9918	0.9674	1.1034	0.9254	1.3451	1.2933	1.4132	1.2528
6	180	maep1	0.9065	0.8757	1.0412	0.8646	1.1874	1.1458	1.2910	1.1172
6	240	maep1	0.9116	0.8832	1.0434	0.8769	1.1020	1.0624	1.2326	1.0528
9	120	maep1	1.0195	0.9930	1.0982	0.9555	1.3777	1.3376	1.4147	1.2783
9	180	maep1	0.9210	0.8784	1.0535	0.8616	1.2004	1.1483	1.2993	1.1223
9	240	maep1	0.9213	0.8830	1.0520	0.8761	1.1135	1.0548	1.2503	1.0489
3	120	rmsep1	1.1790	1.1677	1.3840	1.1484	1.6349	1.6175	1.7879	1.5738
3	180	rmsep1	1.1138	1.0940	1.2216	1.0876	1.5113	1.4909	1.6049	1.4771
3	240	rmsep1	1.1359	1.1158	1.1442	1.1112	1.4592	1.4402	1.5318	1.4394
6	120	rmsep1	1.2427	1.2059	1.3892	1.1538	1.6926	1.6378	1.7755	1.5874
6	180	rmsep1	1.1367	1.0981	1.2982	1.0859	1.5524	1.5060	1.6685	1.4764
6	240	rmsep1	1.1540	1.1137	1.3109	1.1101	1.4769	1.4333	1.6270	1.4321
9	120	rmsep1	1.2738	1.2327	1.3797	1.1811	1.7328	1.6949	1.7874	1.6161
9	180	rmsep1	1.1553	1.1069	1.3141	1.0882	1.5683	1.5059	1.6755	1.4707
9	240	rmsep1	1.1671	1.1148	1.3347	1.1069	1.4874	1.4215	1.6616	1.4220

Source: Own construction

Table 21 – Forecast results one-step-ahead $\phi = 0.5$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	maep1	0.9698	0.9429	1.1053	0.9124	1.2341	1.2291	1.3340	1.1941
3	180	maep1	0.8817	0.8676	0.9635	0.8509	1.1863	1.1651	1.2419	1.1514
3	240	maep1	0.8686	0.8639	0.8865	0.8469	1.1735	1.1530	1.2221	1.1565
6	120	maep1	1.0093	0.9777	1.1131	0.9423	1.2954	1.2326	1.3668	1.1792
6	180	maep1	0.8991	0.8760	1.1140	0.8666	1.2122	1.1700	1.3545	1.1415
6	240	maep1	0.8778	0.8586	1.0009	0.8471	1.1920	1.1525	1.3638	1.1536
9	120	maep1	1.0340	0.9980	1.1303	0.9557	1.3322	1.2921	1.3993	1.1946
9	180	maep1	0.9109	0.8783	1.0949	0.8709	1.2321	1.1756	1.3546	1.1509
9	240	maep1	0.8844	0.8550	1.0470	0.8440	1.2046	1.1538	1.3438	1.1399
3	120	rmsep1	1.2070	1.1799	1.4001	1.1417	1.7232	1.7030	1.8233	1.6659
3	180	rmsep1	1.0935	1.0777	1.2076	1.0600	1.5662	1.5524	1.6399	1.5313
3	240	rmsep1	1.0916	1.0806	1.1224	1.0618	1.6258	1.6040	1.6914	1.6045
6	120	rmsep1	1.2622	1.2276	1.4316	1.1797	1.7928	1.7168	1.8695	1.6541
6	180	rmsep1	1.1137	1.0855	1.3920	1.0780	1.5955	1.5602	1.7728	1.5257
6	240	rmsep1	1.1039	1.0758	1.2773	1.0641	1.6434	1.5997	1.8169	1.6020
9	120	rmsep1	1.2959	1.2518	1.4208	1.2005	1.8499	1.7795	1.9016	1.6730
9	180	rmsep1	1.1266	1.0957	1.3851	1.0817	1.6171	1.5721	1.7718	1.5319
9	240	rmsep1	1.1141	1.0724	1.3428	1.0632	1.6592	1.5981	1.8290	1.5838

Source: Own construction

Table 22 – Forecast results one-step-ahead $\phi = 0.7$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	maep1	0.9489	0.9354	1.1319	0.9159	1.2149	1.2155	1.4718	1.1744
3	180	maep1	0.9063	0.8901	0.9589	0.8898	1.1242	1.1188	1.2034	1.1052
3	240	maep1	0.9111	0.9001	0.9033	0.8850	1.0373	1.0352	1.0693	1.0047
6	120	maep1	0.9916	0.9812	1.2673	0.9429	1.2692	1.2527	1.5118	1.2135
6	180	maep1	0.9299	0.8974	1.1080	0.8948	1.1470	1.1161	1.3774	1.1151
6	240	maep1	0.9237	0.8997	1.0108	0.8861	1.0460	1.0275	1.2156	1.0131
9	120	maep1	1.0346	1.0212	1.3034	0.9573	1.3021	1.3099	1.5682	1.2404
9	180	maep1	0.9420	0.9005	1.2018	0.8919	1.1676	1.1306	1.3804	1.1174
9	240	maep1	0.9294	0.8981	1.1265	0.8821	1.0615	1.0267	1.2765	1.0049
3	120	rmsep1	1.2040	1.1880	1.4382	1.1563	1.5701	1.5809	1.8495	1.5410
3	180	rmsep1	1.1161	1.0968	1.2011	1.0932	1.4709	1.4592	1.5857	1.4486
3	240	rmsep1	1.1322	1.1208	1.1322	1.1037	1.3472	1.3308	1.3997	1.3021
6	120	rmsep1	1.2616	1.2487	1.6314	1.1859	1.6458	1.6171	1.9274	1.5699
6	180	rmsep1	1.1444	1.1028	1.3940	1.0910	1.4909	1.4600	1.7674	1.4633
6	240	rmsep1	1.1484	1.1183	1.2707	1.1010	1.3591	1.3190	1.5626	1.3104
9	120	rmsep1	1.3161	1.3064	1.7143	1.2072	1.6935	1.7057	1.9992	1.6157
9	180	rmsep1	1.1606	1.1093	1.5369	1.0944	1.5161	1.4814	1.7839	1.4696
9	240	rmsep1	1.1574	1.1149	1.4204	1.0998	1.3763	1.3209	1.6198	1.3011

Source: Own construction

Table 23 – Forecast results twelve-step-ahead $\phi = 0.3$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	maeps	0.9584	0.9591	1.0656	0.9385	1.2445	1.2298	1.3486	1.2097
3	180	maeps	0.8925	0.8838	0.9580	0.8739	1.0931	1.0730	1.1842	1.0646
3	240	maeps	0.8605	0.8434	0.8667	0.8374	1.1125	1.1005	1.1413	1.0908
6	120	maeps	1.0136	1.0020	1.1003	0.9617	1.3187	1.2946	1.4066	1.2369
6	180	maeps	0.9157	0.8836	1.0397	0.8758	1.1206	1.0768	1.2440	1.0519
6	240	maeps	0.8748	0.8452	1.0379	0.8355	1.1334	1.0998	1.2608	1.0911
9	120	maeps	1.0487	1.0395	1.1281	0.9778	1.3432	1.3164	1.4528	1.2343
9	180	maeps	0.9292	0.8958	1.0634	0.8786	1.1404	1.0909	1.2509	1.0540
9	240	maeps	0.8845	0.8465	1.0732	0.8422	1.1454	1.0954	1.2691	1.0842
3	120	rmseps	1.2047	1.2015	1.3273	1.1788	1.6086	1.5972	1.7576	1.5697
3	180	rmseps	1.1192	1.1024	1.1939	1.0903	1.4758	1.4470	1.5631	1.4364
3	240	rmseps	1.0685	1.0396	1.0779	1.0334	1.4733	1.4664	1.5004	1.4477
6	120	rmseps	1.2809	1.2579	1.3800	1.2089	1.6977	1.6669	1.8140	1.6007
6	180	rmseps	1.1518	1.1079	1.3141	1.0882	1.5021	1.4452	1.6118	1.4288
6	240	rmseps	1.0858	1.0434	1.2870	1.0305	1.4929	1.4608	1.6443	1.4455
9	120	rmseps	1.3221	1.3066	1.4111	1.2291	1.7313	1.6907	1.8717	1.6021
9	180	rmseps	1.1714	1.1227	1.3432	1.0919	1.5230	1.4612	1.6265	1.4270
9	240	rmseps	1.0971	1.0455	1.3389	1.0412	1.5062	1.4489	1.6556	1.4312

Source: Own construction

Table 24 – Forecast results twelve-step-ahead $\phi = 0.5$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	maeps	0.9637	0.9458	1.1345	0.9169	1.2218	1.2099	1.3786	1.1668
3	180	maeps	0.8916	0.8824	0.9464	0.8714	1.1070	1.0969	1.1474	1.0817
3	240	maeps	0.9095	0.9004	0.9364	0.8958	1.0936	1.0910	1.1679	1.0794
6	120	maeps	1.0078	0.9760	1.1716	0.9496	1.2991	1.2839	1.4164	1.1903
6	180	maeps	0.9083	0.8823	1.0753	0.8702	1.1283	1.0905	1.2598	1.0842
6	240	maeps	0.9253	0.8997	1.0889	0.9039	1.1120	1.0885	1.3248	1.0815
9	120	maeps	1.0639	1.0433	1.2154	0.9565	1.3252	1.3277	1.4477	1.2117
9	180	maeps	0.9272	0.8950	1.0407	0.8695	1.1431	1.0851	1.2508	1.0749
9	240	maeps	0.9293	0.8968	1.1168	0.8995	1.1204	1.0934	1.3247	1.0814
3	120	rmseps	1.2141	1.1946	1.4249	1.1530	1.5777	1.5678	1.7538	1.5172
3	180	rmseps	1.1165	1.1014	1.1772	1.0862	1.4444	1.4361	1.5144	1.4192
3	240	rmseps	1.1414	1.1269	1.1547	1.1182	1.4530	1.4584	1.5472	1.4432
6	120	rmseps	1.2789	1.2349	1.4639	1.1924	1.6804	1.6638	1.8241	1.5587
6	180	rmseps	1.1444	1.1024	1.3594	1.0843	1.4700	1.4338	1.6280	1.4190
6	240	rmseps	1.1629	1.1239	1.3476	1.1235	1.4723	1.4571	1.7223	1.4454
9	120	rmseps	1.3471	1.3225	1.5256	1.2122	1.7135	1.6858	1.8462	1.5581
9	180	rmseps	1.1646	1.1173	1.3334	1.0830	1.4864	1.4228	1.6261	1.4095
9	240	rmseps	1.1691	1.1205	1.3964	1.1216	1.4842	1.4611	1.7309	1.4448

Source: Own construction

Table 25 – Forecast results twelve-step-ahead $\phi = 0.7$

			Normal				t-student			
Lag	Length	Measure	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO	LASSO	adaLASSO	WLadaLASSO	SWLadaLASSO
3	120	maeps	0.9785	0.9806	1.1555	0.9523	1.241	1.2455	1.5119	1.1838
3	180	maeps	0.918	0.9109	0.9914	0.9091	1.1493	1.15	1.247	1.1339
3	240	maeps	0.8584	0.8541	0.9008	0.844	1.0729	1.0825	1.1252	1.0685
6	120	maeps	1.0363	1.0422	1.3321	0.9627	1.3042	1.3086	1.6415	1.2302
6	180	maeps	0.9358	0.9202	1.2236	0.9151	1.162	1.1393	1.4613	1.128
6	240	maeps	0.8688	0.8498	1.0325	0.8456	1.0839	1.0712	1.275	1.0624
9	120	maeps	1.0782	1.1249	1.4096	0.992	1.3603	1.4252	1.7284	1.2723
9	180	maeps	0.9495	0.9182	1.2633	0.9069	1.1747	1.1499	1.4485	1.1272
9	240	maeps	0.8746	0.8505	1.1121	0.8476	1.1013	1.0766	1.3519	1.0543
3	120	rmseps	1.2166	1.2156	1.4617	1.1781	1.5939	1.6043	1.9057	1.5362
3	180	rmseps	1.1459	1.1343	1.2322	1.1325	1.5071	1.5027	1.6218	1.4862
3	240	rmseps	1.0618	1.0591	1.1129	1.0502	1.4382	1.4412	1.4769	1.4303
6	120	rmseps	1.2928	1.2976	1.7048	1.1997	1.6767	1.6899	2.0853	1.5631
6	180	rmseps	1.1719	1.1428	1.5254	1.1391	1.518	1.4823	1.8595	1.4801
6	240	rmseps	1.0761	1.0546	1.2769	1.0458	1.4535	1.4361	1.6672	1.4294
9	120	rmseps	1.3429	1.4057	1.8081	1.2397	1.7451	1.8205	2.2089	1.6297
9	180	rmseps	1.1888	1.1432	1.6122	1.1277	1.5382	1.4901	1.851	1.4774
9	240	rmseps	1.0837	1.0527	1.412	1.0497	1.4702	1.4391	1.7511	1.4195

Source: Own construction

APPENDIX C – DATA BASE FOR EMPIRICAL APPLICATION

Table 26 – Variables for empirical application: part 1

Order	Source	Name in English	Difference
1	IPEA	GDP growth rate - R$ (millions)	0
2	IPEA	Exchange rate - actual effective - INPC - exports index (mean 2010 = 100)	1
3	IPEA	Exchange rate - actual effective - INPC - imports index (mean 2010 = 100)	1
4	IPEA	Exchange rate - R$ / US$ - commercial - buy - mean - R$	1
5	IPEA	Exchange rate - R$ / US$ - commercial - buy - end of period - R$	1
6	IPEA	Exchange rate - R$ / US$ - commercial - sell - mean - R$	1
7	IPEA	Exchange rate - R$ / US$ - commercial - sell - end of period - R$	1
8	IPEA	Purchasing power parity rate	1
9	IPEA	Exports - durable consumer goods - prices - index (mean 2016 = 100)	1
10	IPEA	Exports - non-durable consumer goods - prices - index (mean 2016 = 100)	1
11	IPEA	Exports - intermediate goods - prices - index (mean 2016 = 100)	1
12	IPEA	Exports - capital investments - prices - index (mean 2016 = 100)	1
13	IPEA	Exports - price - index (mean 2006 = 100)	1
14	IPEA	Imports - durable consumer goods - prices - index (mean 2016 = 100)	1
15	IPEA	Imports - non-durable consumer goods - prices - index (mean 2016 = 100)	1
16	IPEA	Imports - intermediate goods - prices - index (mean 2016 = 100)	1
17	IPEA	Imports - capital investments - prices - index (mean 2016 = 100)	1
18	IPEA	Imports - price - index (mean 2006 = 100)	1
19	IPEA	Terms of trade - index (mean 2006 = 100)	1
20	IPEA	Apparent consumption - capital investments - chained index (mean 2012 = 100)	1
21	IPEA	Apparent consumption - intermediate goods - chained index (mean 2012 = 100)	0
22	IPEA	Apparent consumption - consumer goods - chained index (mean 2012 = 100)	0
23	IPEA	Apparent consumption - durable consumer goods - chained index (mean 2012 = 100)	1
24	IPEA	Apparent consumption - semi and non-durable goods - chained index (mean 2012 = 100)	0
25	IPEA	Apparent consumption - general industry - chained index (mean 2012 = 100)	1
26	IPEA	Apparent consumption - manufacturing industry - chained index (mean 2012 = 100)	0
27	IPEA	Balance of trade - (new methodology) - US$ (millions)	0
28	IPEA	Balance of trade - Exports (new methodology) - US$ (millions)	1
29	IPEA	Balance of trade - Imports (new methodology) - US$ (millions)	1
30	IPEA	Financial account - (grants - concessions) (new methodology) - US$ (millions)	1
31	IPEA	Derivatives - (assets - liabilities) (new methodology) - US$ (millions)	0
32	IPEA	Derivatives - Assets (new methodology) - US$ (millions)	1
33	IPEA	Derivatives - Liabilities (new methodology) - US$ (millions)	1
34	IPEA	Reserve assets (new methodology) - US$ (millions)	0
35	IPEA	Loans and long-term securities traded on the foreign market - Amortiz. (new meth.) - US$ (millions)	0
36	IPEA	Portfolio investment - Assets - (new methodology) - US$ (millions)	0
37	IPEA	Portfolio investment - Assets - Revenues (new methodology) - US$ (millions)	0
38	IPEA	Portfolio investment - Assets - Expenses (new methodology) - US$ (millions)	0
39	IPEA	Loans and long-term securities traded on the foreign market - (new methodology) - US$ (millions)	0
40	IPEA	Loans and long-term securities traded on the foreign market - Ingression (new meth.) - US$ (millions)	0
41	IPEA	Portfolio investment - Assets - Fixed income securities - (new methodology) - US$ (millions)	0
42	IPEA	Portfolio investment - Liabilities - (new methodology) - US$ (millions)	0
43	IPEA	Foreign direct investment - Ingression (new methodology) - US$ (millions)	0
44	IPEA	Portfolio investment - Liabilities - Fixed income securities - (new methodology) - US$ (millions)	0
45	IPEA	Foreign direct investment - (new methodology) - US$ (millions)	1
46	IPEA	Foreign direct investment - Ingression (new methodology) - US$ (millions)	1
47	IPEA	Foreign direct investment - Exits (new methodology) - US$ (millions)	1
48	IPEA	Primary income - Revenues (new methodology) - US$ (millions)	1
49	IPEA	Primary income - Expenses (new methodology) - US$ (millions)	1
50	IPEA	Primary income - Direct investment income - Expenses (new methodology) - US$ (millions)	1

Source: Own construction

Table 27 – Variables for empirical application: part 2

Order	Source	Name in English	Difference
51	IPEA	Secondary income - (new methodology) - US$ (millions)	0
52	IPEA	Secondary income - Revenues (new methodology) - US$ (millions)	1
53	IPEA	Secondary income - Expenses (new methodology) - US$ (millions)	1
54	IPEA	International reserves - cash concept - US$ (millions)	1
55	IPEA	International reserves - liquidity concept - US$ (millions)	1
56	IPEA	Services - (new methodology) - US$ (millions)	1
57	IPEA	Services - Revenues (new methodology) - US$ (millions)	1
58	IPEA	Services - Expenses (new methodology) - US$ (millions)	1
59	IPEA	Current transactions - (new methodology) - US$ (millions)	1
60	IPEA	Current transactions - last 12 months (new methodology) - % (GDP)	1
61	IPEA	Electricity consumption - commerce - quantity - GWh	1
62	IPEA	Electricity consumption - industry - quantity - GWh	1
63	IPEA	Electricity consumption - residence - quantity - GWh	1
64	IPEA	Electricity consumption - quantity - GWh - Eletrobras	1
65	IPEA	Bad check - (%)	1
66	IPEA	Actual billing - industry - index (mean 2006 = 100)	1
67	IPEA	SPC - number of consultations - Unity	1
68	IPEA	Nominal sales - industry - index (mean 2006 = 100)	1
69	IPEA	Real sales - industry - index (mean 2006 = 100) - SP	0
70	IPEA	Worked hours - industry - index - (mean 2006 = 100)	0
71	IPEA	Personnel employed - industry - index (mean 2006 = 100)	1
72	IPEA	Worked hours - in production - industry - index (mean 2006 = 100) - SP	1
73	IPEA	Paid hours - industry - index (mean 2006 = 100) - SP	1
74	IPEA	Personnel employed - industry - index (mean 2006 = 100) - SP	0
75	IPEA	Unemployment rate - RMSP - (%)	1
76	IPEA	Level of employment - industry - index (jun. 2005 = 100) - SP	0
77	IPEA	Employment - industry - RJ - index (mean 2006 = 100)	0
78	IPEA	Worked hours - industry - index - RJ - (mean 2006 = 100)	1
79	IPEA	Consumer confidence index	1
80	IPEA	Expectations index	1
81	IPEA	Ibovespa - closing - (% p.m.)	0
82	IPEA	Savings - nominal income - first business day (until May/03/2012) - (% p.m.)	1
83	IPEA	Stock fund - nominal income - (% p.m.)	0
84	IPEA	Savings - nominal income - first business day (until May/04/2012) - (% p.m.)	1
85	IPEA	Commercial dollar - nominal income - (% p.m.)	0
86	IPEA	Interest rate - CDI / Over - (% p .m.)	0
87	IPEA	Interest rate - TJLP - (% p .m.)	0
88	IPEA	Interest rate - Over / Selic - (% p .m.)	0
89	IPEA	Interest rate - TBF - (% p .m.)	0
90	IPEA	Interest rate - TR - (% p .m.)	0
91	IPEA	Number of business days	0
92	IPEA	External debt - states and municipalities - net - (% GDP)	1
93	IPEA	External debt - Federal Government and Central Bank - net - (% GDP)	1
94	IPEA	Tax debt - public sector - net - (% GDP)	1
95	IPEA	Internal debt - states and municipalities - net - (% GDP)	1
96	IPEA	Internal debt - Federal Government and Central Bank - net - (% GDP)	1
97	IPEA	Debt - total - states and municipalities - net - (% GDP)	1
98	IPEA	Debt - total - Federal Government and Central Bank - net - (% GDP)	1
99	IPEA	External debt - states and municipalities - net - R$ (millions)	1
100	IPEA	External debt - Federal Government and Central Bank - net - R$ (millions)	1

Source: Own construction

Table 28 – Variables for empirical application: part 3

Order	Source	Name in English	Difference
101	IPEA	Tax debt - public sector - net - R$ (millions)	2
102	IPEA	Internal debt - states and municipalities - net - R$ (millions)	1
103	IPEA	Internal debt - Federal Government and Central Bank - net - R$ (millions)	2
104	IPEA	Debt - total - states and municipalities - net - R$ (millions)	1
105	IPEA	Debt - total - Federal Government and Central Bank - net - R$ (millions)	1
106	IPEA	Contribution - Finsocial / Cofins - total - gross revenue - R$ (millions)	1
107	IPEA	Social Contribution on Net Profit - total - gross revenue - R$ (millions)	1
108	IPEA	Provisional Contribution on Financial Transactions - gross revenue - R$ (millions)	1
109	IPEA	Contribution - PIS / Pasep - total - gross revenue - R$ (millions)	1
110	IPEA	Federal revenues - gross revenue - R$ (millions)	1
112	IPEA	Tax over Merchandise Circulation - R$ (thousand)	1
113	IPEA	Tax over Imports - total - gross revenue - R$ (millions)	1
114	IPEA	Tax on Financial - total - gross revenue - Transactions	1
115	IPEA	Income tax - total - gross revenue - R$ (millions)	1
116	IPEA	Income tax - legal person - total - gross revenue - R$ (millions)	0
117	IPEA	Income tax - legal entity - total - gross revenue - R$ (millions)	1
118	IPEA	Tax on the Territorial Rural Property - gross revenue - R$ (millions)	0
119	IPEA	Tax on Property of Motor Vehicles - R$ (thousand)	0
120	IPEA	Taxes on Industrialized Products - gross revenue - R$ (millions)	1
121	IPEA	M0 - end of period - R$ (millions)	1
122	IPEA	M1 - end of period - R$ (millions)	1
123	IPEA	M2 - end of period - new concept - R$ (millions)	1
124	IPEA	M3 - end of period - new concept - R$ (millions)	1
125	IPEA	M4 - end of period -new concept - R$ (millions)	1
126	IPEA	Capacity utilization - industry - (%)	0
127	IPEA	Capacity utilization - mineral extraction - (%)	0
128	IPEA	Electricity generation - hydro power - quantity - GWh	1
129	IPEA	Electricity generation - conventional thermal - quantity - GWh	1
130	IPEA	Capacity utilization - industry - RJ - (%)	1
131	IPEA	Inflation - IPC (FIPE) - (% p.m.)	0
132	IPEA	Inflation - INPC - (% p.m.)	0
133	IPEA	Inflation - IPCA - (% p.m.)	0
134	IPEA	Inflation - IGP-DI - (% p.m.)	0
135	IPEA	Inflation - IGP-M - (% p.m.)	0
136	IPEA	Inflation - IPC (FGV) - (% p.m.)	1
137	IPEA	Nominal wage - mean - industry - index (mean 2006 = 100) - SP	1
138	IPEA	Real wage - mean - industry - index (mean 2006 = 100) - SP	1
139	IPEA	Nominal wage - industry - index (mean 2006 = 100) - SP	1
140	IPEA	Real wage - industry - index (mean 2006 = 100) - SP	1
141	IPEA	Mean income - real - wage earners - main work - index (mean 2000 = 100) - RMSP	1
142	IPEA	Real minimum wage - R$	1
143	IPEA	Minimum wage (PPP) - US$	1
144	IPEA	Minimum wage	1
145	FED	10-Year Treasury Constant Maturity Rate, Percent, Monthly, Not Seasonally Adjusted (USA)	1
146	FED	2-Year Treasury Constant Maturity Rate, Percent, Monthly, Not Seasonally Adjusted (USA)	1
147	FED	20-Year Treasury Constant Maturity Rate, Percent, Monthly, Not Seasonally Adjusted (USA)	1
148	FED	3-Year Treasury Constant Maturity Rate, Percent, Monthly, Not Seasonally Adjusted (USA)	1
149	FED	3-Month Treasury Constant Maturity Rate, Percent, Daily, Not Seasonally Adjusted (USA)	1
150	FED	5-Year Treasury Constant Maturity Rate, Percent, Monthly, Not Seasonally Adjusted (USA)	1
151	FED	6-Month Treasury Constant Maturity Rate, Percent, Daily, Not Seasonally Adjusted (USA)	1
152	FED	7-Year Treasury Constant Maturity Rate, Percent, Monthly, Not Seasonally Adjusted (USA)	1

Source: Own construction