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EFFICIENT INTEREST RATE TRACKING IN BRAZIL: SHOULD WE RETHINK MONETARY POLICY RULES?

PORTO ALEGRE

2020

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Dissertação submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para a obtenção do título de Mestre em Economia, com ênfase em Economia Aplicada.

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Resumo

Regras de política monetária são discutidas principalmente em termos de regras de Taylor, que conjecturam que a autoridade monetária escolhe a taxa de juros de curto prazo em respostas a desvios da inflação em relação à meta e do produto a alguma medida de produto potencial. Mostramos que substituir o hiato do produto pela taxa de juros eficiente como principal indicador de atividade real, resultando em uma regra de política monetária que adiante vamos nos referir como regra W, não faz com que modelos DSGE se ajustem melhor aos dados. Regras de Taylor provaram ser consistentemente superiores à regras W equivalentes, sendo capazes de realizar melhores previsões para a taxa Selic e de tornar a taxa eficiente de juros em um indicador de ciclo de negócios mais preciso de acordo com a datação de ciclos CODACE-FGV. Mostramos que a inclusão da taxa eficiente de juros na regra de juros como um intercepto variante no tempo, o que é consistente com a otimalidade de regras de política monetária, pode alterar a dinâmica dos ciclos dos negócios implicada pelo modelo de forma substancial e afetar decisivamente seu desempenho de previsão. O resultado é válido para versões de economia aberta e fechada do modelo de preços rígidos de Calvo com competição monopolística.

Palavras-chave: Regras de juros. modelos DSGE Novos Keynesianos. Pequena economia aberta. Estimação Bayesiana.

Abstract

Monetary policy rules are mostly discussed in terms of Taylor rules, which conjecture that the monetary authority sets the short-term interest rate in response to deviations of inflation from its target and of output to some measure of potential output. We show that replacing the output gap with the efficient interest rate as the main indicator of real activity, resulting in a monetary policy rule that we henceforth refer to as W rule, does not make New Keynesian models fit the data better. Taylor rules proved to be consistently superior to equivalent W rules, being able to deliver better forecasts for the Brazilian federal funds rate (Selic rate) and to turn the efficient interest rate into a more accurate business cycle indicator according to CODACE-FGV dating. We show that the inclusion of the efficient interest rate in the feedback rule as a time-varying intercept, which is consistent with the optimality of monetary policy rules, can substantially change the dynamics of the business cycle implied by the model and decisively affect its forecasting performance. Previous results hold for both closed and small open economy versions of the Calvo model of sticky prices with monopolistic competition.

Keywords: Interest rate rules. New Keynesian DSGE models. Small open economy. Bayesian estimation.

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List of abbreviations and acronyms

FGV	Fundação Getúlio Vargas
CODACE	Dating of economic cycles committee
MAE	Mean absolute error
RMSE	Root mean square error
VAR	Vector autoregression
BVAR	Bayesian vector autoregression
DSGE	Dynamic Stochastic General Equilibrium
US	United States
AR	Autoregressive
CPI	Consumer price index
HP	Hodrick-Prescott
SOE	Small open economy
GDP	Gross domestic product
IPCA	Broad consumer price index

List of symbols

ρ	Interest rate smoothing parameter
ω	Inverse of Frisch elasticity of labor supply
η	Habit persistence parameter
β	Discount factor
α	Calvo probability of firms not reoptimizing their posted price
κ	Substitutability between goods produced in different foreign countries
ν	Parameter of the scheme of price indexation to past inflation
ϵ	Elasticity of substitution between differentiated goods
ζ	Parameter that is inversely related to the degree of home bias in preferences
ψ	Substitutability between domestic and foreign goods
$\bar{\pi}_H$	Long-run domestic inflation target
$\bar{\pi}$	Long-run CPI inflation target
Δs	Stationary terms of trade growth
r	Stationary real interest rate
π^*	Foreign long-run inflation target
r^*	Stationary foreign real interest rate
σ_s	Dynamic IS curve parameter
γ	Stationary productivity growth
η_γ	Function of deep parameters
φ_γ	Function of deep parameters
ξ	Slope of the Phillips curve
ϕ_π	Inflation parameter of the monetary policy rules
ϕ_x	Output gap parameter of the monetary policy rules
ϕ_s	Terms of trade growth parameter of the monetary policy rules

$\phi_{\Delta y}$	Output growth parameter of the monetary policy rules
λ	Hodrick-Prescott filter ratio
ρ_z	Autoregressive parameter of the technology asymmetry process
σ_z	Standard deviation of the technology asymmetry autoregressive process
ρ_γ	Autoregressive parameter of the productivity growth process
σ_γ	Standard deviation of the productivity growth autoregressive process
ρ_u	Autoregressive parameter of the cost-push shock process
σ_u	Standard deviation of the cost-push shock autoregressive process
$\rho_{\bar{\pi}}$	Autoregressive parameter of the time-varying inflation target process
$\sigma_{\bar{\pi}}$	Standard deviation of the time-varying inflation target autoregressive process
ρ_δ	Autoregressive parameter of the preference shock process
σ_δ	Standard deviation of the preference shock autoregressive process
σ_{gdp}^{me}	GDP measurement error standard deviation
σ_{cipca}^{me}	Inflation measurement error standard deviation
σ_{terms}^{me}	Terms of trade growth measurement error standard deviation
σ_i	Standard deviation of the monetary policy shock
σ_{y^*}	Standard deviation of the foreign GDP shock
σ_{i^*}	Standard deviation of the foreign monetary policy shock
σ_{π^*}	Standard deviation of the foreign inflation shock

Contents

1 INTRODUCTION

Quite recent work in macroeconomics involves the development of dynamic stochastic general equilibrium (DSGE) models as policy tools for central bankers. In these more stylized models, often referred to as New Keynesian DSGE models, there are imperfect competition and nominal rigidities, where monetary policy has non-trivial effects on real variables. They can also be used as a tool to describe the conduct of monetary policy through an interest rate rule. These rules usually conjecture that the short-term interest rate reacts to deviations of inflation from its target and of output to some measure of potential output, with some inertia[1]. Along with the non-policy blocks of New Keynesian models, interest rate rules are essential since the equilibrium path of real variables cannot be determined independently of monetary policy. This is why they are provided for both calibration (e.g., Aursland et al. (2020)) and estimation exercises (e.g., Smets and Wouters (2007), Castro et al. (2015), Adolfson et al. (2007), Acocella et al. (2020)). Although it was proposed from a purely empirical perspective, the Taylor rule incorporates several features of an optimal monetary policy, at least for one simple class of optimizing models, in which the monetary authority minimizes a quadratic loss function (e.g., Gali and Monacelli (2005), Palma and Portugal (2014), Paoli (2009)) or a more general function that allows for nonlinear feedback rules (e.g., Sá and Portugal (2015)). Much of the literature has adopted a welfare-based criterion, relying on a second-order approximation to the utility losses experienced by the representative household as a consequence of deviations from the Pareto optimal equilibrium.

The paper is motivated by the findings of Cúrdia et al. (2015), where policy rules in which the interest rate is set to track a measure of the efficient real rate, instead of some measure of the output gap, fit the US data better. Woodford (2001) has already suggested that a desirable rule is likely to require that the intercept be adjusted in response to fluctuations in the efficient interest rate. To our knowledge, this is the first paper to evaluate the empirical plausibility of efficient real interest rate tracking by the Brazilian monetary authority, and it would not be a surprise if we get similar empirical results. In the September 2017 inflation report (COPOM, 2017, p. 55), Brazil's central bank publicly stated that: "...The structural interest rate, also called the neutral interest rate, is a reference point for the conduct of monetary policy. When the real interest rate is below the structural interest rate, it exerts a stimulatory effect – boosting economic activity and contributing to increasing inflation. On the other hand, when the real interest rate is above the structural rate, its effect is contractionary – it contains economic activity and contributes to decreasing inflation...Under the inflation targeting regime, monetary policy must be conducted to keep inflation at the target within the relevant horizon. If the prospective inflation path indicates levels above the target within the relevant horizon for monetary policy, the central bank should conduct its interest rate and

[1] They are the so-called Taylor rules (TAYLOR, 1993). The additional inertia term is consistent with the commitment to a history-dependent behavior of optimal rules (WOODFORD, 2001).

communication policies so that real *ex-ante* interest rates are contractionary, that is, above the neutral rate. The opposite applies if the prospective inflation path points to levels below the target in the relevant horizon."

It was Wicksell who originally formulated the idea that monetary policy is designed to pursue a normal or natural real rate, in which the demand for loan capital and the supply of savings match perfectly (UHR, 1951). Considering that the monetary authority seeks to close the gap of the Selic rate with its efficient real counterpart r_t^e over time, and responds to deviations of inflation π_t from its target $\bar{\pi}_t$, one possible interest rate rule specification is given by

$$i_t = \rho i_{t-1} + (1-\rho)\left(\bar{\pi}_t + r_t^e + \phi_\pi(\pi_t - \bar{\pi}_t)\right) + \epsilon_t^i,$$

which includes the desirable inertial term previously mentioned. We also refer to them as W rules, and we henceforth refer to Taylor rules as T rules. How we define both potential output and the natural real rate makes a huge difference since they are central constructs in our feedback rules. A conventional measure of potential output is the smooth trend resulting from the Hodrick-Prescott filter. Another measure is defined as the level of output that would prevail if prices and wages were flexible[2] in a New Keynesian DSGE model, a hypothetical situation in which the equilibrium interest rate is called the natural interest rate. This equilibrium may or may not coincide with the efficient equilibrium, since optimal allocation can be attained in the highly stylized New Keynesian model if there is an optimal employment subsidy in place along with full stabilization of the price level (GALÍ, 2015)[3].

To avoid the possibility that closed economy models could be underestimating the relevance of the efficient interest rate in the conduct of Brazilian monetary policy, our framework differs from the one in Cúrdia et al. (2015) mainly because of the inclusion of the open economy dimension. As highlighted in Gali and Monacelli (2005), the natural (efficient) levels of output and of the interest rate in a small open economy are generally a function of both domestic and foreign disturbances. The inflation-targeting regime in Brazil represents an additional challenge in the conduct of monetary policy. As pointed by Minella et al. (2003), the significant inflationary pressures stemming from exchange rate or terms of trade volatility can turn the maintenance of price stability into a more challenging task in emerging markets. Our policy rules are coupled with the key building blocks of both New Keynesian closed and open economy models with two sources of suboptimality: the presence of market power in goods markets and infrequent adjustment of prices by firms. In order to improve its ability to fit the data, there is habit persistence in consumption and price indexation to past inflation. Our small open economy analysis features a complete exchange rate pass-through. These models with different policy rules are then estimated within a full information Bayesian strategy and their fit is compared

[2] This is a common feature in much of the recent work in macroeconomics. See for example Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010).

[3] This property holds only in the closed economy version of the basic New Keynesian model. In an open economy, if there is imperfect substitutability between domestic and foreign goods, the optimal employment subsidy is not sufficient to render the flexible price equilibrium allocation optimal (GALI; MONACELLI, 2005). See Paoli (2009) for further details.

using marginal data densities. We include a stochastic unit-root technology shock that induces a common trend in the real variables, allowing us to work with raw data when estimating the models. We show that T rules outperform W rules empirically. They are also more suitable for Selic rate forecasting within our framework and when the Wicksellian efficient interest rate does not drive interest rate decisions, it becomes a more accurate business cycle indicator according to CODACE-FGV dating.

The paper is outlined as follows: Section 2 presents both closed and small open economy models and introduces the monetary policy rules to be evaluated. Section 3 describes the approach to inference and briefly discusses the data and the estimation of the models. Section 4 presents the results for the comparison of policy rules, parameter estimates, and a forecasting exercise with both T and W specifications. Finally, Section 5 presents some conclusions.

2 MODEL OVERVIEW

The interest rate rules are first compared through the simple monetary transmission mechanism - describing the behavior of households and domestic goods firms in a closed economy as proposed by Cúrdia et al. (2015) (henceforth, baseline model) to which we refer the reader for further details.

2.1 THE SMALL OPEN ECONOMY MODEL

The baseline model is then augmented by a continuum of small open economies represented by the unit interval, which shares identical preferences and market structure, as in Gali and Monacelli (2005). The foreign economies are aggregated and treated as an exogenous foreign economy. The behavior of the central bank is captured with an interest rate feedback rule and different specifications are compared when the respective empirical models are brought to the data.

2.1.1 Households

The model is mostly based on the baseline model and augmented according to the stylized New Keynesian small open economy model proposed by Gali and Monacelli (2005). Assume an economy with a continuum of infinitely-lived households with measure one. Every household $h \in (0, 1)$ seeks to maximize

$$\mathbb{E}_0^h \left\{ \sum_{t=0}^{\infty} \beta^t \prod_{s=0}^{t} e^{-\delta_s} \left[\ln(C_t^h - \eta C_{t-1}^h) - \frac{(N_t^h)^{1+\omega}}{1+\omega} \right] \right\} \tag{2.1}$$

subject to

$$P_t C_t^h + \mathbb{E}_t(Q_{t,t+1} D_{t+1}^h) = W_t^h N_t^h + D_t^h + \Pi_t^h, \tag{2.2}$$

where P_t and C_t^h are both composite price and consumption indexes[1], respectively, of domestic and imported goods consumed by the domestic household $h \in (0, 1)$. Wage W_t^h differs across households, but they can fully insure against idiosyncratic wage risk by buying state-contingent securities D_{t+1}^h at price $Q_{t,t+1}$. Household h also earns profits Π_t^h from firms ownership. The aggregate preference shock follows a stationary AR(1) process given by

$$\delta_t = \rho_\delta \delta_{t-1} + \epsilon_t^\delta \tag{2.3}$$

where $\epsilon_t^\delta \sim N(0, \sigma_\delta)$.

[1] More details are shown in the Appendix, along with the equilibrium conditions, steady state computation and the complete log-linearized model.

2.1.2 Firms

A homogeneous good is produced using

$$Y_t^i = \left[\int_0^1 Y_t^i(j)^{1-\frac{1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.4}$$

where $\epsilon > 1$ is the elasticity of substitution between varieties $j \in (0,1)$ produced within any given country. Technology of a typical foreign intermediate producer $j \in (0,1)$ is represented by the production function

$$Y_t^i(j) = A_t^i N_t^i(j) \tag{2.5}$$

where productivity grows at the same rate $\gamma_t^i = \gamma_t^* = \Delta \ln A_t^i$ for every foreign country, where $i \in (0,1)$ is a country-specific index. Labor markets are perfectly competitive, i.e., firms take wages as given. The goods markets, otherwise, are monopolistically competitive. Following Calvo (1983), domestic firms may fully optimize their prices with probability $(1-\alpha)$ in any given period. Thus, in each period, a measure of $(1-\alpha)$ domestic producers reset their prices optimally, while a measure of α adjust their prices according to the indexation scheme[2]

$$P_{H,t}(i) = P_{H,t-1}(i)\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\nu} e^{(1-\nu)\bar{\pi}_H} \tag{2.6}$$

Firms that fully optimize their prices choose $\bar{P}_{H,t}(j)$ to maximize the present discounted value of profits net of sales taxes

$$\mathbb{E}_t\left\{\sum_{s=t}^{\infty} \alpha^{s-t} Q_{t,s} Y_{t,s}(j)\left[(1-\tau_s)\Phi_{t,s} P_{H,t}(j) - MC_s^n(j)\right]\right\} \tag{2.7}$$

subject to

$$Y_{t,s}(j) = \left[\frac{P_{H,t}(j)\Phi_{t,s}}{P_{H,s}}\right]^{-\epsilon}\left(C_{H,s} + \int_0^1 C_{H,s}^i di\right)$$

$$\Phi_{t,s} = \prod_{k=1}^{s-t}\left[e^{\nu\pi_{H,t+k-1}+(1-\nu)\bar{\pi}_H}\right]$$

Despite the assumption of no within-sector firm heterogeneity, it is assumed that there is heterogeneity in technology between domestic and foreign producers. More specifically, it is assumed that $z_t = a_t^i - a_t$ follows a stationary $AR(1)$ process

$$z_t = (1-\rho_z)z + \rho_z z_{t-1} + \epsilon_t^z, \tag{2.8}$$

where $\epsilon_t^z \sim N(0, \sigma_z)$.

[2] The relation between long-run CPI and domestic inflation targets is given by $\bar{\pi} = \bar{\pi}_{H,t} + \zeta\Delta s$, where Δs is the long-term log terms of trade growth.

2.1.3 Foreign Economy

First we model the foreign productivity growth rate as a stationary AR(1) model displaying some persistence

$$\gamma_t^* = (1-\rho_\gamma)\gamma + \rho_\gamma\gamma_{t-1}^* + \sigma_\gamma\epsilon_t^\gamma, \tag{2.9}$$

where $\epsilon_t^\gamma \sim N(0,\sigma_\gamma)$. Let $\vec{x}_t^* = [\hat{y}_t^* \ \hat{\pi}_t^* \ \hat{i}_t^* \ \hat{\gamma}_t^*]'$, where all variables are (log) deviations from their steady state values. We assume that foreign inflation, output and interest rate are exogenously given as

$$\vec{x}_t^* = A\vec{x}_{t-1}^* + C\vec{\varepsilon}_t, \tag{2.10}$$

in a similar structure to the one proposed by Christiano, Trabandt and Walentin (2011). More specifically,

$$\begin{bmatrix} \hat{y}_t^* \\ \hat{\pi}_t^* \\ \hat{i}_t^* \\ \hat{\gamma}_t^* \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & \rho_\gamma \end{bmatrix} \begin{bmatrix} \hat{y}_{t-1}^* \\ \hat{\pi}_{t-1}^* \\ \hat{i}_{t-1}^* \\ \hat{\gamma}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \sigma_{y^*} & 0 & 0 & 0 \\ c_{21} & \sigma_{\pi^*} & 0 & c_{24} \\ c_{31} & c_{32} & \sigma_{i^*} & c_{34} \\ 0 & 0 & 0 & \sigma_\gamma \end{bmatrix} \begin{bmatrix} \epsilon_t^{y^*} \\ \epsilon_t^{\pi^*} \\ \epsilon_t^{i^*} \\ \epsilon_t^{\gamma} \end{bmatrix}, \tag{2.11}$$

where $\vec{\varepsilon}_t \sim N(\vec{0}, I_4)$.

2.1.4 Equilibrium Dynamics: Canonical Representation

Optimal domestic consumption and saving decisions, combined with goods market clearing yields[3]

$$\tilde{x}_t = \mathbb{E}_t\tilde{x}_{t+1} - \zeta\sigma_s\mathbb{E}_t(\tilde{x}_{s,t+1} - \tilde{x}_{s,t}) - (1-\zeta)\varphi_\gamma^{-1}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - r_t^e), \tag{2.13}$$

stating that real activity, measured by $\tilde{x}_t = x_t^e - \eta_\gamma x_{t-1}^e - \beta\eta_\gamma(\mathbb{E}_t x_{t+1}^e - \eta_\gamma x_t^e)$ depends on future expected real activity, on $\tilde{x}_{s,t} = x_{s,t}^e - \eta_\gamma x_{s,t-1}^e - \beta\eta_\gamma(\mathbb{E}_t x_{s,t+1}^e - \eta_\gamma x_{s,t}^e)$ and its future expected value, which links the domestic economy to the rest of the world, and on the gap between ex-ante real interest rate and its efficient level r_t^e, where differently from the baseline model, it is now discounted by $\zeta \in [0,1]$, which is inversely related to the degree of home bias in preferences and is a natural index of openness. A hat denotes the deviation of a log-linearized variable from its stationary equilibrium. The parameter σ_s is a function of deep parameters, $\sigma_s = [\epsilon + \psi(1-\zeta)]$, where ψ is the measure of substitutability between domestic and foreign goods.

Equation 2.13 has additional terms to the baseline counterpart, given by Equation 2.12. Domestic inflation and CPI inflation are linked through the terms of trade channel, and this is why the terms of trade gap affects domestic consumption decisions. Here, $x_t^e = \hat{y}_t - \hat{y}_t^e$ is the efficient output gap and $x_{s,t}^e = \hat{s}_t - \hat{s}_t^e$ is the efficient terms of trade gap. International risk sharing implies

$$x_{s,t}^e = (1-\zeta)^{-1}(\hat{\lambda}_t^e - \hat{\lambda}_t), \tag{2.14}$$

[3] In the baseline model, optimal consumption and saving decisions produce the Euler equation

$$\tilde{x}_t = \mathbb{E}_t\tilde{x}_{t+1} - \varphi_\gamma^{-1}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - r_t^e) \tag{2.12}$$

which is the law of motion of the efficient terms of trade gap, which is at least explicitly independent of foreign variables, where $\hat{\lambda}^e$ is the stationary marginal utility of consumption in the domestic efficient equilibrium. The optimal pricing decisions of domestic firms produce the Phillips curve[4]

$$\tilde{\pi}_t = \xi\left(\omega x_t^e + \zeta x_{s,t}^e + \varphi_\gamma(1-\zeta)^{-1}\tilde{x}_t - \varphi_\gamma\zeta(1-\zeta)^{-1}\sigma_s\tilde{x}_{s,t}\right) + \zeta(\Delta\tilde{s}_t - \beta\mathbb{E}_t\Delta\tilde{s}_{t+1}) + \beta\mathbb{E}_t\tilde{\pi}_{t+1} + u_t, \tag{2.16}$$

where $\tilde{\pi}_t = \hat{\pi}_t - \nu\hat{\pi}_{t-1}$ is a measure of current inflation, $\Delta\tilde{s}_t = \Delta\hat{s}_t - \nu\Delta\hat{s}_{t-1}$ is a measure of current terms of trade in first differences and

$$u_t = \rho_u u_{t-1} + \epsilon_t^u \tag{2.17}$$

is a cost-push shock with $\epsilon_t^u \sim N(0, \sigma_u)$. Equation 2.16 has additional terms to the baseline Phillips curve, given by Equation 2.15, which reflects integration from the domestic economy to both financial and goods international markets[5]. The interaction of the domestic economy with international markets turns possible to represent a theoretical channel of pass-through from foreign inflation to domestic prices, being the terms of trade volatility capable of making it more challenging for the monetary authority to maintain inflation on target.

2.2 MONETARY POLICY

Our analysis is mostly based on the comparison of policy rules proposed by Cúrdia et al. (2015). The baseline Wicksellian rule (henceforth, W rule) supposes that the monetary authority responds to the efficient real interest rate, or the real interest rate that would prevail in the Pareto optimal equilibrium, to the time-varying inflation target, and to inflation, with some inertia:

$$\hat{i}_t = \rho\hat{i}_{t-1} + (1-\rho)\left(\hat{\bar{\pi}}_t + \phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + r_t^e\right) + \epsilon_t^i \tag{2.18}$$

where $\epsilon_t^i \sim N(0, \sigma_i)$. The expression for the efficient real interest rate, in both the baseline and small open economy models, is given by:

$$r_t^e = \mathbb{E}_t\hat{\gamma}_{t+1} + \mathbb{E}_t\hat{\delta}_{t+1} - \omega\mathbb{E}_t\Delta\hat{\hat{y}}_{t+1}^e \tag{2.19}$$

The time-varying inflation target evolves according to:

$$\bar{\pi}_t = (1-\rho_{\bar{\pi}})\bar{\pi} + \rho_{\bar{\pi}}\bar{\pi}_{t-1} + \epsilon_t^{\bar{\pi}}, \tag{2.20}$$

where $\epsilon_t^{\bar{\pi}} \sim N(0, \sigma_{\bar{\pi}})$ and $\rho_{\bar{\pi}} \in (0,1)$. In the baseline Taylor rule (henceforth, T rule), instead of responding to the efficient real interest rate, the monetary authority sets

[4] In the baseline model, optimal pricing decisions produce the Phillips curve

$$\tilde{\pi}_t = \xi(\omega x_t^e + \varphi_\gamma\tilde{x}_t) + \beta\mathbb{E}_t\tilde{\pi}_{t+1} + u_t, \tag{2.15}$$

[5] Even though the terms of trade gap is independent of foreign variables, the law of motion for terms of trade $\hat{s}_t = (1-\zeta)^{-1}(\hat{\lambda}_t^* - \hat{\lambda}_t - \hat{z}_t)$ is a function of world consumption, as shown in the Appendix.

the nominal interest rate in response to the efficient output gap, or the gap between the current level of output and the level of output that would prevail in the Pareto optimal equilibrium:

$$\hat{i}_t = \rho\hat{i}_{t-1} + (1-\rho)\left(\hat{\bar{\pi}}_t + \phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \phi_x x_t^e\right) + \epsilon_t^i \tag{2.21}$$

The widely used Hodrick-Prescott filter (henceforth, HP filter) is also considered for a measure of output gap and a real activity indicator, since it does not require a fully specified model and it is tractable in a rational expectations context (CHRISTIANO; FITZGERALD, 2003). Given observations on $\ln GDP_t$, the HP gap with parameter λ is given by:

$$[1 + \lambda(1-L)^2(1-F)^2]x_t^{hp} = \lambda(1-L)^2(1-F)^2 \ln GDP_t, \tag{2.22}$$

where L and F are backward and forward operators, respectively. The ideal filter is directly applied through rational expectations forecasts, where $\lambda = 1600$ is the suggested ratio for quarterly data. In this case, the Taylor rule with HP filter measure of output gap is given by:

$$\hat{i}_t = \rho\hat{i}_{t-1} + (1-\rho)\left(\hat{\bar{\pi}}_t + \phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \phi_x x_t^{hp}\right) + \epsilon_t^i \tag{2.23}$$

3 BAYESIAN INFERENCE

We use Bayesian methods to characterize the posterior distribution of the structural parameters, combining prior information with the likelihood function, which is based on the vector of observables $[\Delta \ln GDP_t \; CIPCA_t \; SELIC_t \; IPCAT_t]'$. The baseline model measurement equation is given by:

$$\begin{bmatrix} \Delta \ln GDP_t \\ CIPCA_t \\ SELIC_t \\ IPCAT_t \end{bmatrix} = 400 \begin{bmatrix} \gamma \\ \bar{\pi} \\ r + \bar{\pi} \\ \bar{\pi} \end{bmatrix} + 400 \begin{bmatrix} \Delta \hat{y}_t + \hat{\gamma}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{\bar{\pi}}_t \end{bmatrix} + \begin{bmatrix} \epsilon^{me}_{gdp,t} \\ \epsilon^{me}_{cipca,t} \\ 0 \\ 0 \end{bmatrix} \tag{3.1}$$

where GDP_t is real GDP, $CIPCA_t$ is the official exclusion 2 core inflation[1], $IPCAT_t$ is the inflation target, and $SELIC_t$ is the average nominal interest rate (Selic rate), all sampled in quarterly frequency. The constants in Equation 3.1 are the average growth rate of productivity (γ), the long-run inflation target ($\bar{\pi}$), and the average real interest rate (r). The non-observable variables are log-linear deviations from their stationary equilibrium. The sample period spans from 2002:T2 to 2019:T4, after both the Real Plan and inflation-targeting regime adoption. They contributed to a more stable macroeconomic environment and the lower volatility of monetary instance. As argued by Cúrdia et al. (2015), this reasonably homogeneous approach by the monetary authority makes the monetary policy get close to a stable interest rate rule. The main results are not affected by truncating the sample at 2014:T1, before one of the most severe recessions in Brazil's history.

In addition to the baseline model, we use the following time series as observables for the small open economy: terms of trade growth, foreign real GDP growth, foreign inflation and foreign nominal interest rate. Since our purpose is to verify whether the results remain true within another DSGE empirical specification, we take the weighted average of the observable series of the main Brazilian trade partners as a representative time series of the foreign variable[2]. The small open economy model

[1] This less noisy core measure was chosen to be in line with the empirical strategy adopted by Cúrdia et al. (2015), although this choice might be controversial. See Filho and Figueiredo (2011) and Ferreira, Mattos and Ardeo (2017) for an evaluation of Brazilian core inflation measures. The main results are maintained whether we measure inflation by using the extended price index (IPCA).

[2] See Christiano, Trabandt and Walentin (2011) for another example of this procedure.

measurement equation is given by:

$$\begin{bmatrix} \Delta \ln GDP_t \\ CIPCA_t \\ SELIC_t \\ IPCAT_t \\ \Delta \ln WGDP_t \\ WCPI_t \\ WFFR_t \\ TERMS_t \end{bmatrix} = 400 \begin{bmatrix} \gamma \\ \bar{\pi} \\ r+\bar{\pi} \\ \bar{\pi} \\ \gamma \\ \pi^* \\ r^*+\pi^* \\ \Delta s \end{bmatrix} + 400 \begin{bmatrix} \Delta\hat{y}_t + \hat{\gamma}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{\bar{\pi}}_t \\ \Delta\hat{y}^*_t + \hat{\gamma}^*_t \\ \hat{\pi}^*_t \\ \hat{i}^*_t \\ \Delta\hat{s}_t \end{bmatrix} + \begin{bmatrix} \epsilon^{me}_{gdp,t} \\ \epsilon^{me}_{cipca,t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \epsilon^{me}_{terms,t} \end{bmatrix} \tag{3.2}$$

where $WGDP_t$ is the foreign real GDP growth, $WCPI_t$ is the foreign consumer prices inflation, $WFFR_t$ is the foreign nominal interest rate, and $TERMS_t$ is the terms of trade growth. The remaining constants in this equation represent the average foreign growth rate of productivity (γ), the foreign long-run inflation target (π^*), the average foreign real interest rate (r^*), and the long-run terms of trade growth (Δs). The policy rules are evaluated by comparing the fit across models through the Bayes factor as proposed by Kass and Raftery (1995). When two times the log of the Bayes factor is above 10, we consider the existence of very strong evidence in favor of a model.

3.1 ESTIMATION PROCEDURE

We estimate both baseline and small open economy models. The resulting log-linearized equilibrium dynamics make up the rational expectations system, whose solution is approximated to a vector autoregressive process for the model variables and used as state-transition equations. The resulting linear model, when augmented by the measurement equations, composes the state space representation of the DSGE model. Under the assumption of normally distributed innovations, the likelihood function is evaluated with the Kalman filter and it is used to form a posterior density, which is drawn with MCMC methods.

Identification issues are assessed through the local identification condition proposed by Iskrev (2010). Estimation results are obtained using the random walk Metropolis-Hastings algorithm with at least $500,000$ draws, for the baseline model, and at least $3,000,000$ draws, for the small open economy model, with 30 percent burn-in of total draws and four chains in both cases. The acceptance ratio is tuned to be between 0.2 and 0.3, approximately. Within our sample, the posterior mode used to initialize the algorithm and define the jumping distribution is hard to obtain with standard optimization routines, since they often fail to find a minimum with a positive definite Hessian matrix. To avoid this problem, we used a Monte Carlo-based routine that continuously updates the posterior covariance matrix to compute the initial posterior mode. We compute the Bayes factor with the modified harmonic mean estimator proposed by Geweke (1999). Finally, we verify whether the algorithm converged satisfactorily through the convergence diagnosis proposed by Brooks and Gelman (1998), comparing between and within moments of multiple chains[3].

[3] All estimations are made with Dynare v4.6.1

3.2 CHOICE OF PRIORS

The priors for the baseline model are fairly diffuse and are in line with those adopted in previous studies (PALMA; PORTUGAL, 2014; GONÇALVES; PORTUGAL; ARAGÓN, 2016). We calibrate the discount factor as $\beta = 0.98$ consistently with an average annualized real interest rate of 7.06% observed in the data. On the supply side, given our observables, only the slope of the Phillips curve $\xi = (1-\alpha)(1-\alpha\beta)/\alpha$ can be identified. Its prior is centered around 0.01, thus remaining consistent with the degree of price stickiness found in microeconomic studies[4]. The priors for the feedback coefficients ϕ_π and ϕ_x are centered almost like in Cúrdia et al. (2015) but taking Brazilian past estimates into account, i.e., around 1.75 and 0.25, respectively[5]. Educated guesses are given for the prior mean of long-term inflation and real interest rates, given the averages observed in the data and forecasts for the Selic rate for the coming years. In the case of the productivity growth rate, the prior mean is chosen to match the observed rate in the Brazilian economy recently. Measurement errors are permitted to absorb no more than 25 percent of the variance of the inflation time series and 15 percent of the variance of the gross domestic product time series[6]. Prior choices are reported in Table 2.

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4 Given the average price duration obtained from Correa, Petrassi and Santos (2018), we can set $\alpha \approx 0.9$ (fraction of producers not adjusting prices).

5 Equilibrium is determinate in the baseline model if and only if $\phi_\pi + (1-\beta)\phi_x/\xi > 1$ (CÚRDIA et al., 2015 apud WOODFORD, 2011).

6 Inflation measurement error is allowed to have higher variance since it contains additional high-frequency noise (CHRISTIANO; TRABANDT; WALENTIN, 2011).

Table 1 – Prior distributions for the parameters in the baseline model. G, B, N and $IG1$ stand for gamma, beta, normal, and inverse gamma distributions, respectively.

Parameter	Distribution	Lower bound[1]	Upper bound[2]
ω	$G(2, 0.4)$	0	10
ξ	$G(0.01, 0.005)$	0	1
η	$B(0.7, 0.2)$	0	1
ν	$B(0.3, 0.2)$	0	1
ρ	$B(0.6, 0.15)$	0	1
ϕ_π	$N(1.75, 0.3)$	0	10
$4\phi_x$	$N(0.25, 0.3)$	0	10
$400\bar{\pi}$	$N(4, 1)$	-	-
$400r$	$N(3, 1)$	-	-
400γ	$N(1, 0.2)$	-	-
ρ_δ	$B(0.5, 0.2)$	0	1
ρ_γ	$B(0.5, 0.2)$	0	1
ρ_u	$B(0.5, 0.2)$	0	1
$\rho_{\bar{\pi}}$	$B(0.95, 0.04)$	0	1
σ_{gdp}^{me}	$IG1(0.5, 2)$	0	2.50
σ_{cipca}^{me}	$IG1(0.5, 2)$	0	1.17
σ_δ	$IG1(0.5, 2)$	0	10
σ_γ	$IG1(0.5, 2)$	0	10
σ_u	$IG1(0.5, 2)$	0	10
σ_i	$IG1(0.5, 2)$	0	10
$\sigma_{\bar{\pi}}$	$IG1(0.2, 1)$	0	10

Source: Own construction (2020).

[1,2]Optimization is constrained to a feasible subset of the parametric space.

In the small open economy empirical model, the priors are also in line with those adopted in previous studies (PALMA; PORTUGAL, 2014; CHRISTIANO; TRABANDT; WALENTIN, 2011). We calibrate the degree of home bias in preferences as $\zeta = 0.13$, in a way consistent with the import/GDP ratio in Brazil during the sample run. The prior of the parameter σ_s is centered around 1.55 to keep it consistent with estimates for the Brazilian economy. The prior for the feedback coefficient ϕ_s is centered around 0.5[7]. The prior distributions for the long-run foreign inflation target π^*, the long-run foreign real interest rate r^*, and the average growth rate of foreign productivity γ are exactly like in Cúrdia et al. (2015). The prior for the long-run terms of trade growth is centered around 0.6 to match the average observed in the data. In the case of the productivity asymmetry process parameters, we also chose non-informative priors. To the best of our knowledge, this is the first paper considering a unit-root technology process common to Brazilian and foreign economies[8]. Measurement errors are permitted to absorb no more than 25 percent of

[7] It is close to the central bank real exchange rate preference parameter estimate by Palma and Portugal (2014). In our model, there is a direct relation between terms of trade and the effective real exchange rate $\hat{q}_t = \int_0^1 q_{i,t} di = (1-\zeta)\hat{s}_t$. See Gali and Monacelli (2005) for further details.

[8] For safety, the prior means are close to the ones in Adolfson et al. (2007).

the variance of the observable time series for terms of trade and inflation. Finally, all parameters describing the stochastic processes driving the foreign variables share the same prior distributions as those in Christiano, Trabandt and Walentin (2011).

Table 2 – Prior distributions for the parameters in the small open economy model. G, B, N and $IG1$ stand for gamma, beta, normal, and inverse gamma distributions, respectively.

Parameter	Distribution	Lower bound[1]	Upper bound[2]
ω	$G(2, 0.4)$	0	10
ξ	$G(0.01, 0.005)$	0	1
σ_s	$G(1.55, 1)$	-	-
η	$B(0.7, 0.2)$	0	1
ν	$B(0.3, 0.2)$	0	1
ρ	$B(0.6, 0.15)$	0	1
ϕ_π	$N(1.75, 0.3)$	0	10
$4\phi_x$	$N(0.25, 0.3)$	0	10
ϕ_s	$N(0.5, 0.3)$	0	10
$400\bar{\pi}$	$N(4, 1)$	-	-
$400r$	$N(3, 1)$	-	-
400γ	$N(3, 0.2)$	-	-
$400\pi^*$	$N(2, 1)$	-	-
$400r^*$	$N(2, 1)$	-	-
$400\Delta s$	$N(0.6, 0.5)$	-	-
ρ_δ	$B(0.5, 0.2)$	0	1
ρ_γ	$B(0.5, 0.2)$	0	1
ρ_u	$B(0.5, 0.2)$	0	1
ρ_z	$B(0.5, 0.2)$	0	1
$\rho_{\bar{\pi}}$	$B(0.95, 0.04)$	0	1
σ^{me}_{gdp}	$IG1(1, 2)$	0	2.50
σ^{me}_{cipca}	$IG1(0.5, 2)$	0	1.17
σ^{me}_{terms}	$IG1(2, 2)$	0	5.70
σ_δ	$IG1(0.5, 2)$	0	10
σ_γ	$IG1(0.5, 2)$	0	10
σ_u	$IG1(0.5, 2)$	0	10
σ_i	$IG1(0.5, 2)$	0	10
σ_z	$IG1(0.5, 2)$	0	10
σ_{y^*}	$IG1(0.5, 2)$	0	10
σ_{i^*}	$IG1(1.5, 2)$	0	10
σ_{π^*}	$IG1(0.5, 2)$	0	10
$\sigma_{\bar{\pi}}$	$IG1(0.2, 1)$	0	10
Other foreign VAR parameters	$N(0, 0.5)$	-	-

Source: Own construction (2020).

[1,2]Optimization is constrained to a feasible subset of the parametric space.

4 RESULTS

We introduce the results with a brief comment on some of the parameter estimates of the T specification reported in Table 3[1].

[1] We turn back to emphasize that this is the first paper to assess the empirical plausibility of efficient real interest rate tracking by the Brazilian monetary authority. Therefore, there is no way to evaluate the estimates of the Wicksellian specifications.

4.1 POSTERIOR PARAMETER VALUES

Table 3 – Estimation results for T, W, and W&T specifications.

Statistics	W	SOE W	T	SOE T	W&T	SOE W&T
ML	-568.71	-1158.69	-561.65	-1155.39	-564.47	-1156.78
Parameter	Mean		Mean		Mean	
ω	2.19	2.13	2.04	1.94	2.12	2.17
100ξ	1.92	1.06	0.94	0.57	1.54	0.90
σ_s	-	0.76	-	0.64	-	0.80
η	0.52	0.63	0.60	0.73	0.40	0.61
ν	0.25	0.25	0.20	0.15	0.19	0.20
ρ	0.79	0.63	0.81	0.79	0.79	0.66
ϕ_π	1.76	1.75	2.33	2.34	1.65	1.82
$4\phi_x$	-	-	0.72	0.36	0.65	0.54
ϕ_s	-	0.12	-	0.29	-	0.12
$400\bar{\pi}$	4.98	4.92	4.53	4.50	4.79	4.94
$400r$	3.39	3.86	3.99	4.40	3.45	3.91
400γ	1.05	2.97	1.05	2.98	1.05	2.97
$400\pi^*$	-	1.96	-	1.96	-	1.97
$400r^*$	-	0.58	-	0.57	-	0.54
$400\Delta s$	-	0.78	-	0.82	-	0.75
ρ_δ	0.97	0.92	0.91	0.95	0.94	0.90
ρ_γ	0.93	0.56	0.85	0.71	0.79	0.56
ρ_u	0.80	0.52	0.52	0.46	0.81	0.57
ρ_z	-	0.96	-	0.76	-	0.96
ρ_π	0.86	0.84	0.85	0.83	0.86	0.85
σ_δ	3.26	2.14	2.64	1.01	3.09	2.32
σ_γ	0.94	1.88	1.40	1.89	1.63	1.87
σ_u	0.65	1.24	0.78	1.15	0.52	1.02
σ_i	0.82	0.46	1.08	1.32	0.80	0.47
σ_z	-	5.53	-	5.43	-	5.55
σ_{y^*}	-	0.23	-	0.20	-	0.24
σ_{i^*}	-	0.12	-	0.11	-	0.11
σ_{π^*}	-	1.08	-	1.09	-	1.09
σ_π	0.67	0.67	0.67	0.67	0.67	0.67
σ^{me}_{gdp}	0.44	2.46	0.42	2.47	0.52	2.46
σ^{me}_{cipca}	0.73	0.47	0.44	0.33	0.53	0.45
σ^{me}_{terms}	-	5.58	-	5.52	-	5.60

Source: Own construction (2020).

SOE stands for the small open economy version of each specification.

The posterior mean of the inverse of Frisch elasticity of labor supply is quite higher than the estimates that can be found in Silveira (2008), especially when there is both habit formation in consumption and price indexation, which is an unexpected result. However, they are consistent with the estimates found in Palma and Portugal (2014) and closer to other estimates in the international empirical literature. The posterior mean of the habit persistence parameter is very close to those found in Silveira (2008) and Castro et al. (2015). The interest rate smoothing coefficient is overall consistent with past estimates, like ϕ_π and ϕ_x are consistent with the values found in Castro et al. (2015). In the case of parameter ϕ_x, our estimate is slightly lower, then we must take into account the latter sample, among other factors.

The slope of the Phillips curve varies across the different specifications and its posterior is concentrated next to the prior in some cases. As we will see later, relative to W rules, the T rules fit the data better, and their closed economy specification yield a very satisfactory estimate for this parameter, although the SOE T specification has a lower value for ξ than expected[2]. Our results also differ substantially from the results obtained for US data, when the estimates of the slope of the Phillips curve were concentrated near low values in all cases. Cúrdia et al. (2015) suggested that it indicates an identification issue since in their Taylor specification both the parameters ν and ρ_u had bimodal posterior distributions.[3] Interestingly, the reverse happens to the Brazilian data (when the economy is closed), strengthening this argument. The parameters associated with the productivity growth process have a bimodal distribution in the W specification, which has a worse empirical fit. However, with the inclusion of the open economy dimension, the argument is weakened. In this case, the parameters associated with the technological asymmetry process have a bimodal distribution in the T specification, which has a better fit. It is worth remembering that the technological asymmetry process ends up defining the face of the domestic productivity growth process.

The degree of indexation to past inflation is also lower than that found in empirical studies. Our open economy models yield even lower estimates for this parameter. This is not a surprise, since in our model ν also accounts for how much both the domestic producers that reset or not their prices optimally seek to achieve the long-run inflation target, which plays an important role in the price indexation scheme. The endogenous persistence mechanism is less relevant in the Taylor specifications, suggesting that past inflation plays a less significant role if we conclude that this model fits the data better. This isn't convenient since ν posterior mean wouldn't be closer to the estimates found in Silveira (2008) and Palma and Portugal (2014). One can conclude that because of the later sample, domestic producers updated their beliefs about the Central Bank's ability to achieve the long-term inflation target.

Estimates of GDP measurement error differ significantly with the inclusion of the open economy dimension. This may mean that the open economy model is forcing an unobserved connection between the output of domestic and foreign economies[4]. There may be two possible reasons. First, the foreign economy is theoretically

[2] Regarding the SOE T specification, we have $\xi = 0.0057$, which implies $\alpha \approx 0.9357$ and means that our estimates are consistent with a higher average price duration.

[3] Posterior distributions for selected parameters can be verified in the Annex.

[4] See Justiniano and Preston (2010) for an example of the inability of this kind of model in explaining some features of the data. The GDP forecasting performance of our models deteriorates dramatically with the inclusion of the open economy dimension, although there is an improvement in the inflation forecasting performance.

a representation of the rest of the world, and not of a bunch of countries[5]. In addition, both of our models are stylized and abstract from capital accumulation and management, non-competitive features in the labor market, and a more detailed description of the imports and exports sectors, as well as financial and employment frictions, to name a few. Neither of our models will be able to reproduce the main features of the observed time series in the best way possible, especially in the case of a small open economy, but they will provide a reasonable description of the data. This argument is strengthened by the fact that the estimated standard deviation of the terms of trade measurement error reaches its upper bound. In Table **??**, we present the standard deviations for the observed time series and the ones implied by our models.

4.2 WHICH REAL ACTIVITY INDICATOR SHOULD WE CHOOSE?

The comparison of the estimated marginal likelihoods and the implied Bayes factors are reported in Table 4. First of all, the Wicksellian rule shows the worst empirical performance relative to any other feedback rule into consideration[6]. It means that the policy rule that includes some measure of the output gap characterizes more adequately the conduct of monetary policy by the Brazilian monetary authority in the inflation-targeting regime. This result is maintained if we remove the time-varying inflation target from the feedback rules. In our framework, if the actual real interest rate matches its efficient counterpart, there is no output gap. To exploit the possibility that both efficient output gap and efficient real interest rate may be useful complements as real activity indicators, we estimate a combined W&T rule given by

$$\hat{i}_t = \rho\hat{i}_{t-1} + (1-\rho)\left(\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + r_t^e + \phi_x x_t^e\right) + \epsilon_t^i \tag{4.1}$$

We can conclude little about this specification since our Bayes factor ratio is too small when the behavior of households and firms is described by the New Keynesian small open economy. The improvement obtained in Cúrdia et al. (2015) for US data is maintained only for the baseline model when there is strong evidence in favor of the hybrid policy rule over the W rule. Another change we could make in the policy rule would be to consider the output growth as a real activity indicator:

$$\hat{i}_t = \rho\hat{i}_{t-1} + (1-\rho)\left(\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + r_t^e + \phi_{\Delta y}\Delta y_t\right) + \epsilon_t^i \tag{4.2}$$

Concerning the output growth policy rule, we should consider the possibility that the Brazilian monetary authority prioritized growth over closing the output gap, which coincides with moments in the history of Brazil's monetary policy when possible inflationary pressures were neglected in favor of immediate results in terms of growth[7].

[5] Even if they are responsible for almost half of the total of exports and imports.

[6] If two times the difference between the log-marginal likelihoods of two models is greater than 10, then there is very strong evidence in favor of one model. See Kass and Raftery (1995) for details.

[7] One of the mechanisms whereby this can occur is through political cycles, in which there may be negligence in stabilizing inflation in favor of political agents.

This is also a plausible option given the competitive fit to the data relative to the other policy rules specifications. Finally, by including the open economy dimension, the main results remain unchanged, although the evidence of better empirical fit of the Taylor rules weakens. Also, any evidence in favor of the Hodrick-Prescott policy rule over the Taylor policy rule ceases to exist.

Table 4 – Estimation results for policy rule specifications. The policy rules are defined as $\hat{i}_t = \rho\hat{i}_{t-1} + (1-\rho)\Omega_t + \epsilon_t^i$, where Ω_t is the systematic response to the state of the economy.

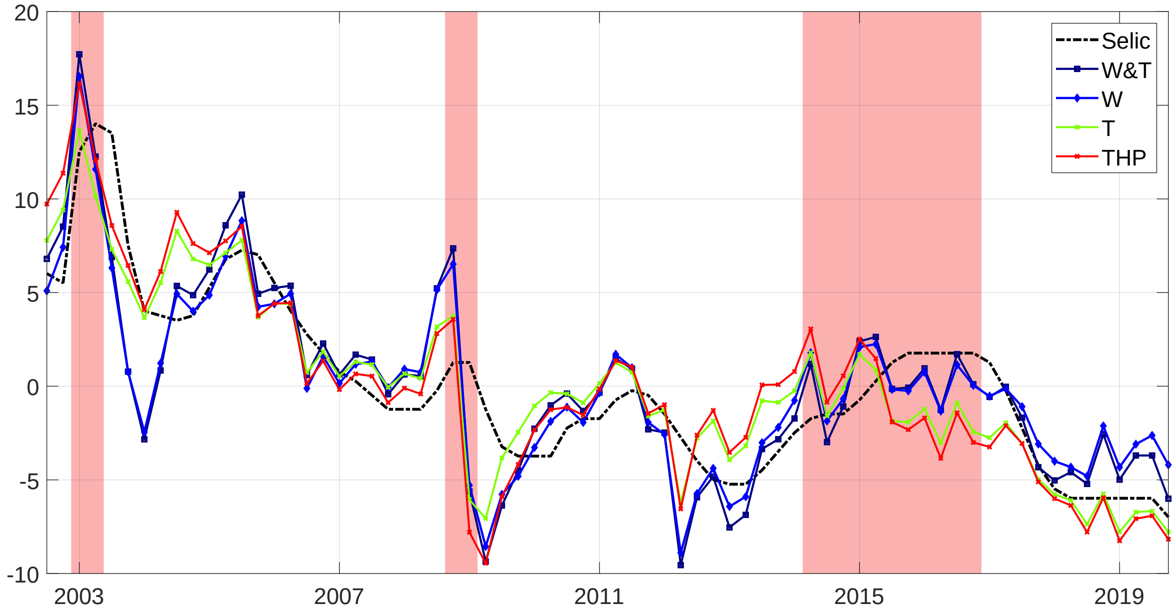
Specification	Ω_t	$2\ln BF$	ML
W	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + r_t^e$	-	-568.71
T	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_x x_t^e$	14.11	-561.65
W&T	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_x x_t^e + r_t^e$	8.47	-564.47
T with HP gap	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_x x_t^{HP}$	16.91	-560.25
T with growth	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_{\Delta y}\Delta y_t$	12.27	-562.57
SOE W	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_s\Delta\hat{s}_t + r_t^e$	-	-1158.69
SOE T	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_s\Delta\hat{s}_t + \phi_x x_t^e$	6.60	-1155.39
SOE W&T	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_s\Delta\hat{s}_t + \phi_x x_t^e + r_t^e$	3.82	-1156.78
SOE T with HP gap	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_s\Delta\hat{s}_t + \phi_x x_t^{HP}$	6.58	-1155.40
SOE T with growth	$\phi_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + \hat{\bar{\pi}}_t + \phi_s\Delta\hat{s}_t + \phi_{\Delta y}\Delta y_t$	9.56	-1153.91

Source: Own construction (2020).

Figure 1 illustrates the estimated behavior of the efficient interest rate over time in the baseline model. As clear as in Cúrdia et al. (2015), this business cycle indicator rises during booms and drops in recessions. Even if r_t^e don't affect the interest rate setting, falls in r_t^e tend to precede falls in the Selic rate, and increases in r_t^e tend to precede rises in the Selic rate[8]. In these closed economy models, there is some homogeneity regarding the behavior of the Wicksellian rate of return. It is worth emphasizing the similarity between the W and W&T specifications. Thus, we can conclude that adding the output gap to the monetary policy rule does little to change the behavior of the efficient interest rate when it affects monetary policy decisions. This fact is confirmed by looking at Figure 2 when even with the addition of the open economy dimension this result stands still.

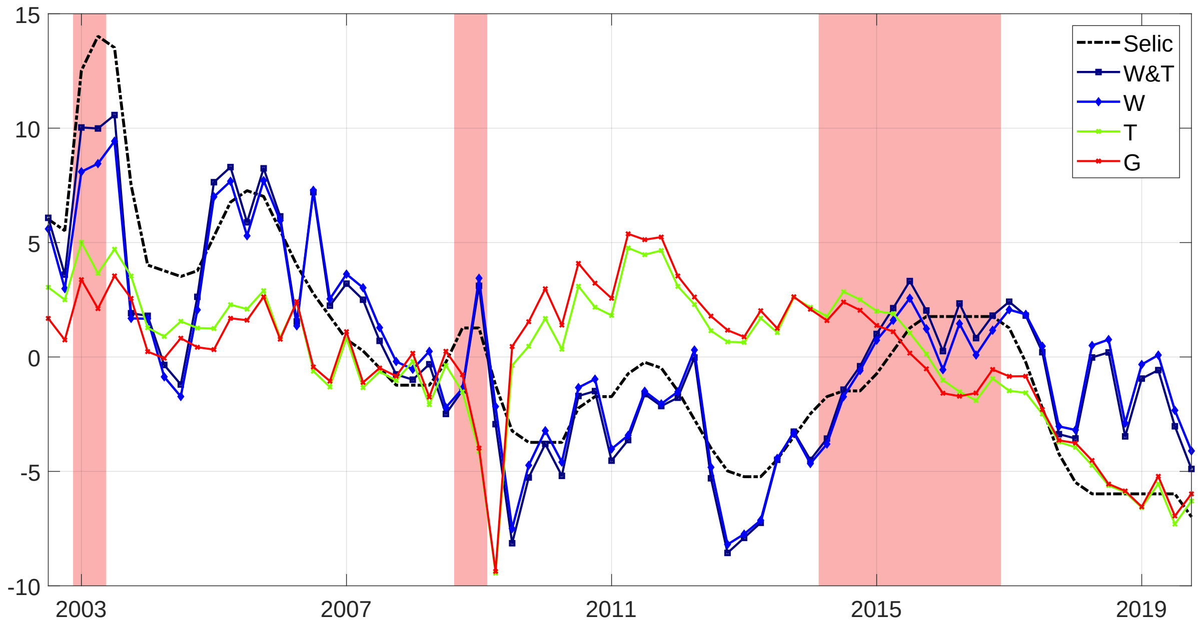
[8] Observations on the nominal interest rate also carry information on the efficient interest rate.

Figure 1 – Selic rate and smoothed estimates of the efficient interest rate across the baseline main specifications. All rates are demeaned and expressed in annualized percentages. Red shades mark CODACE-FGV recessions in the Brazilian economy during the sample run.



Source: Own construction (2020).

Figure 2 – Selic rate and smoothed estimates of the efficient interest rate across the small open economy main specifications. All rates are demeaned and expressed in annualized percentages. Red shades mark CODACE-FGV recessions in the Brazilian economy during the sample run.

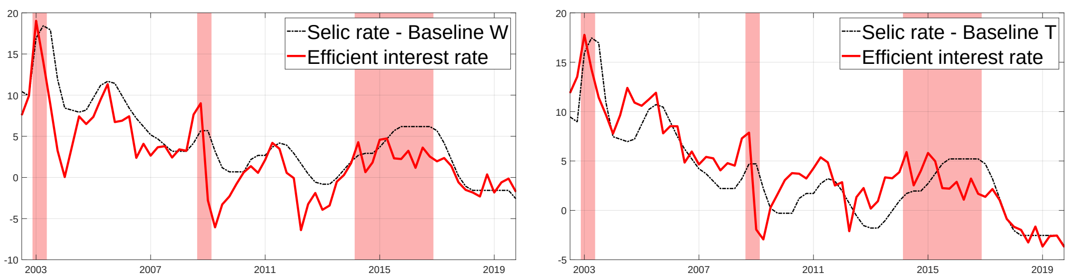


Source: Own construction (2020).

Figure 2 illustrates the estimated behavior of the efficient interest rate over time in the small open economy model. The first impression that the efficient interest rate is a great business cycle indicator remains, with reservations. The efficient interest rate estimates aren't consistent with the liquidity crisis of 2002, for example, where the inflationary pressure caused the Selic rate to move inconsistently with the business cycle since the monetary authority was struggling to maintain inflation on target. At that time, in addition to the reversal of capital flows, there was uncertainty about the future of the economic policy to be conducted by the newly elected administration and also about the sustainability of the public debt. These dynamics aren't captured by our model, which abstracts from financial factors. In the following quarters, the Brazilian economy improved its performance, for several reasons, and it was possible to gradually lower the Selic rate without further inflationary pressures, which is captured by the sharp drop in r_t^e. The estimates also capture well the dynamics of the business cycle during the global financial crisis of 2007-2008.

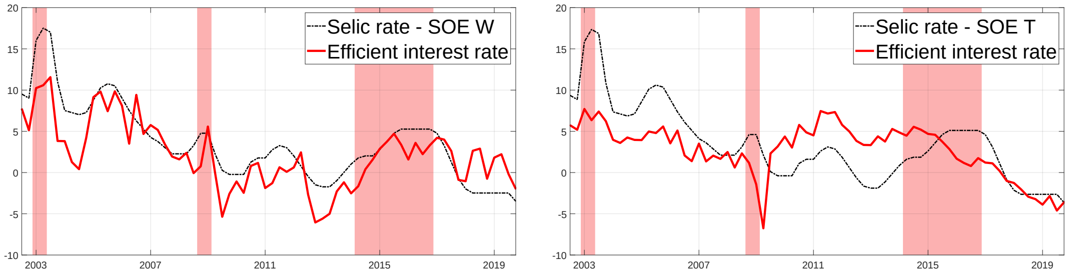
During the last recession, mainly in the T specification (Figure 2), the efficient interest rate has detached itself from the previous pattern around the Selic rate, suggesting a structural change in the model. This could be related to the recent record-low level of the Selic rate. Figures 3 and 4 also make it clear that the Brazilian monetary policy maybe was misguided between 2011 and 2016. Even though the Selic rate was below its efficient level between 2008 and 2011 (shortly after the global financial crisis), this behavior is consistent with the monetary policy delay inherent in the impossibility of observing the efficient level of interest rates in real-time. The monetary authority appears to have lowered the Selic rate with some caution as a reaction to the global financial crisis, just as it increased the Selic rate to a higher level with a similar delay, as a reaction to the rise of the efficient interest rate after the crisis. However, in Taylor's specifications, the Selic rate remains below its efficient level for too long a period, which could explain the prolonged inflationary pressure observed in the period, even if measured by the core, as it is precisely the case. This is consistent with the discretionary shift that resulted in the breaking of monetary policy rules after 2011 (CORTES; PAIVA, 2017).

Figure 3 – Selic rate and smoothed estimates of the efficient interest rate across the baseline main specifications. All rates are deviations from their stationary equilibrium and expressed in annualized percentages. Red shades mark CODACE-FGV recessions in the Brazillian economy during the sample run.



Source: Own construction (2020).

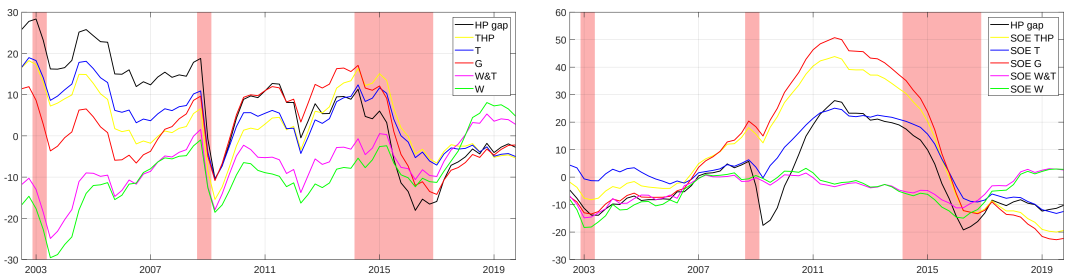
Figure 4 – Selic rate and smoothed estimates of the efficient interest rate across the small open economy main specifications. All rates are deviations from their stationary equilibrium and expressed in annualized percentages. Red shades mark CODACE-FGV recessions in the Brazillian economy during the sample run.



Source: Own construction (2020).

In addition to the statistical standpoint, the feasibility of the W rules is unlikely due to the more coherent reading that can be made of monetary policy conduction during the sample run through other rules. As previously mentioned, the behavior of the efficient interest rate in the W rules is inconsistent[9] with the inflationary pressure observed between 2011 and 2016, during and after the unconventional monetary policy put in place by the administration of Alexandre Tombini. The possibility that there was no commitment to the long-run inflation target can be justified by the intervention in the group of administered prices, which is related to price adjustment contracts for items such as gasoline and electricity. The credibility of the monetary policy, which means a commitment to conventional monetary policy, was restored by Ilan Goldfajn, who was able to control inflation, that was at risk of not converging within the bounds established by the inflation targeting regime. This credible performance is captured by both closed economy specifications, but in the case of open economies, it is only consistent with the T specification, as can be seen in Figure 4.

Figure 5 – Smoothed estimates of the output gap across different specifications. Red shades mark CODACE-FGV recessions in the Brazillian economy during the sample run. HP gap stands for the HP filter implied gap measure. The other measures are the efficient output gap across all specifications.



Source: Own construction (2020).

[9] When we say that monetary policy is consistent with the inflation targeting regime, it means that if it is the case that there is inflationary pressure, the real interest rate is above its efficient level.

Figure 5 illustrates the estimated output gaps. As highlighted by Cúrdia et al. (2015), this is an important reality check, given the possibility of an unreasonable gap measure. This is not our case. Both the efficient output gap and Hodrick-Prescott filter implied gap measure captures well the dynamic of the business cycles. However, under the small open economy W and W&T specifications, the output gap is also clearly less cyclical than in the T specification. This figure is a perfect representation of the feasibility of the hypothesis that the conduct of monetary policy was incompatible with the inflation targeting regime after the global financial crisis, during Tombini's presidency. As the Phillips curve suggests, one of the main components of inflation is the output gap, which remains high between 2011 and 2016 in the specifications that got a better empirical fit.

4.3 RELATIVE FORECASTING PERFORMANCE

To deepen the comparison between the W and T rules, we perform a forecasting exercise with the models and compare the respective performances with those of an unrestricted model. More specifically, we compare both out-of-sample root mean square errors (RMSE) and mean absolute errors (MAE) of the DSGE models with point forecasts from the posterior mean parameter values. The unconstrained vector autoregressions are estimated in the same dataset, and their lag orders are chosen based on a set of information criteria and marginal likelihoods. As we should expect a bad empirical performance of unconstrained VARs, since overparameterized models typically perform poorly in out-of-sample forecasts, we consider the hierarchical Bayesian VAR approach proposed by Giannone, Lenza and Primiceri (2015). Every model considered is estimated in a subsample, 2002Q3-2015Q4, of the available data, and the remaining observations, 2016Q1-2019Q4, are used for forecast evaluation. We choose both W and T models to have an additional tool for evaluating the fit when the efficient interest rate is included in the feedback rule.

Both Tables 5 and 6 reinforce already established results for closed economies (e.g. Smets and Wouters (2007)). First, the BVAR models outperform the VAR model, especially for GDP forecasting. When considering the four-quarter horizon forecasts, the most relevant horizon for monetary policy decisions, both DSGE models outperform the VAR model. Moreover, the DSGE models outperform the BVAR models in almost every forecast horizon for all observable time series. Some aspects of the result must be highlighted when comparing the baseline W and T models. The baseline T model seems to have superior forecasting ability over the baseline W model for every observable time series. It also has the ability to outperform the W specification in the interest rate forecasts, as it is precisely our goal to understand the logic behind the monetary authority decisions concerning to the choice of Selic rates. Finally, there is a forecasting performance tradeoff that arises from the inclusion of r_t^e or x_t^e in the feedback rule, which is highlighted in the combined W&T specification. In this hybrid model, there are better forecasts only for the one-step-ahead GDP growth relative to the Taylor specification. There are also better forecasts for GDP growth and inflation, but worst forecasts for the Selic rate, relative to the Wicksell specification.

Table 5 – Out-of-sample root mean square errors (RMSE) relative to the VAR(4) model.

		Model					
Quarter	Obs	VAR(4)	BVAR(2)	BVAR(4)	Wicksell	Taylor	W&T
	Δy_t	1.00	0.67	0.67	0.79	0.73	0.76
1Q	π_t	1.00	0.95	1.05	0.82	0.75	0.78
	i_t	1.00	0.76	0.72	0.67	0.60	0.73
	Δy_t	1.00	0.42	0.47	0.37	0.37	0.39
2Q	π_t	1.00	0.99	1.07	0.70	0.59	0.62
	i_t	1.00	0.82	0.78	0.74	0.64	0.78
	Δy_t	1.00	0.80	0.69	0.74	0.57	0.60
3Q	π_t	1.00	0.86	0.92	0.51	0.33	0.32
	i_t	1.00	0.97	0.95	0.85	0.75	0.89
	Δy_t	1.00	0.50	0.53	0.58	0.33	0.50
4Q	π_t	1.00	0.89	0.93	0.62	0.43	0.54
	i_t	1.00	1.13	1.08	0.95	0.83	0.97

Source: Own construction (2020).
The models are reestimated each year, starting with 2002Q3-2015Q4 to compute from one to four-step-ahead forecasts. Intuitively, the last subsample runs from 2002Q3 to 2018Q4. A number greater than unity indicates that the model in that column makes worse forecasts than does the VAR(4) model. Obs stands for observable time series of GDP growth (Δy_t), core IPCA inflation (π_t), and Selic rate (i_t).

Table 6 – Out-of-sample mean absolute errors (MAE) relative to the VAR(4) model.

		Model					
Quarter	Obs	VAR(4)	BVAR(2)	BVAR(4)	Wicksell	Taylor	W&T
	Δy_t	1.00	0.74	0.85	0.96	0.91	0.89
1Q	π_t	1.00	1.00	1.01	0.81	0.72	0.78
	i_t	1.00	0.68	0.61	0.65	0.64	0.65
	Δy_t	1.00	0.34	0.38	0.40	0.41	0.43
2Q	π_t	1.00	1.06	1.04	0.69	0.56	0.63
	i_t	1.00	0.63	0.66	0.69	0.61	0.67
	Δy_t	1.00	0.85	0.72	0.68	0.54	0.55
3Q	π_t	1.00	0.95	0.89	0.53	0.32	0.32
	i_t	1.00	0.86	1.08	0.89	0.80	0.92
	Δy_t	1.00	0.46	0.65	0.57	0.30	0.50
4Q	π_t	1.00	0.92	0.91	0.62	0.42	0.53
	i_t	1.00	1.09	1.23	0.99	0.89	0.99

Source: Own construction (2020).
The models are reestimated each year, starting with 2002Q3-2015Q4 to compute from one to four-step-ahead forecasts. Intuitively, the last subsample runs from 2002Q3 to 2018Q4. A number greater than unity indicates that the model in that column makes worse forecasts than does the VAR(4) model. Obs stands for observable time series of GDP growth (Δy_t), core IPCA inflation (π_t), and Selic rate (i_t).

Tables 7 and 8 evaluate the forecasting performance of the New Keynesian small open economy models. Contrary to what happened when we used only the observables used to estimate the closed economy baseline models, the BVAR models outperform the VAR model only for GDP forecasting. Moreover, the BVAR models also outperform the New Keynesian models only for GDP forecasting. In the hybrid model, the inclusion of the output gap in the monetary policy rule has almost no impact on the forecasting performance relative to the W model, which still has a forecasting ability that is not as good as the one of the T model.

In general, T models have better forecasting performance than W models regardless of the addition of the open economy dimension, which implies the use of a larger set of observables and augmenting the dimension of the system of rational expectations equations that defines the equilibrium. We present, therefore, evidence that points to a better fit to the Brazilian data by the New Keynesian models whose monetary policy rule is of the Taylor type. Also, the evidence converges in the sense of excluding the possibility that the Brazilian monetary authority may use the Wicksellian rate of return as an indicator of real activity, even if it is used only as a complement to the output gap.

Table 7 – Out-of-sample root mean square errors (RMSE) relative to the VAR(5) model.

		Model					
Quarter	Obs	VAR(5)	BVAR(2)	BVAR(5)	SOE W	SOE T	SOE W&T
	Δy_t	1.00	0.76	0.98	1.33	1.06	1.32
1Q	π_t	1.00	0.79	0.90	0.90	0.67	0.91
	i_t	1.00	1.02	1.11	1.26	1.04	1.25
	Δy_t	1.00	0.35	0.44	0.51	0.46	0.52
2Q	π_t	1.00	1.46	1.61	1.27	0.74	1.28
	i_t	1.00	1.44	1.49	1.33	1.18	1.33
	Δy_t	1.00	0.27	0.31	0.61	0.55	0.58
3Q	π_t	1.00	1.32	1.40	0.82	0.44	0.83
	i_t	1.00	1.55	1.46	1.33	1.13	1.32
	Δy_t	1.00	0.26	0.33	0.75	0.72	0.73
4Q	π_t	1.00	0.93	0.92	0.61	0.32	0.64
	i_t	1.00	1.67	1.47	1.28	1.09	1.32

Source: Own construction (2020).
The models are reestimated each year, starting with 2002Q3-2015Q4 to compute from one to four-step-ahead forecasts. Intuitively, the last subsample runs from 2002Q3 to 2018Q4. A number greater than unity indicates that the model in that column makes worse forecasts than does the VAR(4) model. Obs stands for observable time series of GDP growth (Δy_t), core IPCA inflation (π_t), and Selic rate (i_t).

Table 8 – Out-of-sample mean absolute errors (MAE) relative to the VAR(5) model.

		Model					
Quarter	Obs	VAR(5)	BVAR(2)	BVAR(5)	SOE W	SOE T	SOE W&T
1Q	Δy_t	1.00	0.85	0.66	1.21	1.01	1.20
	π_t	1.00	0.80	0.84	0.90	0.65	0.92
	i_t	1.00	0.98	1.06	1.26	0.89	1.21
2Q	Δy_t	1.00	0.30	0.38	0.47	0.43	0.47
	π_t	1.00	1.51	1.54	1.24	0.71	1.30
	i_t	1.00	1.13	1.25	1.24	0.91	1.21
3Q	Δy_t	1.00	0.32	0.23	0.67	0.63	0.65
	π_t	1.00	1.70	1.64	0.91	0.45	0.99
	i_t	1.00	1.07	1.40	1.23	0.94	1.20
4Q	Δy_t	1.00	0.32	0.44	1.05	1.04	1.03
	π_t	1.00	1.35	1.27	0.87	0.41	0.93
	i_t	1.00	1.57	1.66	1.35	1.05	1.38

Source: Own construction (2020).
The models are reestimated each year, starting with 2002Q3-2015Q4 to compute from one to four-step-ahead forecasts. Intuitively, the last subsample runs from 2002Q3 to 2018Q4. A number greater than unity indicates that the model in that column makes worse forecasts than does the VAR(4) model. Obs stands for observable time series of GDP growth (Δy_t), core IPCA inflation (π_t), and Selic rate (i_t).

5 CONCLUSIONS

How should we model Brazil's central bank behavior? Since Taylor (1993), central banks have been described as setting short-term interest rates according to an interest rate rule. These rules usually conjecture that the short-term interest rate reacts to deviations of inflation from its target and of output from some measure of potential output, with some inertia. This paper evaluates an alternative view of what are the real factors driving interest rate decisions: the hypothesis that the interest rate is set to track the efficient interest rate. It was Wicksell who originally formulated the idea that monetary policy is designed to pursue a normal or natural real rate. This is why we refer to these interest rate rules as W rules. Since the efficient interest rate is a counterfactual object, we conducted our empirical investigation within a New Keynesian DSGE framework through Bayesian methods. The main finding is that T rules proved to be consistently superior to equivalent W rules, justifying their popularity.

The result implied by the baseline models is antagonistic to the one found by Cúrdia et al. (2015) for US data. In addition to the influence of the foreign economy on domestic prices, which assumes a different magnitude in Brazil, the inflation-targeting regime represents an additional challenge in the conduct of monetary policy. As pointed by Minella et al. (2003), the significant inflationary pressures stemming from the exchange rate or terms of trade volatility can make the maintenance of price stability a more challenging task in emerging markets. It could be the case that closed economy models are underestimating the relevance of the efficient interest rate in the conduct of Brazilian monetary policy. As previously mentioned, the dynamics of the Wicksellian rate of return depends heavily on foreign shocks. However, our result also holds for the small open economy version of the Calvo sticky price model with monopolistic competition, meaning that the better performance of the T rules is robust to the addition of foreign disturbances. More work across different models would be desirable. Both our models are stylized and abstract from capital accumulation and management, incomplete asset markets, non-competitive features in the labor market, and a more detailed description of the imports and exports sectors, as well as financial and employment frictions, to name a few.

Woodford (2001) suggested that a desirable rule is likely to require that the intercept be adjusted in response to fluctuations in the efficient interest rate. Although the Wicksellian natural rate of interest is a reference point for the conduct of monetary policy, we do not find any evidence that we should include it as a time-varying intercept in monetary policy rules for Brazilian data. Unlike in our models, the efficient real interest rate is not observable in practice, being this one justification of our results. This limitation represents a major challenge for central banks (GALÍ, 2015). Therefore, researchers need to be careful when proposing a monetary policy rule within the New Keynesian framework. As we have shown, the inclusion of a neutral interest rate in the feedback rule as a time-varying intercept can substantially change the dynamics of the business cycle implied by the model and decisively affect its forecasting performance. Furthermore, when r_t^e does not affect monetary policy decisions, it becomes a great business cycle indicator according to CODACE-FGV dating. The co-movement between the Selic rate and the estimates of the efficient interest rate in the W specifications raise the concern that observations on the nominal interest rate explain the estimates of r_t^e.

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APPENDIX A – EQUILIBRIUM CONDITIONS, STEADY STATE COMPUTATION AND LOG-LINEARIZED MODEL

Our world economy consists of a continuum of small open economies mostly based on Gali and Monacelli (2005), which is used as an aggregation tool for the closed economy model proposed by Cúrdia et al. (2015). Therefore, our consumption index is given by

$$C_t^h = \left[(1-\zeta)^{\frac{1}{\psi}}(C_{H,t}^h)^{\frac{\psi-1}{\psi}} + \zeta^{\frac{\psi-1}{\psi}}(C_{F,t}^h)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}} \tag{A.1}$$

where $C_{H,t}^h$ is the household $h \in (0,1)$ index of consumption of domestic goods given by

$$C_{H,t}^h = \left[\int_0^1 C_{H,t}^h(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{A.2}$$

where $j \in (0,1)$ denotes the good variety, $C_{F,t}^h$ is an index of imported goods given by

$$C_{F,t}^h = \left[\int_0^1 (C_{i,t}^h)^{\frac{\kappa-1}{\kappa}} di\right]^{\frac{\kappa}{\kappa-1}}, \tag{A.3}$$

where $C_{i,t}^h$ is an index of the quantity of goods imported from country i consumed by household $h \in (0,1)$ given by

$$C_{i,t}^h = \left[\int_0^1 C_{i,t}^h(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{A.4}$$

A.1 HOUSEHOLDS

In addition to consumption and labor supply decision, the household also must decide how to allocate its consumption among differentiated goods. This requires the consumption index of each category to be maximized for any given level of expenditures. In this case, we have[1]

$$\int_0^1 P_{H,t}(j)C_{H,t}^h(j)dj + \int_0^1\int_0^1 P_{i,t}(j)C_{i,t}^h(j)djdi = P_{H,t}C_{H,t}^h + P_{F,t}C_{F,t}^h = P_tC_t^h \tag{A.5}$$

The household consumption and saving optimal decision problem can be rewritten as:

$$\max_{C_t^h, N_t^h} \mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\prod_{s=0}^{t}e^{-\delta_s}\left[\ln(C_t^h - \eta C_{t-1}^h) - \frac{(N_t^h)^{1+\omega}}{1+\omega}\right]\right\} \; s.t. \; P_tC_t^h + \mathbb{E}_t(Q_{t,t+1}D_{t+1}^h) = W_t^hN_t^h + D_t^h + \Pi_t^h \tag{A.6}$$

[1] Details can be found in Gali and Monacelli (2005).

The logarithmic utility function ensures the existence of a balanced growth path, since our technological progress is non-stationary. The households solve the following Lagrangian problem:

$$\max_{C_t^h, N_t^h, D_{t+1}^h} \mathbb{E}_0^h \sum_{t=0}^{\infty} \beta^t \prod_{s=0}^{t} e^{-\delta_s} [\mathcal{L}],$$

$$\mathcal{L} = \left\{ \left[\ln(C_t^h - \eta C_{t-1}^h) - \frac{(N_t^h)^{1+\omega}}{1+\omega} \right] + \Xi_t \left[W_t^h N_t^h + D_t^h + \Pi_t^h - P_t^h C_t^h - \mathbb{E}_t(Q_{t,t+1} D_{t+1}^h) \right] \right\} \tag{A.7}$$

Consumption will be no more indexed by h because the existence of state-contingent securities ensures that, in equilibrium, consumption and asset holdings are the same for all households[2]. The marginal utility of nominal consumption can be written as:

$$P_t \Xi_t = \frac{1}{C_t - \eta C_{t-1}} - \eta \beta \mathbb{E}_t \left\{ \frac{e^{-\delta_{t+1}}}{C_{t+1} - \eta C_t} \right\} \tag{A.8}$$

Euler equation:

$$\Xi_t = \beta \mathbb{E}_t \left\{ \frac{\Xi_{t+1}}{Q_{t,t+1}} e^{-\delta_{t+1}} \right\} = \beta R_t \mathbb{E}_t \left(\Xi_{t+1} e^{-\delta_{t+1}} \right) \tag{A.9}$$

or

$$1 = \mathbb{E}_t \left\{ \frac{M_{t,t+1}}{Q_{t,t+1}} \frac{P_t}{P_{t+1}} e^{-\delta_{t+1}} \right\} = R_t \mathbb{E}_t \left\{ \frac{M_{t,t+1} P_t}{P_{t+1}} e^{-\delta_{t+1}} \right\}, \tag{A.10}$$

where $M_{t,t+1} = \beta \frac{\Xi_{t+1}}{\Xi_t} \frac{P_{t+1}}{P_t}$ is a real version of the stochastic discount factor and $R_t = (\mathbb{E}_t Q_{t,t+1})^{-1}$ is the gross return on securities paying off one unit of domestic currency at $t+1$. The intratemporal optimality condition is given by:

$$W_t \Xi_t = N_t^{\omega}, \tag{A.11}$$

implying that

$$\frac{W_t}{P_t} = N_t^{\omega} \left[\frac{1}{C_t - \eta C_{t-1}} - \eta \beta \mathbb{E}_t \left(\frac{e^{-\delta_{t+1}}}{C_{t+1} - \eta C_t} \right) \right]^{-1} \tag{A.12}$$

A.2 FIRMS

Each country $i \in (0,1)$ produces a continuum of differentiated goods, represented by the unit interval. A homogeneous good Y_t^i is produced accordingly to

$$Y_t^i = \left[\int_0^1 Y_t^i(j)^{1-\frac{1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{A.13}$$

where $Y_t^i(j)$ is a variety $j \in (0,1)$ from country $i \in (0,1)$. Under the assumed price-setting environment, the dynamics of the domestic price must be computed. Let $O^N(t) \subseteq (0,1)$ be the set of firms not reoptimizing their posted price at t and $O(t) \subseteq (0,1)$ be the set of firms that fully optimized their prices, i.e., they all choose an identical price $\bar{P}_{H,t}(j)$. In this case, the aggregate price level is given by:

[2] More details in Justiniano, Primiceri and Tambalotti (2010)

$$P_{H,t} = \left\{ \int_{O^N(t)} \left[P_{H,t-1}(j) \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\nu} e^{(1-\nu)\pi_H} \right]^{1-\epsilon} dj + \int_{O(t)} [\bar{P}_{H,t}(j)]^{1-\epsilon} dj \right\}^{\frac{1}{1-\epsilon}}$$

In the Calvo staggered price setting environment, the first set of firms has probability α of occurring and the second has probability $(1-\alpha)$. Intermediate firms are distributed uniformly over the real line, i.e., the measure of firms is the Lebesgue measure, so considering that the distribution of prices among these firms not adjusting at t is the same as at $t-1$, with total mass reduced to α, we have:

$$P_{H,t} = \left\{ \alpha \left[P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\nu} e^{(1-\nu)\pi_H} \right]^{1-\epsilon} + (1-\alpha)(\bar{P}_{H,t})^{1-\epsilon} \right\}^{\frac{1}{1-\epsilon}} \tag{A.14}$$

Dividing both sides of the previous equation by P_{t-1},

$$\Pi_{H,t}^{1-\epsilon} = \alpha \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\nu(1-\epsilon)} e^{(1-\epsilon)(1-\nu)\pi_H} + (1-\alpha) \left(\frac{\bar{P}_{H,t}}{P_{H,t-1}} \right)^{1-\epsilon}, \tag{A.15}$$

where $\Pi_{H,t} = P_{H,t+1}/P_{H,t}$. The intermediate domestic producer's optimization problem is to choose the optimal price, given by:

$$\bar{P}_{H,t}(j) = \arg\max \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} Q_{t,s} Y_{t,s}(j) \left[(1-\tau_s) \Phi_{t,s} P_{H,t}(j) - MC_s^n(j) \right] \right\}$$

$$s.t. \quad Y_{t,s}(j) = \left[\frac{P_{H,t}(j)\Phi_{t,s}}{P_{H,s}} \right]^{-\epsilon} \left(C_{H,s} + G_{H,s} + \int_0^1 C_{H,s}^i di \right) \tag{A.16}$$

$$\Phi_{t,s} = \prod_{k=1}^{s-t} \left[e^{\nu \pi_{H,t+k-1} + (1-\nu)\pi_H} \right]$$

where the stochastic discount factor $Q_{t,s}$ is given by $\beta^{s-t}\Xi_s/\Xi_t$ and $MC_s^n(j)$ is the nominal marginal cost. The first-order condition associated with the optimization problem above takes the form:

$$\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \mathbb{E}_t \left\{ \frac{\Xi_s \bar{Y}_{t,s}(j)}{\Xi_t} \left[(1-\tau_s) \Phi_{t,s} \bar{P}_{H,t}(j) - \mu MC_s^n(i) \right] \right\} = 0 \tag{A.17}$$

$$\mu = \epsilon(\epsilon - 1)^{-1}$$

With no price rigidities ($\alpha = 0$), the above optimality condition reduces to $\bar{P}_{H,t}(i) = \mu(1-\tau_t)^{-1} MC_t^n(i)$, which suggests that $\mu(1-\tau_t)^{-1}$ is the desired markup in a frictionless environment. Price rigidities vanish in the long run:

$$\sum_{s=t+1}^{\infty} (\alpha\beta)^{s-t} \mathbb{E}_t \left\{ \frac{\Xi_s \bar{Y}_{t,s}(j)}{\Xi_t} \left[(1-\tau_s) \Phi_{t,s} \bar{P}_{H,t}(j) - \mu MC_s(j) \right] \right\}$$

$$+\bar{Y}_{t,t}(j)\left[(1-\tau_t)\bar{P}_{H,t}(j)-\mu MC_t^n(j)\right]=0 \tag{A.18}$$

This implies that in steady state[3]

$$\bar{P}_H(j)=\mu(1-\tau)^{-1}MC^n(j), \tag{A.19}$$

Aggregate domestic price level dynamics can be rewritten as:

$$\pi_{H,t}=\alpha(1-\nu)\pi_H+\alpha\nu\pi_{H,t-1}+(1-\alpha)\left(\ln\bar{P}_{H,t}(i)-\ln P_{H,t}+\pi_{H,t}\right) \tag{A.20}$$

This implies that in the long run domestic inflation target steady state

$$\frac{\bar{P}_H(j)}{P_H}=1. \tag{A.21}$$

Market clearing in the labor market requires

$$N_t=\int_0^1 N_t(j)dj=\int_0^1\left(\frac{Y_t(j)}{A_t}\right)dj \tag{A.22}$$

A.3 STATIONARY EQUILIBRIUM

In our case, technological progress $\{A_t\}_{t=0}^{\infty}$ is non stationary. In the symmetric long run inflation target environment, we define normalized stationary variables as:

$$\begin{aligned}
\tilde{\bar{P}}_H &= \bar{P}_H/P_H\\
\tilde{Y} &= Y/A\\
\tilde{C} &= C/A\\
\tilde{C}^i &= C^i/A^i\\
\tilde{Y}^i &= Y^i/A^i\\
\tilde{W} &= W/(AP)\\
MC &= MC^n/P_H\\
\lambda &= \Xi AP\\
\lambda^i &= \Xi^i A^i P^i
\end{aligned}$$

Marginal utility of consumption:

$$\lambda_t=\frac{e^{\gamma_t}}{e^{\gamma_t}\tilde{C}_t-\eta\tilde{C}_{t-1}}-\eta\beta\mathbb{E}_t\left\{\frac{e^{-\delta_{t+1}}}{e^{\gamma_{t+1}}\tilde{C}_{t+1}-\eta\tilde{C}_t}\right\} \tag{A.23}$$

Euler equation:

$$\lambda_t=\beta R_t\mathbb{E}_t\left\{\lambda_{t+1}e^{-\gamma_{t+1}-\pi_{t+1}-\delta_{t+1}}\right\} \tag{A.24}$$

Intratemporal optimality condition:

$$\tilde{W}_t=N_t^{\omega}\left[\frac{e^{\gamma_t}}{\tilde{C}_te^{\gamma_t}-\eta\tilde{C}_{t-1}}-\eta\beta\mathbb{E}_t\left(\frac{e^{-\delta_{t+1}}}{\tilde{C}_{t+1}e^{\gamma_{t+1}}-\eta\tilde{C}_t}\right)\right]^{-1}=\frac{N_t^{\omega}}{\lambda_t} \tag{A.25}$$

[3] The exogenous sales taxes process $\{\tau_t\}_{t=0}^{\infty}$ is assumed to be stationary.

Real marginal cost for intermediate good producer:

$$MC_t = \tilde{W}_t P_t / P_{H,t} \tag{A.26}$$

Price-setting equation for domestic firms changing prices:

$$\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \mathbb{E}_t \left\{ \lambda_s \tilde{\bar{Y}}_{t,s}(j) \left[(1-\tau_s) \tilde{\Phi}_{t,s} \tilde{\bar{P}}_{H,t}(j) - \mu MC_s(j) \right] \right\} = 0 \tag{A.27}$$

$$\tilde{\Phi}_{t,s} = \prod_{k=1}^{s-t} \left[e^{\nu(\pi_{H,t+k-1} - \pi_H) - (\pi_{H,t+k} - \pi_H)} \right]$$

$$\tilde{\bar{Y}}_{t,s}(j) = \left(\tilde{\bar{P}}_{H,t}(j) \tilde{\Phi}_{t,s} \right)^{-\epsilon} \tilde{Y}_s$$

Final goods market clearing condition:

$$\tilde{Y}_t = \left(\frac{P_{H,t}}{P_t} \right)^{-\psi} \left[(1-\zeta) \tilde{C}_t + \zeta e^{z_t} \int_0^1 \left(S_t^i S_{i,t} \right)^{\kappa-\psi} \mathcal{Q}_{i,t}^{\psi} \tilde{C}_t^i di \right] \tag{A.28}$$

A condition analogous to the one above holds for all foreign countries:

$$\tilde{Y}_t^i = \left(\frac{P_{i,t}}{P_t^i} \right)^{-\psi} \left[(1-\zeta) \tilde{C}_{i,t} + \zeta \int_0^1 \left(S_t^i S_{i,t} \right)^{\kappa-\psi} \mathcal{Q}_{i,t}^{\psi} \tilde{C}_t^i di \right] \tag{A.29}$$

A.4 INTERNATIONAL RISK SHARING

Under the assumption of complete securities markets, a domestic household first-order condition must hold for any household in any other country i. In equilibrium, stochastic discount factors in both countries must be equal:

$$\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{A_t P_t}{A_{t+1} P_{t+1}} = Q_{t,t+1} = \frac{\lambda_{t+1}^i}{\lambda_t^i} \frac{A_t^i P_t^i}{A_{t+1}^i P_{t+1}^i} \frac{\varepsilon_{i,t}}{\varepsilon_{i,t+1}} \tag{A.30}$$

Thus

$$\frac{\lambda_t}{A_t P_t} = v_i \frac{\lambda_t^i}{A_t^i P_t^i} \frac{1}{\varepsilon_{i,t}}, \quad \forall i \in (0,1) \tag{A.31}$$

and for all t, where $v_i = 1$ implying symmetric initial conditions. Taking logs on both sides:

$$\lambda_t = \lambda_t^i \exp\{-(a_t^i - a_t) - (p_t^i + e_{i,t} - p_t)\} \tag{A.32}$$

where $a_t^i = \ln A_t^i$ and $a_t = \ln A_t$. Despite the assumption of no within-sector firm heterogeneity, heterogeneity in technology is assumed between domestic and foreign producers. More specifically, it is assumed that $z_t = a_t^i - a_t$ follows a stationary $AR(1)$ process

$$z_t = (1-\rho_z) z + \rho_z z_{t-1} + \epsilon_t^z, \tag{A.33}$$

where z is the steady state technology asymmetry. The uncovered interest parity condition is given by:

$$\mathbb{E}\left\{\frac{\Xi_{t+1}}{\Xi_t}\left[R_t - R_t^i\left(\frac{\varepsilon_{i,t+1}}{\varepsilon_{i,t}}\right)\right]\right\} = 0 \tag{A.34}$$

Log-linearizing around the perfect foresight steady state, and aggregating over i, yields the same linear expression obtained in Gali and Monacelli (2005):

$$i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1} \tag{A.35}$$

A.5 LOG-LINEARIZED EQUILIBRIUM

Steady state marginal utility of consumption:

$$\lambda = \frac{1-\eta_\gamma \beta}{1-\eta_\gamma}\tilde{C}^{-1} \tag{A.36}$$

Steady state Euler equation:

$$i = \rho + \gamma + \pi, \tag{A.37}$$

where $i = -\ln Q$ is the steady state nominal interest rate and $\rho = -\ln \beta$. The steady state production function for intermediate good producer is given by:

$$\tilde{Y} = N \tag{A.38}$$

Steady state real marginal cost:

$$MC = \tilde{W} \tag{A.39}$$

Combining the previous equation with A.21 and A.19:

$$\tilde{W} = \frac{1-\tau}{\mu} \tag{A.40}$$

Log-linear deviations from the steady state are defined as $\hat{x}_t = \ln(X_t/X)$, $\hat{\gamma}_t = \gamma_t - \gamma$ and $\hat{\pi}_{H,t} = \pi_{H,t} - \pi_H$. Euler equation becomes:

$$\hat{\lambda}_t = \mathbb{E}_t \hat{\lambda}_{t+1} + \hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \mathbb{E}_t \hat{\gamma}_{t+1} - \mathbb{E}_t \hat{\delta}_{t+1} \tag{A.41}$$

Marginal utility of consumption:

$$\hat{\lambda}_t = -\varphi_\gamma[\hat{\tilde{c}}_t + \eta_\gamma(\hat{\gamma}_t - \hat{\tilde{c}}_{t-1})] + \varphi_\gamma \beta \eta_\gamma \mathbb{E}_t[(1-\eta_\gamma)\delta_{t+1} + \hat{\tilde{c}}_{t+1} + \hat{\gamma}_{t+1} - \eta_\gamma \hat{\tilde{c}}_t] \tag{A.42}$$

or

$$\hat{\lambda}_t = -\varphi_\gamma(\hat{\tilde{c}}_t - \eta_\gamma \hat{\tilde{c}}_{t-1}) + \varphi_\gamma \eta_\gamma \beta(\mathbb{E}_t \hat{\tilde{c}}_{t+1} - \eta_\gamma \hat{\tilde{c}}_t) + \varphi_\gamma \eta_\gamma(\beta \mathbb{E}_t \hat{\gamma}_{t+1} - \hat{\gamma}_t) + \frac{\beta \eta_\gamma}{1-\beta\eta_\gamma}\mathbb{E}_t \hat{\delta}_{t+1} \tag{A.43}$$

Intratemporal optimality condition:

$$\hat{\tilde{w}} = \omega \hat{n}_t - \hat{\lambda}_t \tag{A.44}$$

Aggregate relationship between production and hours, valid up to a first order approximation:

$$\hat{\hat{y}}_t = \hat{n}_t \tag{A.45}$$

Real marginal cost for intermediate good producer:

$$\hat{mc}_t = \hat{\hat{w}}_t + \zeta \hat{s}_t \tag{A.46}$$

Price-setting equation for firms changing prices:

$$\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \mathbb{E}_t \left\{ \hat{\mu}_s + \hat{\bar{\hat{p}}}_t - \hat{mc}_s + \sum_{k=1}^{s-t} (\nu \hat{\pi}_{H,t+k-1} - \hat{\pi}_{H,t+k}) \right\} = 0 \tag{A.47}$$

$$\hat{\mu}_s = \frac{1-\tau_s}{\mu} - \frac{1-\tau}{\mu}$$

Solving for $\hat{\bar{\hat{p}}}_t$

$$\frac{1}{1-\alpha\beta} \hat{\bar{\hat{p}}}_t = \hat{mc}_t - \hat{\mu}_t + \sum_{s=t+1}^{\infty} (\alpha\beta)^{s-t} \mathbb{E}_t \left\{ \hat{mc}_s - \hat{\mu}_s - \sum_{k=1}^{s-t-1} (\nu \hat{\pi}_{H,t+k-1} - \hat{\pi}_{H,t+k}) \right\}$$

$$= \hat{mc}_t - \hat{\mu}_t +$$

$$(\alpha\beta)\mathbb{E}_t \left\{ \frac{-(\nu \hat{\pi}_{H,t} - \hat{\pi}_{H,t+1})}{1-\alpha\beta} + \sum_{s=t+1}^{\infty} (\alpha\beta)^{s-t-1} \mathbb{E}_{t+1} \left[\hat{mc}_s - \hat{\mu}_s - \sum_{k=1}^{s-t-1} (\nu \hat{\pi}_{H,t+k} - \hat{\pi}_{H,t+k+1}) \right] \right\}$$

$$= \hat{mc}_t - \hat{\mu}_t + \frac{\alpha\beta}{1-\alpha\beta} \mathbb{E}_t \left\{ \hat{\bar{\hat{p}}}_{t+1} - (\nu \hat{\pi}_{H,t} - \hat{\pi}_{H,t+1}) \right\} \tag{A.48}$$

Aggregate price level dynamics:

$$\hat{\bar{\hat{p}}}_t = -\alpha(1-\alpha)^{-1} (\nu \hat{\pi}_{H,t-1} - \hat{\pi}_{H,t}) \tag{A.49}$$

Finally, combining A.48 and A.49 yields the inflation equation

$$\tilde{\pi}_{H,t} = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} (\hat{mc}_t - \hat{\mu}_t) + \beta \mathbb{E}_t \tilde{\pi}_{H,t+1}, \tag{A.50}$$

where $\tilde{\pi}_{H,t} = \hat{\pi}_{H,t} - \nu \hat{\pi}_{H,t-1}$. Bilateral terms of trade are defined as $S_{i,t} = P_{i,t}/P_{H,t}$. Thus, the effective terms of trade are given by

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}, \tag{A.51}$$

which can be approximated up to first order[4] by

$$s_t = \int_0^1 s_{i,t} di \tag{A.52}$$

[4] $s_t = \frac{1}{1-\epsilon} \ln \left(\int_0^1 e^{(1-\epsilon)s_{i,t}} di \right) \approx \frac{1}{1-\epsilon} \ln \left(\int_0^1 e^{(1-\epsilon)0} di \right) + \left(\int_0^1 s_{i,t} di - 0 \right)$, where $s_{i,t} = 0$ holds in that symmetric steady state.

The CPI formula can also be log-linearized around the symmetric steady state, where PPP condition ($P_{H,t} = P_{F,t}$) holds and is expressed in terms of inflation rates:

$$\pi_t = \pi_{H,t} + \zeta \Delta s_t \tag{A.53}$$

Since the law of one price holds for individual goods at all times, we have $P_{i,t} = \varepsilon_{i,t} P^i_{i,t}$. By substituting into the definition of $P_{F,t}$, log-linearizing around the symmetric steady state[5] yields

$$p_{F,t} = \int_0^1 (e_{i,t} + p^i_{i,t}) di = e_t + p^*_t \tag{A.54}$$

where e_t is the (log) effective nominal exchange rate and p^*_t is the (log) world price index. Combining the previous result with the terms of trade definition yields

$$s_t = e_t + p^*_t - p_{H,t}, \tag{A.55}$$

or

$$\Delta \hat{s}_t = (1-\zeta)^{-1}(\Delta \hat{e}_t + \hat{\pi}^*_t - \hat{\pi}_t) \tag{A.56}$$

where π^*_t is the world inflation. The (log) effective real exchange rate is given by[6]

$$q_t = \int_0^1 q_{i,t} di = (1-\zeta) s_t \tag{A.57}$$

International risk sharing condition, when aggregating over i:

$$\hat{\lambda}_t = \hat{\lambda}^*_t - \hat{z}_t - (1-\zeta)\hat{s}_t \tag{A.58}$$

Final goods market clearing yields

$$\hat{\tilde{y}}_t = (1-\zeta)\hat{\tilde{c}}_t + \zeta(\hat{\tilde{c}}^*_t + \hat{z}_t) + \zeta[\kappa + \psi(1-\zeta)]\hat{s}_t \tag{A.59}$$

The above condition holds for all countries (without the productivity asymmetry term). Thus, it can be rewritten indexed by i. Aggregating over all countries yields the world market clearing condition:

$$\hat{\tilde{y}}^*_t = \hat{\tilde{c}}^*_t \tag{A.60}$$

Marginal utility of consumption combined with goods market clearing yields

$$\hat{\lambda}_t = -\varphi_\gamma(1-\zeta)^{-1}(\hat{\tilde{y}}_t - \eta_\gamma \hat{\tilde{y}}_{t-1}) + \varphi_\gamma \eta_\gamma \beta (1-\zeta)^{-1}(\mathbb{E}_t \hat{\tilde{y}}_{t+1} - \eta_\gamma \hat{\tilde{y}}_t) + \varphi_\gamma \eta_\gamma (\beta \mathbb{E}_t \hat{\gamma}_{t+1} - \hat{\gamma}_t)$$

$$+ \frac{\beta \eta_\gamma}{1 - \beta \eta_\gamma} \mathbb{E}_t \hat{\delta}_{t+1} + \varphi_\gamma \zeta (1-\zeta)^{-1}(\hat{\tilde{y}}^*_t - \eta_\gamma \hat{\tilde{y}}^*_{t-1}) - \varphi_\gamma \eta_\gamma \beta \zeta (1-\zeta)^{-1}(\mathbb{E}_t \hat{\tilde{y}}^*_{t+1} - \eta_\gamma \hat{\tilde{y}}^*_t)$$

---

5 $p_{F,t} = \frac{1}{1-\epsilon} \ln\left[\int_0^1 e^{(1-\epsilon)(e_{i,t} + p^i_{i,t})} di\right] \approx \frac{1}{1-\epsilon} \ln\left[\int_0^1 e^{(1-\epsilon)(e^{ss}_{i,t} + p^{i,ss}_{i,t})} di\right] + \int_0^1 (e_{i,t} - e^{ss}_{i,t}) di + \int_0^1 (p^i_{i,t} - p^{i,ss}_{i,t}) di$, where $p^{i,ss}_{i,t}$ and $e^{ss}_{i,t}$ stands for steady state values for the (log) country i's domestic price index and (log) bilateral nominal exchange rate, respectively. Since in this symmetric steady state $p^{i,ss}_{i,t} = p^{j,ss}_{i,t}$, $\forall i \neq j$, it is the case that $e^{ss}_{i,t} = 0$, $\forall i \in (0,1)$ and A.54 remains valid even if there is positive inflation in every country that composes the world economy.

6 Details in Gali and Monacelli (2005).

$$+\varphi_\gamma\zeta(1-\zeta)^{-1}\sigma_s(\hat{s}_t-\eta_\gamma\hat{s}_{t-1})-\varphi_\gamma\eta_\gamma\beta\zeta(1-\zeta)^{-1}\sigma_s(\mathbb{E}_t\hat{s}_{t+1}-\eta_\gamma\hat{s}_t)$$

$$+\varphi_\gamma\zeta(1-\zeta)^{-1}(\hat{z}_t-\eta_\gamma\hat{z}_{t-1})-\varphi_\gamma\eta_\gamma\beta\zeta(1-\zeta)^{-1}(\mathbb{E}_t\hat{z}_{t+1}-\eta_\gamma\hat{z}_t) \tag{A.61}$$

where $\sigma_s=\kappa+\psi(1-\zeta)$. For any other country i, we have $\hat{\lambda}_t^i$, which can be aggregated over i to yield

$$\hat{\lambda}_t^*=-\varphi_\gamma^*(\hat{\tilde{y}}_t^*-\eta_\gamma^*\hat{\tilde{y}}_{t-1}^*)+\varphi_\gamma^*\eta_\gamma^*\beta(\mathbb{E}_t\hat{\tilde{y}}_{t+1}^*-\eta_\gamma^*\hat{\tilde{y}}_t^*)+\varphi_\gamma^*\eta_\gamma^*(\beta\mathbb{E}_t\hat{\gamma}_{t+1}^*-\hat{\gamma}_t^*)+\frac{\beta\eta_\gamma^*}{1-\beta\eta_\gamma^*}\mathbb{E}_t\hat{\delta}_{t+1} \tag{A.62}$$

where[7] $\eta_\gamma^*=\eta e^{-\gamma^*}$ and $\varphi_\gamma^*=[(1-\eta_\gamma^*)(1-\beta\eta_\gamma^*)]^{-1}$.

A.5.1 Efficient Equilibrium

Exactly like in Cúrdia et al. (2015), in the efficient equilibrium, the marginal rate of substitution between hours and consumption equals the marginal rate of transformation

$$\frac{\lambda_t^e}{(\tilde{Y}_t^e)^\omega}=1 \tag{A.63}$$

which can be log-linearized and combined with goods market equilibrium to yield

$$\omega\hat{\tilde{y}}_t^e+\varphi_\gamma(1-\zeta)^{-1}(\hat{\tilde{y}}_t^e-\eta_\gamma\hat{\tilde{y}}_{t-1}^e)-\varphi_\gamma\eta_\gamma\beta(1-\zeta)^{-1}(\mathbb{E}_t\hat{\tilde{y}}_{t+1}^e-\eta_\gamma\hat{\tilde{y}}_t^e)=\varphi_\gamma\eta_\gamma(\beta\mathbb{E}_t\hat{\gamma}_{t+1}-\hat{\gamma}_t)$$

$$+\frac{\beta\eta_\gamma}{1-\beta\eta_\gamma}\mathbb{E}_t\hat{\delta}_{t+1}+\varphi_\gamma\zeta(1-\zeta)^{-1}(\hat{\tilde{y}}_t^*-\eta_\gamma\hat{\tilde{y}}_{t-1}^*)-\varphi_\gamma\eta_\gamma\beta\zeta(1-\zeta)^{-1}(\mathbb{E}_t\hat{\tilde{y}}_{t+1}^*-\eta_\gamma\hat{\tilde{y}}_t^*)$$

$$+\varphi_\gamma\zeta(1-\zeta)^{-1}\sigma_s(\hat{s}_t^e-\eta_\gamma\hat{s}_{t-1}^e)-\varphi_\gamma\eta_\gamma\beta\zeta(1-\zeta)^{-1}\sigma_s(\mathbb{E}_t\hat{s}_{t+1}^e-\eta_\gamma\hat{s}_t^e)$$

$$+\varphi_\gamma\zeta(1-\zeta)^{-1}(\hat{z}_t-\eta_\gamma\hat{z}_{t-1})-\varphi_\gamma\eta_\gamma\beta\zeta(1-\zeta)^{-1}(\mathbb{E}_t\hat{z}_{t+1}-\eta_\gamma\hat{z}_t) \tag{A.64}$$

The intertemporal Euler equation holds in the efficient equilibrium and can be log-linearized to yield

$$r_t^e=\mathbb{E}_t\hat{\gamma}_{t+1}+\mathbb{E}_t\hat{\delta}_{t+1}-\omega(\mathbb{E}_t\hat{\tilde{y}}_{t+1}^e-\hat{\tilde{y}}_t^e) \tag{A.65}$$

A.5.2 Flexible Price Equilibrium

In the flexible price equilibrium:

$$\hat{mc}_t=\hat{\mu}_t=\omega\hat{\tilde{y}}_t^n-\hat{\lambda}_t^n+\zeta\hat{s}_t^n \tag{A.66}$$

Combining this law of motion with marginal utility of consumption and goods market clearing condition yields

[7] Since $\gamma_t=\gamma_t^*-(z_t-z_{t-1})$, it is the case that $\gamma=\gamma^*$, or $\eta_\gamma^*=\eta_\gamma$.

$$\omega\hat{\tilde{y}}_t^n - \hat{\mu}_t + \zeta\hat{s}_t^n + \varphi_\gamma(1-\zeta)^{-1}(\hat{\tilde{y}}_t^n - \eta_\gamma\hat{\tilde{y}}_{t-1}^n) - \varphi_\gamma\eta_\gamma\beta(1-\zeta)^{-1}(\mathbb{E}_t\hat{\tilde{y}}_{t+1}^n - \eta_\gamma\hat{\tilde{y}}_t^n) = \varphi_\gamma\eta_\gamma(\beta\mathbb{E}_t\hat{\gamma}_{t+1} - \hat{\gamma}_t)$$

$$+\frac{\beta\eta_\gamma}{1-\beta\eta_\gamma}\mathbb{E}_t\hat{\delta}_{t+1} + \varphi_\gamma\zeta(1-\zeta)^{-1}(\hat{\tilde{y}}_t^* - \eta_\gamma\hat{\tilde{y}}_{t-1}^*) - \varphi_\gamma\eta_\gamma\beta\zeta(1-\zeta)^{-1}(\mathbb{E}_t\hat{\tilde{y}}_{t+1}^* - \eta_\gamma\hat{\tilde{y}}_t^*)$$

$$+\varphi_\gamma\zeta(1-\zeta)^{-1}\sigma_s(\hat{s}_t^n - \eta_\gamma\hat{s}_{t-1}^n) - \varphi_\gamma\eta_\gamma\beta\zeta(1-\zeta)^{-1}\sigma_s(\mathbb{E}_t\hat{s}_{t+1}^n - \eta_\gamma\hat{s}_t^n)$$

$$+\varphi_\gamma\zeta(1-\zeta)^{-1}(\hat{z}_t - \eta_\gamma\hat{z}_{t-1}) - \varphi_\gamma\eta_\gamma\beta\zeta(1-\zeta)^{-1}(\mathbb{E}_t\hat{z}_{t+1} - \eta_\gamma\hat{z}_t) \tag{A.67}$$

A.5.3 Equilibrium Dynamics: Canonical Representation

Optimal domestic consumption and saving decisions combined with goods market clearing yields

$$\tilde{x}_t = \mathbb{E}_t\tilde{x}_{t+1} - \zeta\sigma_s\mathbb{E}_t(\tilde{x}_{s,t+1} - \tilde{x}_{s,t}) - (1-\zeta)\varphi_\gamma^{-1}(\hat{\imath}_t - \mathbb{E}_t\hat{\pi}_{t+1} - r_t^e) \tag{A.68}$$

$$\tilde{x}_t = x_t^e - \eta_\gamma x_{t-1}^e - \beta\eta_\gamma(\mathbb{E}_t x_{t+1}^e - \eta_\gamma x_t^e)$$

$$x_t^e = \hat{\tilde{y}}_t - \hat{\tilde{y}}_t^e$$

$$\tilde{x}_{s,t} = x_{s,t}^e - \eta_\gamma x_{s,t-1}^e - \beta\eta_\gamma(\mathbb{E}_t x_{s,t+1}^e - \eta_\gamma x_{s,t}^e)$$

$$x_{s,t}^e = \hat{s}_t - \hat{s}_t^e$$

International risk sharing implies

$$\hat{s}_t = (1-\zeta)^{-1}(\hat{\lambda}_t^* - \hat{\lambda}_t - \hat{z}_t) \tag{A.69}$$

where $\hat{\lambda}_t^*$ is the marginal utility of consumption in terms of world consumption index $\hat{\tilde{c}}_t^*$ and foreign productivity growth rate shock $\hat{\gamma}_t^*$. In terms of domestic efficient equilibrium:

$$\hat{s}_t^e = (1-\zeta)^{-1}(\hat{\lambda}_t^* - \hat{\lambda}_t^e - \hat{z}_t) \tag{A.70}$$

Previous equations combined yield

$$x_{s,t}^e = \hat{s}_t - \hat{s}_t^e = (1-\zeta)^{-1}(\hat{\lambda}_t^e - \hat{\lambda}_t), \tag{A.71}$$

which is the law of motion of the efficient terms of trade gap. The real marginal cost is given by

$$\hat{mc}_t = \omega\hat{\tilde{y}}_t - \hat{\lambda}_t + \zeta\hat{s}_t = \omega x_t^n + \hat{\mu}_t + \zeta x_{s,t}^n + \varphi_\gamma(1-\zeta)^{-1}\tilde{x}_t^n - \varphi_\gamma\zeta(1-\zeta)^{-1}\sigma_s\tilde{x}_{s,t}^n \tag{A.72}$$

The optimal pricing decisions of domestic firms produce the Phillips curve

$$\tilde{\pi}_{H,t} = \xi\left(\omega x_t^n + \zeta x_{s,t}^n + \varphi_\gamma(1-\zeta)^{-1}\tilde{x}_t^n - \varphi_\gamma\zeta(1-\zeta)^{-1}\sigma_s\tilde{x}_{s,t}^n\right) + \beta\mathbb{E}_t\tilde{\pi}_{H,t+1} \tag{A.73}$$

$$\tilde{\pi}_{H,t} = \hat{\pi}_{H,t} - \nu\hat{\pi}_{H,t-1}$$

$$\xi = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$$

$$\tilde{x}_t^n = x_t^n - \eta_\gamma x_{t-1}^n - \beta\eta_\gamma(\mathbb{E}_t x_{t+1}^n - \eta_\gamma x_t^n)$$

$$x_t^n = \hat{\tilde{y}}_t - \hat{\tilde{y}}_t^n$$

$$\tilde{x}_{s,t}^n = x_{s,t}^n - \eta_\gamma x_{s,t-1}^n - \beta\eta_\gamma(\mathbb{E}_t x_{s,t+1}^n - \eta_\gamma x_{s,t}^n)$$

$$x_{s,t}^n = \hat{s}_t - \hat{s}_t^n$$

Rewriting the Phillips curve with the welfare-relevant gap:

$$\tilde{\pi}_{H,t} = \xi\left(\omega x_t^e + \zeta x_{s,t}^e + \varphi_\gamma(1-\zeta)^{-1}\tilde{x}_t - \varphi_\gamma\zeta(1-\zeta)^{-1}\sigma_s\tilde{x}_{s,t}\right) + \beta\mathbb{E}_t\tilde{\pi}_{H,t+1} + u_t \tag{A.74}$$

$$u_t = \rho_u u_{t-1} + \epsilon_t^u$$

Finally, the CPI inflation Phillips curve is given by

$$\tilde{\pi}_t = \xi\left(\omega x_t^e + \zeta x_{s,t}^e + \varphi_\gamma(1-\zeta)^{-1}\tilde{x}_t - \varphi_\gamma\zeta(1-\zeta)^{-1}\sigma_s\tilde{x}_{s,t}\right) + \zeta(\Delta\tilde{s}_t - \beta\mathbb{E}_t\Delta\tilde{s}_{t+1}) + \beta\mathbb{E}_t\tilde{\pi}_{t+1} + u_t \tag{A.75}$$

$$\tilde{\pi}_t = \hat{\pi}_t - \nu\hat{\pi}_{t-1}$$
$$\Delta\tilde{s}_t = \Delta\hat{s}_t - \nu\Delta\hat{s}_{t-1}$$

APPENDIX B – PERFECT FORESIGHT STEADY STATE

Invoking symmetry among all countries (other than the home country), our (stationary) international risk sharing condition is given by[1]

$$\tilde{C}^* = \tilde{C}e^{-z}\mathcal{Q}^{-1} \tag{B.1}$$

Goods market equilibrium, when evaluated at the steady state, implies[2]

$$\tilde{Y} = \left(\frac{P_H}{P}\right)^{-\psi}\left[(1-\zeta)\tilde{C} + \zeta e^z \int_0^1 (S^i S_i)^{\kappa-\psi}\mathcal{Q}_i^{\psi}\tilde{C}^i di\right]$$

$$= \left(\frac{P_H}{P}\right)^{-\psi}\left[(1-\zeta)\tilde{C} + \zeta e^z \int_0^1 (S^i S_i)^{\kappa-\psi}\mathcal{Q}_i^{\psi}\tilde{C}e^{-z}\mathcal{Q}_i^{-1} di\right]$$

$$= h(S)^{\psi}\tilde{C}\left[(1-\zeta) + \zeta S^{\kappa-\psi}\mathcal{Q}^{\psi-1}\right]$$

$$= h(S)^{\psi}\tilde{C}\left[(1-\zeta) + \zeta S^{\kappa-\psi}q(S)^{\psi-1}\right]$$

where $h'(S) > 0$ and $q'(S) > 0$. Imposing the world market clearing condition $\tilde{C}^* = \tilde{Y}^*$ we get

$$\tilde{Y} = e^z\left[(1-\zeta)h(S)^{\psi}q(S) + \zeta S^{\kappa-\psi}h(S)^{\psi}q(S)^{\psi}\right]\tilde{Y}^*$$

$$= e^z\left[(1-\zeta)h(S)^{\psi}q(S) + \zeta h(S)^{\kappa}q(S)^{\kappa}\right]\tilde{Y}^*$$

or

$$\tilde{Y} = v(S)\tilde{Y}^* \tag{B.2}$$

with $v(S) > 0$, $v'(S) > 0$ and $v(1) = e^z$. Labor market clearing in the steady state yields

$$\tilde{W} = \frac{\tilde{Y}^{\omega}}{\lambda} = \frac{(1-\tau)}{\mu}\frac{P_H}{P},$$

implying that

[1] Stationary marginal utility of consumption can be used to yield $\tilde{C}^i = (1-\beta\eta_\gamma)(1-\eta_\gamma)^{-1}(\lambda_t^i)^{-1}$.

[2] We have $\mathcal{Q} = \dfrac{SP_H}{P} = \dfrac{S}{h(S)} = q(S)$ and $h(S) = \dfrac{P}{P_H} = \left[(1-\zeta) + \zeta S^{1-\psi}\right]^{\frac{1}{1-\psi}}$.

$$\frac{\tilde{Y}^{\omega}}{(1-\beta\eta_{\gamma})(1-\eta_{\gamma})^{-1}\tilde{C}^{-1}} = \frac{(1-\tau)}{\mu}\frac{1}{h(S)}$$

Previous result combined with international risk sharing condition yields

$$\tilde{Y} = \left[\frac{(1-\tau)(1-\beta\eta_{\gamma})}{\mu(1-\eta_{\gamma})Se^{z}\tilde{Y}^{*}}\right]^{\frac{1}{\omega}} \tag{B.3}$$

Conditional on $\tilde{Y}^{*}$ and z, equations B.2 and B.3 constitute a system of two equations in $\tilde{Y}$ and S, with a unique solution given by

$$\tilde{Y} = e^{z}\tilde{Y}^{*} = \left[\frac{(1-\tau)(1-\beta\eta_{\gamma})}{\mu(1-\eta_{\gamma})}\right]^{\frac{1}{1+\omega}} \tag{B.4}$$

and

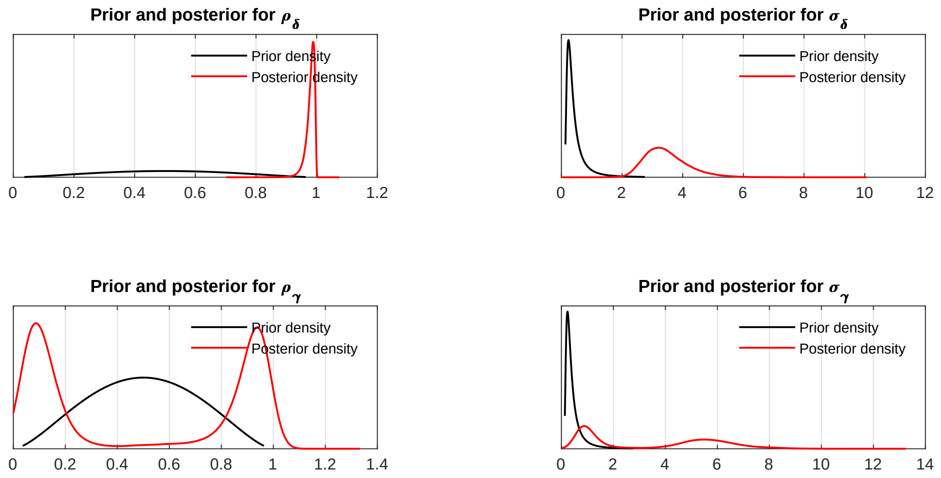
$$S = 1 \tag{B.5}$$

implying $S_i = 1$ for all i. There are some similarities between the stationary equilibrium implied by our model and the one implied in Gali and Monacelli (2005). The terms of trade of both models are uniquely pinned down in the perfect foresight steady state. The home economy's output also coincides with that in the rest of the world $Y = Y^{*}$, despite the addition of some frictions and a stochastic unit-root world technology shock[3].

[3] Given symmetric initial conditions, i.e., by assuming $a_0 = a_0^i$ this implies that the technology levels must be the same in the steady state ($z = 0$).

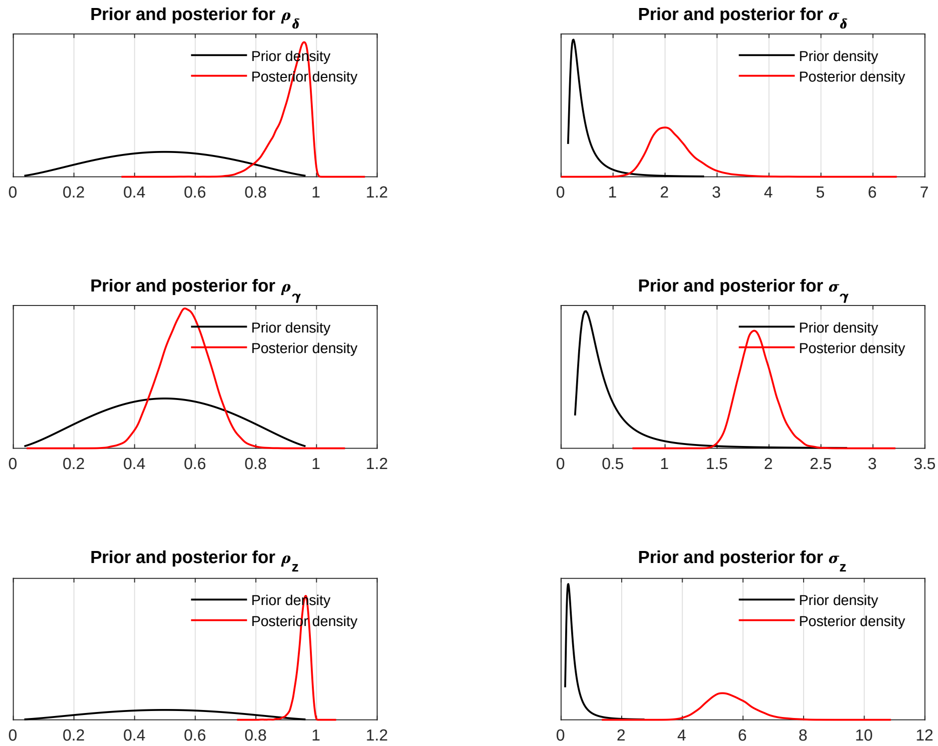
ANNEX A – PRIOR-POSTERIOR PLOTS

Figure 6 – Prior and posterior distributions for selected parameters under the baseline W specification



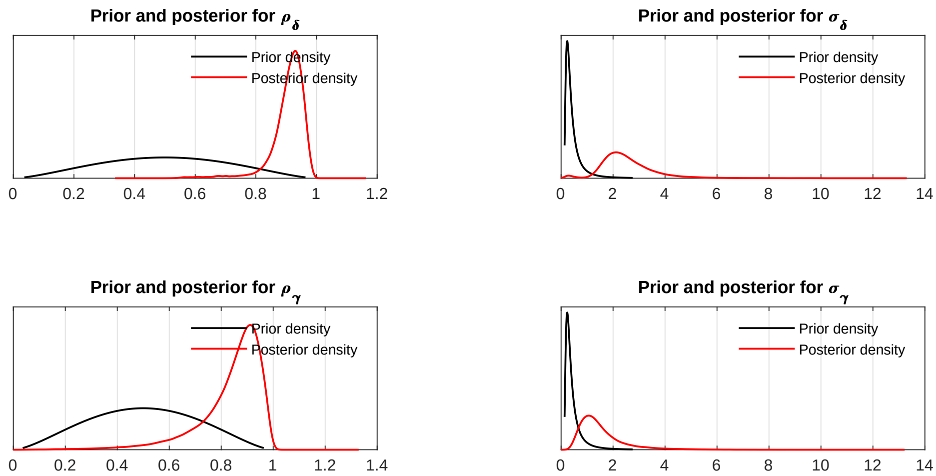
Source: Own construction (2020).

Figure 7 – Prior and posterior distributions for selected parameters under the SOE W specification



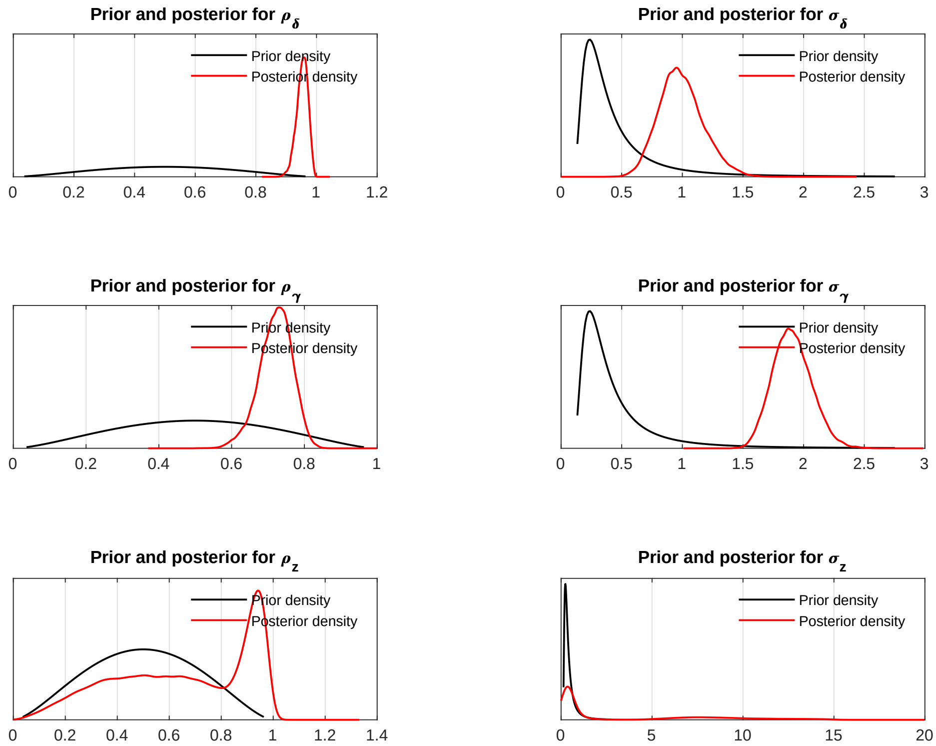
Source: Own construction (2020).

Figure 8 – Prior and posterior distributions for selected parameters under the baseline T specification



Source: Own construction (2020).

Figure 9 – Prior and posterior distributions for selected parameters under the SOE T specification



Source: Own construction (2020).