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A GAME-THEORETICAL MODEL OF DOMESTIC WARS ON DRUGS: THE CASE OF BRAZILIAN FAVELAS

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Dissertação apresentada ao Programa de Pós-Graduação em Economia da Universidade Federal do Rio Grande do Sul como requisito parcial à obtenção do título de Mestre em Economia. Área de concentração: Economia Aplicada.

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RESUMO

Desde o início dos anos 80, diversos países adotaram a guerra às drogas como principal política de combate ao tráfico de drogas. Esta abordagem inclui um conjunto de políticas anti-drogas com o objetivo de desencorajar o mercado dessas substâncias, porém este método tem se mostrado pouco efetivo, havendo vários exemplos de bem sucedidos impérios de drogas em operação. No caso do Brasil, o comércio ilegal de drogas permitiu a criação de um verdadeiro Estado paralelo em algumas cidades como Rio de Janeiro e São Paulo, atuando dentro do limite de favelas, onde poderosas facções de drogas suprem a necessidade de bens públicos no lugar do Estado. De tal forma, a ação mais observável do governo nessas localidades ocorre através dos confrontos com as facções, podendo gerar efeitos perversos na comunidade (por exemplo, por meio de balas perdidas). Construímos um modelo de guerra às drogas doméstica para este contexto, considerando um jogo dinâmico de informação perfeita da interação entre um governo, uma facção de drogas e cidadãos residentes de favelas. A sequência do jogo é a seguinte. Primeiro, o governo escolhe o nível de repressão (a certo custo) a utilizar contra a facção, podendo causar externalidades negativas na comunidade ao redor. Ao observar essa externalidade, os indivíduos decidem entre aderir à atividade ilegal associada à facção ou ao mercado formal de trabalho. Finalmente, a facção de drogas maximizadora de lucros recruta os indivíduos que optam por participar e decidem o investimento em armas. Nossos resultados apontam para a importância de se considerar o contexto social ao se decidir a política a ser executada no combate às drogas. Mostramos que, se os cidadãos da favela são míopes, têm baixa aversão ao risco ou são muito sensíveis à repressão, a ação do governo pode contribuir para aumentar o número de pessoas recrutadas pela facção. Sob certas condições, em equilíbrio, os lucros da facção podem até crescer à certo nível de repressão governamental. Ainda, a performance da economia do país também pode impactar este resultado, ao contribuir ou não para aumentar as chances dos indivíduos encontrarem empregos formais.

Palavras-chave: Guerra às Drogas. Teoria dos Jogos. Brasil.

ABSTRACT

Since the beginning of the 80s, several countries have launched a war on drugs as their main policy to fight drug trafficking. Such an approach includes a set of drug policies that are intended to discourage the market of those substances, but this method has not been very effective. There are several examples of successful drug empires still operating. In Brazil, for instance, the illegal drug market created a parallel State in some cities, such as Rio de Janeiro and São Paulo, actuating from the inside of favelas (slums). In those places, powerful drug factions supply some public goods the State fail to arrange. The most visible government action is violent confronts with those factions, which may have perverse effects on civilians (e.g stray bullets). We build a model of domestic wars on drugs in those social contexts. Through a dynamic game with perfect information, the interaction between a government, a drug faction, and citizens that live in a favela is studied. The timing is as follows. First, the government chooses the extent of (costly) repression used against the drug faction, which may cause negative externalities on the citizens of the favela. Those individuals must then decide whether to join the faction or the formal labor market. The larger the negative impact from the government's repression on their lives, the more prone to becoming a criminal they are. Finally, the profit-maximizer drug faction recruits those citizens willing to become drug dealers and chooses the number of guns to buy. Our findings highlight the importance of taking into account the social context of the region when one decides how and to what extent to fight drug trafficking. We show that, if citizens of the favela are myopic, have low risk aversion or are very sensitive to repression, the government's actions to fight crime may increase the number of people recruited by the faction. In fact, under certain conditions, in equilibrium, the faction's profit may be increasing in the magnitude of repression. Moreover, the general economic performance of the city or country is another important aspect to be considered, by affecting the individual chances to find a formal job.

Keywords: War on Drugs. Game Theory. Brazil.

SUMÁRIO

1 INTRODUCTION

In 2017, Brazil presented an alarming number of 63,895 murderers, figuring as the 13th on the world ranking of homicide (MUGGAH; TUBóN, 2018). It is estimated that 50% of those murderers are linked with the war on drugs (IGARAPé, 2015). The illicit drug market in Brazil is mostly commanded by organized crime groups, usually concentrated in slums – so-called favelas (WAISELFISZ, 2016), low-income irregular neighborhoods of big cities where the presence of non-state factions correlates with the location of homicidal violence (GARZóN-VERGARA, 2016).

According to 2010's Brazilian census, Brazil has 11,4 million people living in slums (IBGE, 2010). Particularly, in the city of Rio de Janeiro, 22.2% of the residents are living in slums, whereas in São Paulo this proportion is 11%. Those localities are also the poorest of the cities where violent confronts between drug-factions and the police take place. Those circumstances put residents in a social vulnerability scenario, where impoverished young men are acting directly in confront with the police (MEIRELLES; GOMEZ, 2009). The vulnerability associated with guns fire in Rio de Janeiro's slums harms the educational conditions of the community, which impacts their future perspectives of finding a formal job (MONTEIRO; ROCHA, 2017).

Moreover, the Brazilian police force is the most important organization in the anti-traffic enforcement and has one of the highest fatality rates in the world (MIRAGLIA, 2016). In Rio's metropolis, 5.993 cases of gun-fire occurred in 2017, most of them motivated by police operations. The neighborhoods affected the most were two large slums, Cidade de Deus and Complexo do Alemão (MATULJA; ZIONI, 2018). Hence, the State seems to be an important agent of violence on those localities.

Besides Brazil, drug dealing is a critical problem in many countries of the West, moving large amounts of money and supporting other criminal activities. When it comes to the U.S., street gangs activities are closely related to drug distribution and sale, on a large scale and especially in street-level (FBI, 2015). The drug smuggling and traffic relates, in both countries, with smaller crimes like thievery and robbery, but also with huge ones, such as gun traffic, violence, murderers and illicit enrichment.

Nevertheless, it seems that the way those countries are trying to combat the drug problem is not leading to the solution. They are trying to inhibit the drug factions by prohibiting and chasing the leaders, while the demand keeps the same. Paradoxically, the enforcement approach seems to be actually aggravating the violence related to the drugs market, as shown by Werb et al. (2011). This argument is evidenced by the large number of 3,009 people killed in confrontations with on-duty police officers in Brazil in 2014 (GARZóN-VERGARA, 2016).

At this work, we create a game-theoretical model of the war on drugs in consumer

countries. As a starting point, we apply and rethink a model of counter-terrorism created by Mesquita (2005). In our model, we study the interaction between government, drug-traffickers factions and society. The model is original in the discussion of the war on drugs for considering the possibility that the government repression may have the reverse effect than expected, even contributing to the faction's growth.

Since in Brazil selling drugs is illegal, the government uses its power to promote law enforcement and repress this activity. Nevertheless, the profitability of the market is a big incentive for its existence. Brazilian big urban cities are organized in a way that slums (favelas) share space with noble neighborhoods but have profound differences in organization. Those are the shelter of the biggest drug-dealing factions since those locations are neglected by the State presence and its provision of public goods, such as security.

The citizens who live in slums are from lower social classes, with restrict access to good education, usually becoming workers of low productivity and low income. For that reason, they have a lower reserve wage, which implies little opportunity cost to join illicit activities. Since the slums are usually placed on difficult access zones, making it hard for the police to get in, that helps the local dominion of the factions and close interaction with the community.

When the government decides to combat the traffic, the police and the army have to get inside the slum by force, which usually leads to firefights that scare the community and can make innocent victims. That attitude may repress the traffic at some level, but also can increase the anger of the population against the government and create incentives to join the faction. In other words, the police action may increase the appeal of its own enemy.

The faction wants as many allies as possible and to expand its territory, but when there is government intervention the faction loses territory, guns and even followers (due to deaths). The government wants to end or minimize the traffic and the power of the factions. At the same time, the government does not want to take excessive violent actions that will damage its public image with the community (negative externalities).

The model developed in this thesis considers this complex situation and analyze the best response to each player (faction, government, individuals) and if their rational response is efficient in diminishing the criminality or the illegal drug market. The model consists of a sequential game of perfect information that follows the narrative explained above. To model the utility functions of each player we make use of assumptions established over the study of Rio de Janeiro and São Paulo's scenarios and criminal literature. We solved the game by backward induction in order to find the Subgame Perfect Equilibrium (SPE).

The solution of the faction's problem generated a drug production function that

depends on the number of individuals willing to participate. This production function leads to the faction's profit level. The individuals of the favela observe this profit and decide to adhere to the faction or to enter the legal market, based on their personal characteristics and the faction success. The government wants to promote law enforcement by reducing the faction's profits (a proxy of criminal activity), but this may impact the individual decision in a non-desirable way. The solution of the game suggests that, under some conditions on the parameters, the government action may reduce or raise the profit of the faction through its impact on the individuals' decision to join crime. We call those phenomenons "deterrence effect" and "encouragement effect", respectively.

We, then, proceed to the analyses of some comparative statics. We show that an increase in drug prices might have a dubious response from the government, depending on its efficiency on the crackdown – an efficient government may increase the level of repression when drug price is higher since the new price might attract more citizens. Furthermore, we study the effect of a better economic context in the government strategy. We show that, in booming economies, the government may act more aggressively against the faction, since more individuals will be in the legal market and the crackdown will affect less the community.

The war on drugs has been an understudied issue in the field of economic theory, even though it is still an important problem in many countries, especially in Latin America. We intended to bring a new perspective to this topic, in light of the game theory perspectives considering the incentives and strategies of a diverse pool of rational agents.

For that purpose, we make the ingenious feature of adapting a counter-terrorism model, since models of terrorism are more traditionally debated in the field and seem to hold a great resemblance with the anti-drugs efforts (as argued in the next section). By doing so, our approach helps us to understand if the usual strategies executed by governments are really having an impact on criminality as expected. We intend to emerge this important matter as a new subject to the formal theory's agenda.

We hope to provide insights that may support policymaking in countries living under the war on drugs and violent conflicts between the State and the organized criminal activity. This work intends to contribute with both the discussion of a political problem faced worldwide and approximating this under-discussed problem to the economic theory field. This work discussed urgent political decisions and bring new perspectives with the advent of formal modeling.

The complexity of the theme provides a great opportunity to increment the model developed here in future works. A possible extension that moves the issue closely with the political economy would be to consider how this violent anti-drugs policy may impact the valence observed of the incumbent politician and affect his chances in the electoral process. Also, a more precise model could include more than one faction competing for

the market and territory and also the friendly relation of the faction with some members of the state through corruption.

The remainder of this dissertation is structured as follows. In Section 2 we deepen in the literature of the war on drugs in the economic field, approximating to the studies that use the game theory approach. To motivate the problem we are studying and provide more context to the reader, in Section 3 we describe the configuration of the war on drugs in two big cities of Brazil, Rio de Janeiro and São Paulo, stipulating assumptions to feed the model to come. In Section 4 we construct the model, describing the payoff of each player, the timing and, then, the equilibrium of the game, followed for some comparative statics. In Section 5 we discuss the results of the model and conclude. The formal proofs of the propositions are presented in the Appendix.

2 RELATED LITERATURE

A seminal effort to introduce criminality and strategic behavior in the economy field is made by Becker (1968), by assuming rational individuals weight the costs and benefits of the decision to engage or not in illegal activities. The author proposes a model relating the pros and cons of committing a felony – such as his income in the legal market, probability of convictions, punishment if convicted. The final choice of the individual will result from his expected utility. This approach has been used to confront the drug problem in several empirical papers (ENTORF; WINKER, 2008; CORMAN; MOCAN, 2000).

Later on, Murphy, Grossman e Becker (2006) models the war on drugs by the market perspective, analyzing supply and demand elasticity of the substances. That paper presents the government and the suppliers as key strategic actors, maximizing their own welfare subjected to the specificity of the commodity. It differs from what it's been proposed here for considering more of the market dynamic and less of the subjective interests of the parts.

When it comes to game-theoretical analyses, the war on drugs has not been of much concern yet. In this sense, Mejía e Restrepo (2008) work in a sequential game of the war on drugs, considering the interaction between rich countries, that can't control their domestic demand for drugs, and drugs producer from poor countries that take advantage from it. To do so, the authors present the anecdotal case Plan Colombia, executed between 2000 and 2006. The model works on two fronts, the government and the drug trafficker, but doesn't consider the interaction with society.

Grossman e Mejía (2008) present a partial equilibrium game theory model of war on drug producers, considering the state action in two fronts, (1) instigating conflict between production groups for the control of croplands, and (2) attempting to eradicate crops and interdicting drug shipment. The model is calibrated with Colombia well-documented case in Plan Colombia, with the Colombian government as the state and the United States as an interested outsider considering if subside the strategy (1) or (2).

Those papers (and others like Castillo, Mejia e Restrepo (2013)) are focused on the view of the drug-producer country, like Colombia and Mexico, which may use some different enforcement strategies from those drug-consumer countries, like Brazil and the USA. For a detailed examination of the structure in those drug-producer countries, see the study of Filippone (1994) about the Medellin cartel.

We will focus on a structure closer to those mentioned, but focusing on consumer countries, where smuggling is more present in slums, and closer to impoverished communities. For that reason, we are focusing on an inside-country approach, excluding possible outside suppliers from the model.

As for drug consumer countries, Bertolai e Scorzafave (2018) propose a game, in

light of economics of conflict literature, for the drug-dealing activity, considering inter-gangs competition to rule the illicit drug market. The authors comprehend the prize of the conflict as the control over the retailer drugs market and they conclude that the hegemony of one gang ruling the drug market is capable to generate decreases on violence. The paper shows the motivating case of São Paulo's fall on violent crimes after the 2000s, as imprisonment policy got more aggressive and one single trafficker gang consolidated control over the prison system.

Closer to our proposal, Poret e Tejedo (2006) present a model for the market of prohibited drugs with three types of agents, the drug law enforcement authorities, traffickers and buyers. The model states government authorities decide the level of enforcement in two fronts, the apprehension of the trafficker and the sanction paid by the trafficker in case of arrest and conviction. Traffickers choose the quantities of drugs they are going to sell and decide if they allow new entrants or expand their market share. The model analysis shows the counter-intuitive result that increases in law enforcement lead to a higher number of traffickers on the market since it gives incentives for traffickers to permit new entrants.

There is a scarce number of papers studying the war on drugs by the perspective of political economy and game theory, especially in the case of consumer countries. That's why, as presented in the first section, to relate strategic interaction between government, drug factions and society we will borrow some insights from terrorism models in the political economy. That artifice is justified by some similarities both problems have, as pointed by Kenney (2003) and Björnehed (2004).

Kenney (2003) presents an important link between the drug war and terrorism by showing how American enforcement strategy in Colombia in the 1990s had an impact on the counter-terrorism approach years later. The paper points out similarities between the illegal drugs market and terrorism by comparing al-Qaeda and the drug cartels in Colombia. Some of the similarities exposed are that both have flat decision-maker structures, have semi-autonomous cells to carry out the most dangerous activities, plan their illicit operation with meticulous care, reward intelligence, and learn from experience and adapting its strategies to new situations. The author also explains how easy it is for both types of organization to replace a leader to another in order to maintain its operation.

In the same sense, Björnehed (2004) explains that the term narco-terrorism originate from actions of terrorism executed by drug groups and the fact that some terrorist groups may use drug traffic to afford their actions. More recently, though, the understanding that drug trafficking and terrorism are interconnected overcame the particularities of the two situations above, and the term is now seen as the state of a link between the two problems.

With that in mind, the model we develop in this thesis is an adaptation of the terrorism model presented by Mesquita (2005) for the war on drugs. Mesquita (2005)

builds a model relating government, terrorist groups and potential volunteers to engage in the terrorist activity (terrorist sympathizers), considering their choice as endogenous. The paper intends to explain the evidence that, even with economic factors being determinants of the mobilization, terrorists have an educational level in or above average. The author suggests that seemly contradictory evidence can be explained by a screening process engaged by terrorist organizations.

The similarities already observed between terror and drugs organizations, and the government approach to combat them, encourage us to apply this model for the dynamics of those drug-consumers countries. The model of counter-terror seems to fit well on this new feature of the war on drugs, especially considering the agents involved (the government, a para-state organization, the individuals in between) and their strategies. As in Mesquita (2005)'s work, for a drug-consumer country undertaken by drug-market factions, the government faces a trade-off between combat the criminal organizations and, with that, try not to push risk-population to join those factions.

Also like proposed by Mesquita (2005), we consider a pool of potential volunteers or sympathizers, as those who live close to the criminal activity and have a low opportunity cost in joining it. The government choice will impact this decision by increasing violence in their neighborhood and also, as pointed by Poret e Tejedo (2006), by making criminality more profitable for new entrants. This assumption is consistent with Kenney (2003) argument against the kingpin strategy that the conviction of a leader elevates the opportunity of other potential leaders.

Last but not least, the faction evaluates the impact of the government crackdowns on their operation and recruits the new volunteers. That part of our proposal differs from Mesquita (2005)'s because we could not find evidence of screening on drug factions organizations. It seems that in a slum ruled by a faction, one can choose only to stay out of it or to collaborate, and that is enough to be part of it. Also, unlike terrorist groups, the drug faction is not only ideologically, but profit-motivated (consistent with Poret e Tejedo (2006)). In the next section, we will introduce the Brazilian case for two cities, Rio de Janeiro and São Paulo, and the structure of the game will be presented in Section 4.

3 CRIMINAL DRUG FACTIONS: THE CASES OF RIO DE JANEIRO AND SÃO PAULO

The formal approach developed further in this work is based on solid real evidence of many localities of Brazil and worldwide. For the purpose of illustrating the model that follows we will lean over the cases of São Paulo and Rio de Janeiro's drug factions. This choice is justified for both the cities being marked with the presence of favelas ruled by powerful factions and the police struggle to coerce them.

The analysis made here cover other drug consumer countries with the presence of organized crime in the retail market of drugs, such as the United States of America. An emblematic foreign case is the drug selling gangs of Chicago, as studied by Levitt e Venkatesh (2000), that relates to the Brazilian case in many ways, such as by affecting the impoverish juvenile and being a very risky activity. Nevertheless, in this section, we will understand the reality of the mentioned Brazilian cities and the contingencies that influence the behavior of government, drug factions, and risk areas individuals.

3.1 RIO DE JANEIRO

Favelas are the Brazilian name for slums, impoverish neighborhoods inside of great metropolis, neglected by government assistance, with low access of high-quality schools, health care or even basic sanitation. In Rio de Janeiro, those favelas are also situated in hills, making access to government agents more difficult. This scenario leaves the local population in a reality of vulnerability, living under a parallel state, such as a drug-related faction in power of the location that forces its own rules. The most important factions of Rio de Janeiro are *Comando Vermelho (CV)*, *Primeiro Comando (PC)* and *Amigos dos amigos (AMA)*.

As presented by Dowdney (2003), in Rio de Janeiro the factions may dominate more than a specific favela, having a representative member of the group responsible to organize the traffic on each favela, the so-called owner (*dono*, in Portuguese). This member works autonomously for the faction and rules all the faction activity at one or more favelas. He is assisted with other members, such as the managers (*gerentes gerais*), who organize the other members that pack (*endolador*), transport (*avião*) and sell (*vapor*) the drugs, look for invaders of the favela (*olheiros*), and fight when the invaders get in (*soldado*). Dowdney (2003, p.51) stipulates that "factions have economic advancement as their primary objective and all employ similar structures to achieve it", what evidence the relevance of our model to come.

Assumption 1. *The drug faction primary objective is to maximize profits.*

Carvalho e Soares (2016) present detailed work relating a survey made by an NGO called *Observatório de Favelas* within Rio de Janeiro faction's members in the years of

2004 to 2006 with the 2000 Brazilian census to trace the incentives behind the individuals choice of joining a faction. The authors understand, as we do, that the workers of the faction are recruited from the young men leaving in a favela with a drug faction active, *and any young men can join, based strictly in their own decision.* If so, then they investigate, what are the incentives that lead individuals to this decision.

Assumption 2. *The drug faction doesn't screen from the pool of individuals willing to participate.*

The authors identify that faction-members earn more than 100% compared to what they would in the legal sector since they have many discriminated characteristics in the labor market, such as being black, illiterate and from poorer families. Carvalho e Soares (2016) also find there are great risks involved in the factions activity, with two-thirds of the members that had to be part of gunfights with the police and great mortality rate. They understand there are some behavioral aspects related to the decision to adhere to the faction, such as a lower risk-aversion and time inconsistency. Also, the years of schooling do not seem to aggregate on the returns of the members, but loyalty, experience, and bravery do.

Assumption 3. *The individuals who decide to join the faction have higher levels of risk-passion and associate a lower value to the future.*

Deepen in the Observatório de Favelas' (2006) research, we can extract other valuable information. Most of the faction members left school between the ages of 11 and 14, and they point out the biggest reason the difficulty related to education and their necessity to have immediate economic returns. Nevertheless, many members had other experiences in the labor market before joining the traffic. The researchers understand the reasons for that experience do not stand might be related to the economic environment. Also, in the two years of the research, 20% of the sample died, of which 64% are related to police confrontation.

Assumption 4. *The economic environment, that impacts the prospecting of finding a formal job, affects the decision to join the faction.*

Meirelles e Gomez (2009) developed a survey with 30 former members of a drug faction in Rio de Janeiro, pursuing the reasons that made them adhere to the criminality and abandon it afterward. The authors found that the decision to ingress is not only based on economic, but also symbolic returns. The young men join the faction seeking to be respected in the community, to get access to consumer goods, status, and power.

The government strategies to combat the drug-factions activity is developed by Military and Civil Police. As well established by Dowdney (2003), even though there are

some enterprises to provide a more regular presence of the police inside favelas, such as police posts or police patrols, they are not really representative, since their presence is not so strong, leading to corruption and compromise of those government members. Thus, the actual policing in favela communities is made through "invasions" and "occupations" motivated by strategic reasons, usually followed by confrontation and gunfights.

The author presents interview excerpts with a figure of the military force explaining the government strategy approximate to a military unit entering into enemy territory in a context of war. This approach makes it dangerous to stay in favelas, not only for the involved parts but also to the regular citizens. Dowdney (2003) points out how this creates a hostile relationship between the police and the favela's community, as best explained in the excerpt that follows.

> Policemen are seen by favela residents as violent, dangerous and abusive. Poor police-community relations are worsened by the fact that drug traffickers punish community members that speak to the police. However, police violence and impunity serves to strengthen the belief of favela residents in the 'laws of drug trafficking' as their best option for social order. In effect, this strengthens the traffickers' position within the community. The community fears, distrusts and dislikes the police and fails to view them as the protectors of their rights as Brazilian citizens. Instead, they are caught between inter-faction armed conflicts and repressive, violent and corrupt policing. (DOWDNEY, 2003, p. 81)

That approach paints a picture of 9,651 firefights in the Rio's metropolis in 2018 of which 1,154 are related with confrontation and police action, making several deaths, among them four kids of at least 12 years old, 42 teenagers between 12 and incomplete 18 years old and 20 elderly with more than 60 years old (MATULJA; ZIONI, 2018).

Assumption 5. *The government crackdown in favelas generates negative externalities for the whole community.*

The approach adopted by the police force of "shooting-first" makes it difficult to select those related to criminal activity, usually failing to protect the regular innocent residents. Summed with the low government assistance provided by the government, such as health and education, this scenario "create(s) revolt amongst favela children and adolescents and a hatred of the police that encourages them to join factions as a method of revenge" (DOWDNEY, 2003, p. 89).

Assumption 6. *The government crackdown may increase the willingness of risk-citizens to join a faction.*

Hirata e Grillo (2019) recollect the recent federal intervention in Rio de Janeiro at the beginning of 2018, with the Brazilian military force to provide a back up for the city's

police. The authors highlight that "the number of deaths due to law enforcement's use of lethal force increased dramatically (up to four deaths per day), whereas there was no significant decline in criminal occurrence" (HIRATA; GRILLO, 2019, p.124).

3.2 SÃO PAULO

The case of São Paulo has its own particularities, with the criminal agenda being set almost exclusively by PCC (for *Primeiro Comando da Capital*), the biggest and more powerful drug-related faction, that aroused in prisons in 1993 and further spreading through favelas, aiming to create a network of members that respect each other and are united against the police and the State action.

Hirata e Grillo (2019) make an ethnographic study comparing the violence scenarios of conflict between the police and the criminal groups in Rio de Janeiro and São Paulo. The authors understand the issue in those cities highly correlates the State and the criminal activity, since these policies affects directly the illegal market and the criminal dynamic, ultimately affecting the death of poor citizens. The authors explain that São Paulo opted to privilege a strategy of mass incarceration while Rio's approach is more violent. Nevertheless, both cities have the strong presence of drug factions, actuating with the drug trade as their main source of income.

Hirata e Grillo (2019) explain that, in both cities, the figure of an 'other' of whom the faction can protect its members, is an important instrument to produce a collective identity. This 'other' may be a rival faction, the prison system or the law enforcement agents, i.e. the police. When the community also perceives the police as an 'other', this may influence the decision of joining the criminal activity.

The authors observe that PCC has a stronger political character, and goes beyond only the drug trafficking, being involved also in robbery and other crimes. Also, besides the fact that PCC celebrates the entrance of a new member with a baptism, from which point they start to be considered 'brothers'. The faction also includes those that are not members, but contribute in any way or only relates with the members as 'cousins' with reciprocal loyalty. This proximity indicates there are not many boundaries for those who want to join the faction, strengthen **Assumption 2**.

The authors emphasize the relation between PCC and the State as more of an agreement in São Paulo than real violent confrontation. We are not modeling this type of relationship in this paper, which might appear in some future extensions. Nevertheless, the idea of confrontation and its implications still exist in the city, as we can assert with some anecdotal cases, such as the favela of Paraisópolis' case in 2019 when the military force invaded the favela in a funk ball and the commotion caused nine deaths (MORTES..., 2019).

Biderman et al. (2019) point out the case of São Paulo as different from Rio de Janeiro's since PCC has territory monopoly of the city, not only in the wholesale but also in the security of the illegal workers. The authors argue that Rio de Janeiro has to deal not only with the corrupt interaction of factions with some policemen and the confront with those non-corrupt involved with the drug war, but also disputes between the three large factions for territory. But, PCC, for being a monopolist of the drugs retail and prisons in São Paulo, may minimize conflict and create a less violent scenario.

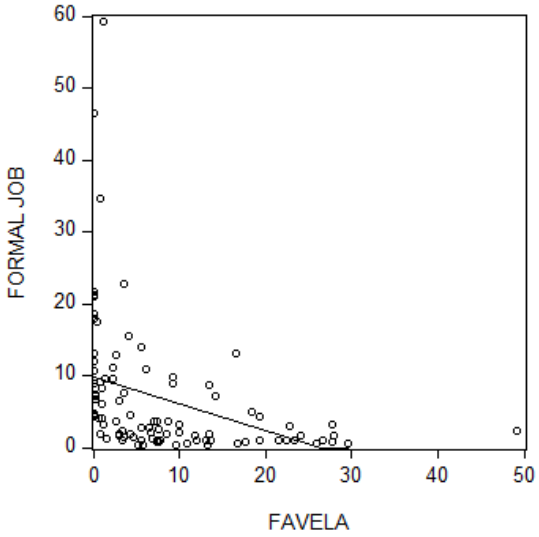
Biderman et al. (2019) also understand that this type of organization creates barriers of entry based on the threat of violence, and this regulation of violence is compatible with the profit-maximizing behavior of a monopolist, recalling our previously established **Assumption 1**. This description fits well our model to be presented in the next section, that comprehends the interaction of a unique criminal group with the law enforcement government.

The PCC aroused as a response to the violence from the police to fight criminals, bringing a discourse of "peace among the bandits", working as a kind of "crime union" and propounding strategies to survive the reality they are inserted (MANSO; DIAS, 2017). PCC even has implemented its own conduct code that must be followed by its members and created a sophisticated system to collect debts and intermediate conflicts (BIDERMAN et al., 2019). The PCC code establishes the union and loyalty of the members and the punishment of those members that commit robbery, rape or extortion inside the system. The main message of the code is the revolt with Brazilian prison conditions, inciting an ideal of revolution against political authorities related to jail conditions.

When it comes to the individuals' decision to adhere to the crime in São Paulo, we will study some contingency factors that may be involved. As argued previously in Rio de Janeiro's literature and further in the model, citizens of favelas and poorest localities may have more trouble finding a formal job. We use data from the *Mapa da Desigualdade* (2019) of São Paulo to trace a partial correlation between the number of residences in favelas per municipality of São Paulo metropolis and the percentage of individuals on formal jobs to see if this relation stands. As shown in Figura 1, we found a strong negative relation between the variables – resembling an inverse function – suggesting that members of favelas may have more difficulty to find jobs on the formal market.

Another important issue for the further construction of the formal model is that residents of impoverish areas dominated by organized crime may have a lower life expectancy. To see this relation, we can also trace, based on the same database of the Inequality Map of São Paulo, a partial correlation between the number of residences in favelas for each municipality and its average age of death. The correlation is presented in Figura 2, revealing a negative relation between the variables and suggesting the more residences in favelas a municipality has, the lower is the average age of death. This result relates to our

Figura 1 – Partial Correlation between number of residences in favelas per municipality and percentage of individuals in formal jobs



Fonte: Developed by the author based on data from Mapa da Desigualdade (2019).

model through two means: (1) the lower life expectancy generates a lower opportunity cost to adhere to the crime; (2) the local presence of crime pushes the life expectancy down. Also, why would one bother to study if she doesn't expect to be alive to yield the results?

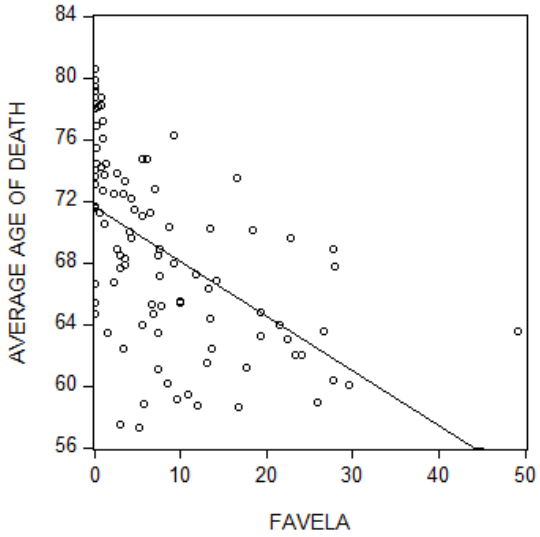
Both the results endorse a tacit idea presented on the Rio de Janeiro's studied literature that we will step back to postulate:

Assumption 7. *Individuals residing in favelas have a lower opportunity cost to adhere to drug factions.*

The observation of a reality where the individual has more trouble to find an insertion on the formal market added by the expectancy of not living very long may increase the chances of the individual to choose the "easier" way. Those results relate with **Assumption 3** in the lower value attributed to the future and with **Assumption 4**, such that the prospecting of finding a formal job affects the decision to join the faction. This econometric feature is not supposed to be exhausting, just aiming to present some important relations through the help of disposable data, and fill the gap of those analyses in other sources.

Manso e Dias (2018) describe in detail the history of the configuration of the PCC and explain the many episodes of slaughter in São Paulo's prisons and the tug of war between the police and the faction. The authors present many events of confronting between the parts since the creation of the faction. These episodes culminated in rebellions and deaths

Figura 2 – Partial Correlation between number of residences in favelas per municipality and average age of death



Fonte: Developed by the author based on data from Mapa da Desigualdade (2019).

on both sides, making innocent victims in the way. This timeline of confronting explicitly generates negative externality in communities around, reinforcing our **Assumption 5**.

4 THE MODEL

In this section, we build a model of the domestic war on drugs based on the characteristics of places such as the favelas of Rio de Janeiro and São Paulo. Although our framework is simple, we believe that it describes the interaction between government, factions, and citizens of favelas well enough, highlighting its main aspects.

4.1 ENVIRONMENT

Consider a domestic war on drugs in a specific social context, namely the one as described in the previous sections, such as Brazilian favelas. In this context, drug traffickers and citizens interact daily through many activities, given that the first often assume some state responsibilities (e.g. provision of public security). Due to the complexity and dangers of urban warfare, government actions in favelas can have negative effects on the chances of their inhabitants in the formal labor market. As a consequence, citizens may be attracted to crime for reasons related to its relative advantages when compared to legal jobs.

In this section we use the insights of the literature about Rio de Janeiro and São Paulo presented above to propose a dynamic game between a government (G), whose only objective is to combat drug trafficking; one criminal faction (F), which rules the favela, and wants to maximize its profits from the drug dealing activity; and, a risk group (R) composed by favela's citizens, who live near high-intensity drug trafficking areas, in poor housing conditions and with low access to public goods and education. The timing of the game is as follows.

1. The government chooses the extent of repression (crackdown) to use against the faction in order to fight drug trafficking, $a \in [0, 1]$; this creates negative economic externalities over citizens of the favela, namely a damage $\tau(a)$, which impacts their chances of joining the formal labor market and finding a legal job as well as the price of the guns the faction must buy;

2. After observing the government action and feeling its consequences, the risk group chooses between joining the faction (becoming a criminal) and working in a legal job;

3. The faction then maximizes its profits by recruiting those citizens who are willing to join the crime organization and choosing the number of guns to buy.

Let us start by presenting the citizen's payoff. We assume that there is a continuum of mass one of citizens living in the favela controlled by the faction. The payoff of citizen i when she chooses to get a legal job – and, therefore, not to get involved in drug trafficking

– is

$$U_i^E = (2 - \tau(a))f(\gamma)\delta, \tag{4.1}$$

where γ measures the current economic performance of the country, state and city in which the citizen lives in, which impacts her prospects to find a formal job, as postulated in **Assumption 4**. The function f measures the impacts from the level of economic activity on the citizen's opportunity in the labor market. We assume that $f' > 0$, such that better economic conditions increase the chances of getting a legal job that pays a good salary, which in turn increases U_i^E.

The variable $\delta \in (0, 1)$ represents her intertemporal discount factor, such that individuals that value more the future (e.g. living longer) will prefer to follow the legal market. This idea is best contextualized in Section 3, culminating on **Assumption 3**. Moreover, as the return of the formal labor market is, in general, higher in the long run and requires a substantial investment of time and effort (e.g. scholarly time, studies, in job experience), the more patient the citizen who chooses this option, the higher her utility.

We also assume that the negative externality generated by the government crackdown is uncertain. Formally, $\tau(\cdot)$ is a random variable with distribution function $T(a)$, whose support is $[0, 1]$. For the sake of simplicity, define $\tau(a) = a^2 + \varepsilon$, where ε is a zero-mean shock with support $[-1, 1]$. Observe that such a function is increasing in a, such that large crackdowns generate high negative externality on citizen's life. The shock, however, allows for unpredictable outcomes, which may be due to variables outside the model. It is possible, for instance, that an extreme repression ($a = 1$) has no negative effect on the citizen's welfare, which occurs when $\varepsilon = -1$. Finally, notice in (4.1) that, although we have indexed the utility by i, it does not vary with it. This implies that all citizens of the favela have the same utility from choosing a legal job[1].

As presented in **Assumption 5**, there are several ways through which government repression can have perverse effects on the life of the honest citizens of the favela (those who choose to work in a legal job). First, there are the direct consequences of the violence related to the drug faction, such as stray bullets, widespread fear, and the impossibility of moving around. This reality affects all the community, making it harder to children go to school and increase their future chances on the legal market, and to adults to search for a job – in general, it is necessary to go outside the favela to increase the chances – when one is afraid or unable to move around. Second, the confront can even serve to create a revolt from the citizens against the police action, increasing their willingness to join the criminality (**Assumption 6**).

Now, suppose that the citizen i decides to join the faction. In this case, her payoff

[1] This is not a strong assumption as there is another source of heterogeneity, namely the parameter λ.

is:

$$U_i^J = e(\pi, \lambda_i) + u(a) + w, \tag{4.2}$$

where w is the monetary compensation paid by the faction for its members, such as a "salary" for the activity. This salary is higher than the legal market's when considering the peers of the faction members (same scholarly level and sociocultural background), as presented by Carvalho e Soares (2016). Also, those authors understand there is a personal inclination to crime, related to the individual risk-passion, which was established in our **Assumption 3** and is captured in the model by the variable $\lambda_i \in [0, 1]$. The idea is that, in general, people who are more risk-passionate (with high values for λ_i) are also more prone to joining crime.

The benefit from becoming a member of the drug faction is captured by $e(\pi, \lambda_i)$, which measures the ego-rent from getting power over other citizens of the favela and the prestige of being a member of a group with large financial and military power. This benefit is increasing in the faction's size, measured by its profit, π, and in λ_i: the more powerful the faction, the larger the prestige and the power of its members over the other citizens; the more risk lover the citizen, the larger is her welfare of taking part in such dangerous and adventurous activity. As the citizen's choice is binary, there is no need for concavity, such that it suffices a linear form such as $e(\pi, \lambda_i) = \pi + \lambda_i$.

The government repression has a direct negative effect on the citizen who becomes a drug dealer, which is represented by the function $u(a)$ in (4.2). The larger the extent of the repression the more severe the impacts on the criminals of the favela, such that $u^{'} < 0$ – it is a disutility – and $u^{''} > 0$. In addition, $u^{'}(0) \to -\infty$ and $u^{'}(1) \to 0$. Crackdowns may result in arrest, injuries and even death of those involved in drug trafficking. This is particularly true for social context such as the Brazilian favelas.

Consider now the faction's payoff. As it is argued in the previous sections, drug factions besides their particularities actuate as profit-maximizing organizations (see **Assumption 1**). Thus, its payoff is given by its profits, or its revenues deduced by its costs,

$$U^F = \pi = pq\left(K, L\right) - C\left(q\left(K, L\right)\right).$$

The first term of U^F represents the faction's incomes, where p is the drug price and $q\left(L, K\right)$ is the profit-maximizing drug production function; the second term, $C\left(q\right)$, represents the cost function associated with this production level. We follow Murphy, Grossman e Becker (2006) by assuming that the drug market is perfectly competitive, which implies that the faction is a price-taker, such that $p > 0$ is a constant. Observe that we also assume that only two inputs are required for drug production, namely labor (workforce from citizens of the favela, $\overline{L}$) and capital, which we interpret as the amount of weapons and all the faction's military apparatus (hidden accommodations, cars, etc). Henceforth, we refer K simply as "guns".

The faction's play occurs in two steps. First, it minimizes the cost by choosing the number of guns and recruiting all the citizens willing to join the organization. The market of guns is assumed to be perfectly competitive as well, with the price given by $g(a)$. One can notice that the only source of variation is the government crackdown. For example, whenever the police and other enforcement agencies seizure firearms and ammunition, the faction needs to buy new ones, spending more money. We model such an effect as it increases the price of the guns, that is, $g^{'} > 0$. Moreover, we assume that $g(\cdot)$ is concave in a and $g^{'}(0) \to +\infty$ and $g^{'}(1) \to 0$.

It is important to highlight that the faction hires all citizens who are willing to become its members, in consonance with the discussion in the last section and **Assumption 2**. It is worth noticing that there is no evidence of "unemployment" among drug traffickers, especially in Brazilian favelas. In fact, factions' job structures ensure that even young boys have a positions according to their skills (e.g. *avião*, *olheiro* and *vapor*). The second step of the faction's problem is to choose the quantity of output (drugs) to produce and sell, which generates their production function, $q(K, L)$. In the next section, we present the decision process in detail.

The drug production function that we assumed has some standard properties. First, it presents decreasing returns to scale. Second, it satisfies the principle of diminishing marginal productivity, such that $\frac{\partial^2 q}{\partial K^2} < 0$ and $\frac{\partial^2 q}{\partial L^2} < 0$. Third, guns (and other military apparatus) and labor are complements in the production, which implies that $\frac{\partial^2 q}{\partial K \partial L} > 0$. Finally, if either input is zero, there is no production. A Cobb-Douglas function, $q = L^{\alpha} K^{\beta}$, with $\alpha, \beta \in (0, 1)$ and $\alpha + \beta = 1$ satisfies all the above requirements, for example. For the sake of simplicity, henceforth we consider that the production function is a Cobb-Douglas. The choice of other more general functional forms, such as a CES, would not change our results.

An important characteristic of our model is the potential ambiguous effect from the government repression (a) on the faction's profit. On the one hand, a crackdown has the direct effect of increasing the price of one of the inputs, namely guns, which affects negatively the profits. On the other hand, the availability of the other input necessary to produce, namely labor, is also a function of a, given that the number of citizens willing to join the criminal activity responds to the extent of the repression. This indirect effect, as we will see below, can be either positive or negative, depending on the other parameters.

Finally, the government wants to minimize drug trafficking, which may be represented by minimizing the faction's profit. Crackdowns and repression are costly, given that they require the use of limited resources (e.g. weapons and soldiers or police officers) and may result in loss of part of military apparatus and even in the loss of men. We represent this cost by function $c(a)$, which is increasing and convex on the extent of the repression chosen by the government, $c^{'}(a) > 0$, $c^{''}(a) > 0$. Moreover, we assume that $c^{'}(0) = 0$ and

$c^{'}(1) \to +\infty$. Formally, we have

$$U^G = -\pi - c(a).$$

The next section solves the game by using backward induction in order to find the Subgame Perfect Equilibrium (SPE).

4.2 EQUILIBRIUM

Given the timing of the game described above, we start by finding the faction's best response to both the number of citizens willing to become criminals and the level of government crackdown. For, first we must consider its minimization problem:

$$\begin{gathered}\min_K wL + g(a)K \\ \text{subject to } L = \bar{L} \text{ and } L^\alpha K^\beta = Q,\end{gathered}$$

where the first constraint highlights the assumption that the factions does not screen workers and hire all citizens willing to join it. As it is shown in the appendix, the demand for guns and the cost function are $K\left(w, g(a), Q\right) = \left(\frac{Q}{\bar{L}^\alpha}\right)^{\frac{1}{\beta}}$ and $C\left(w, g(a), Q\right) = w\bar{L} + g(a)\left(\frac{Q}{\bar{L}^\alpha}\right)^{\frac{1}{\beta}}$, respectively.

We can now state the faction's profit maximization problem:

$$\max_Q \pi = pQ - \left[w\bar{L} + g(a)\left(\frac{Q}{\bar{L}^\alpha}\right)^{\frac{1}{\beta}}\right],$$

whose solution is

$$Q\left(a, \bar{L}\right) = \left(\frac{\beta p \bar{L}^{\frac{\alpha}{\beta}}}{g(a)}\right)^{\frac{\beta}{1-\beta}}. \tag{4.3}$$

For future use, notice that the optimal profit can also be rewritten as a function of only a and $\bar{L}$, that is

$$\pi\left(a, \bar{L}\right) = \left(\frac{\beta p \bar{L}^{\alpha}}{g(a)^\beta}\right)^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{\beta}\right) - w\bar{L}. \tag{4.4}$$

Our first proposition analyzes the response of the optimal drug production to changes in the number of workers and in the level of government repression.

Proposition 1. *The optimal drug production, given by equation* (4.3)*, is increasing in the number of citizens willing to join the faction* $\bar{L}$ *and decreasing in the level of crackdown* a*.*

The two effects described above are somehow expected. As labor is an input in the production function, the higher the number of citizens working in the faction – recall that all individuals willing to join the organization are hired – the higher the drug production. Furthermore, the more expensive the guns are, the higher is the total cost, which impacts

negatively the optimal level of production. As we can see above, the constraint $\bar{L} = L$ implies that that the demand for guns is independent on their price, such that the only effect $g(a)$ has on profit is through the direct increase in the cost. We can also see that (4.3) is a standard supply curve as it is increasing in the drug price p.

Observe in the faction's best response that, if the only effect of crackdown on $Q\left(a, \bar{L}\right)$ were the partial one, it would be unambiguous: more repression would imply fewer drugs. As the next result shows, similar reasoning can be applied to the faction's profit, namely the partial effect of a in π is negative. As we will show below, there is also an indirect effect, whose channel is how the decisions of the citizens of the favela change in response to government action. The next result also establishes standard conditions under which the faction's profit is increasing in the number of workers (citizens).

Proposition 2. *The faction's optimal profit, given by* (4.4)*, is decreasing in the extent of government repression, that is* $\frac{\partial \pi}{\partial a} < 0$*. In addition, it is increasing in the number of citizens willing to join the faction, that is* $\frac{\partial \pi}{\partial \bar{L}} > 0$*, if and only if the marginal productivity of labor is larger than the real salary* $\frac{w}{p}$*.*

Based on the evidence discussed in the previous sections, namely that drug factions hire any individual willing to work in the drug business, we make the following further assumption:

Assumption 8. *The marginal productivity of labor is larger than* $\frac{w}{p}$ *for all set of parameters* $(w, \alpha, \beta, p, \delta, \gamma)$*.*

A simple way to guarantee that the above assumption holds is by supposing that the real salary is sufficiently low, which can occur if the nominal salary is low or the drug price is high enough.

Let us now analyze the decision of the citizens of the favela. Each citizen i chooses to join the faction if and only if $U_i^J \geq U_i^E$[2]. Backward induction implies that she takes into account the faction's best response function (4.3) – and its profit (4.4) as well – in the moment of her decision. Thus, the above condition can be written as

$$\pi(a, \overline{L}) + \lambda_i + u(a) + w \geq (2 - \tau(a)) f(\gamma)\delta.$$

By rewriting the above expression, we find that only citizens with a propensity to crime (those with risk aversion low enough) above a given threshold choose to join the faction. By taking the expected value of $\tau(a)$, the negative externality generated by the government action, we can show that citizen i of the favela chooses to join the faction if and only if:

$$\lambda_i \geq (2 - a^2) f(\gamma)\delta - \pi(a, \overline{L}) - u(a) - w = \lambda\left(a, \overline{L}\right).$$

[2] For the sake of simplicity, we assume that whenever the citizen is indifferent between joining the faction and entering into the formal labor market, the choice is to join.

Let λ_i be distributed according to a cumulative distribution function $H : \left[0, \bar{\lambda}\right] \to [0, 1]$ and probability density function $h(\cdot)$. The mass of citizens who join the faction is, therefore, given by $1 - H\left(\lambda\left(a, \bar{L}\right)\right)$. We can now find the number of citizens willing to become drug traffickers as a function of the government crackdown.

As we can notice below, the function that gives us how the number of citizens willing to become a criminal responds to government repression is implicitly defined. The existence of $\bar{L}(a)$ solving such an implicit equation (see (4.5) below) is showed in the appendix and does not require any further assumption to hold. Instead, its uniqueness is guaranteed only if we assume the technical condition below.

Proposition 3. *Suppose that the probability distribution function $H(\cdot)$ and the set of parameters $(w, \alpha, \beta, p, \delta, \gamma)$ are such that the $h(\cdot)\frac{\partial\pi\left(a,\bar{L}\right)}{\partial\bar{L}} < 1$ for all $\left(a, \bar{L}\right) \in [0, 1] \times [0, \infty)$. Then, the mass of citizens of the favela who chooses to join the drug faction $\bar{L}(a)$, implicitly defined by the equation*

$$\bar{L} = 1 - H\left((2 - a^2)f(\gamma)\delta - \pi(a, \bar{L}) - u(a) - w\right), \tag{4.5}$$

exists and is unique.

In order to have an intuition of the above requirement, observe that, if the variance of λ_i is sufficiently large, then the probability density function is limited. In other words, there is a $k > 0$, a constant small enough such that $h(\cdot) < k$. Thus, if there is a sufficiently large dispersion of the individual parameter that measures the propensity to crime, the above assumption holds. We can now state the main comparative statics result of this section.

Proposition 4. *The number of citizens of the favela who are willing to join the faction – and, as we have seen, hired by them – is strictly increasing in the level of government repression if and only if the marginal effect of such an action on the faction's profit is sufficiently low. In particular, for $a \approx 0$ we have $L^{'}(a) < 0$, and for $a \approx 1$ we have $L^{'}(a) > 0$.*

The first thing to notice – and perhaps the most important result of this paper – in the above proposition is that crackdowns may be counterproductive. Given that repression affects negatively both U_i^J and U_i^E, it may be the case that the marginal impact on the payoff from getting a legal job, due to the negative externality, is higher than the one from becoming a criminal. Recall that one of the components of the latter is the marginal impact on the citizen's ego-rent (e.g. from getting power over the favela), which in turn depends on the faction's profit. If the negative impact on the profit is lower than the direct disutility from repression (e.g. possibility of injuries and being arrest) plus the marginal effect on the formal labor market prospects than there is an increase in the number of

citizens who join the drug faction when the crackdown becomes more aggressive. Instead, if the faction's profit is sufficiently sensitive to repression, government actions have a large impact on the ego-rent of those citizens who join the faction, which implies that the number of citizens becoming drug traffickers is decreasing in the extent of the crackdown.

Although we cannot state a monotone behavior for $L^{'}(a)$, it is possible to show that there is a "deterrent effect" when the level of repression is very low and a "encouragement effect" when it is very high. The first is explained by the large marginal disutility (recall that $u^{'}(0) \to -\infty$) and the large marginal impact on the faction's profit ($\frac{\partial \pi(0,\bar{L})}{\partial a} \to -\infty$) when a is very low. The latter, on the other hand, occurs because both the marginal disutility and the marginal impact on profits approaches zero when the repression is severe enough.

As it is the first to play, the government maximizes its payoff by taking into account the best response functions of both the citizens, $\bar{L}(a)$ and the faction, $Q\left(a, \bar{L}(a)\right)$ – and its profit $\pi\left(a, \bar{L}(a)\right)$ as well. Its optimization problem is, therefore, given by

$$\max_{a} U^{G} = -\pi(a, \overline{L}(a)) - c(a).$$

The first order condition is

$$-\left[\frac{\partial \pi}{\partial a} + \frac{\partial \pi}{\partial \overline{L}} L^{'}(a)\right] - c'(a) = 0. \tag{4.6}$$

The existence of a unique interior solution of the above equation is discussed in the next result.

Proposition 5. *Suppose that $(w, \alpha, \beta, p, \delta, \gamma)$ are such that the second order conditions of the government problem is satisfied, that is, U^G is strictly concave. Then, there exists a unique $a\,(w, \alpha, \beta, p, \delta, \gamma) \in (0, 1)$ that solves the government's optimization problem.*

One way to understand the above result is through the intuition of the Theorem of Intermediate Value, used in its proof (see appendix). The existence of a positive and unique optimal level of repression means that, while the marginal benefit is larger than the cost when there is no repression ($a = 0$), the opposite happens when the government implements the maximum level ($a = 1$). In other words, the marginal (negative) impact of crackdowns on the faction's profit is higher than the cost associated with this policy whenever a is low. Recall that, whenever the extent of the crackdown against traffickers is sufficiently low, we have $L^{'}(a) < 0$ (deterrent effect). This implies that, in this case, the two partial effects on profit are positive ($-\frac{\partial \pi}{\partial a} > 0$ and $-\frac{\partial \pi}{\partial \overline{L}} L^{'}(a) > 0$) while the marginal cost is null ($c^{'}(0) = 0$). On the other hand, whenever the crackdown is severe enough, the cost of repression is sufficiently high, such that it overcomes the marginal benefit – in this case it is only $-\frac{\partial \pi}{\partial a} > 0$ as $L^{'}(a) > 0$ (encouragement effect) for a high enough. The concavity of U^G guarantees the monotonicity of this behavior.

As we can see in the proof of the proposition (see appendix), some assumptions are essential for the result. If, for example, $g^{'}(1) = g > 0$ and $c^{'}(1) = c > 0$ are finite, then $\frac{\partial \pi(1,\bar{L})}{\partial a} < 0$ is also finite, and the existence of a interior solutions is no longer guaranteed. If fact, if $\left|\frac{\partial \pi(1,\bar{L})}{\partial a}\right|$ is sufficiently large, we have a corner solution, with $a = 1$. Clearly, this is the case where repression is cheap and has a huge negative impact on the faction. It is also possible to have $a = 0$ as long as $g^{'}(0) = \bar{g} > 0$ and $c^{'}(0) = \bar{c} > 0$ are finite and $\left|\frac{\partial \pi(0,\bar{L})}{\partial a}\right|$ is sufficiently low. This is the opposite case: repression is relatively expensive and has a small impact of the traffickers.

4.3 SOME COMPARATIVE STATICS

The next proposition summarizes the equilibrium of the game:

Proposition 6. *The SPE of the game played by the government, the faction and citizens of the favela is the following:* $\left(s^G, s^R, s^F\right) = \left(a\left(w, \alpha, \beta, p, \delta, \gamma\right), \bar{L}(a), Q\left(a, \bar{L}\right)\right)$*, where* s^j *is the strategy of the player* $j = G, R, F$*, and* $a\left(w, \alpha, \beta, p, \delta, \gamma\right)$*,* $\bar{L}(a)$ *and* $Q\left(a, \bar{L}\right)$ *are defined by* (4.6)*,* (4.5) *and* (4.3)*, respectively.*

We can now perform some comparative statics exercise. Henceforth, we only consider the case of interior solution, that is, the one in which $a\left(w, \alpha, \beta, p, \delta, \gamma\right) > 0$. Thus, we continue to assume that U^G is strictly concave. Our first result concerns the impacts of changes in the drug price on the level of government repression.

Proposition 7. *Suppose that crackdowns deter citizens from joining the faction, that is,* $\bar{L}^{'}(a) < 0$*, and that the higher the drug price the higher the (absolute) marginal effect of crackdowns on "labor supply", that is,* $\frac{\partial \bar{L}^{'}(a)}{\partial p} < 0$*. Then, the higher the drug price the larger the optimal level of the government crackdown. Furthermore, if crackdowns generate the "encouragement effect" (*$\bar{L}^{'}(a) > 0$*), then the optimal level crackdown is decreasing in* p *if the above effect is sufficiently large.*

Proposition 7 states that, in response to a rise in the drug price, the government must become more aggressive fighting drug trafficking whenever two conditions are satisfied. First, there must be a "deterrence effect" of crackdowns. Second, such an effect must be increasing in the drug price: the more expensive the drug sold by the faction the more effective crackdowns are in deterring citizens to become criminals. We can summarize this by stating that $\frac{\partial a(w,\alpha,\beta,p,\delta,\gamma)}{\partial p} > 0$ whenever the government is sufficiently efficient in discouraging people from joining the faction. On the other hand, if the government is inefficient, such that there is a sufficiently large "encouragement effect" due to crackdowns, it must decrease repression as a response to a rise in the drug price. The mechanism behind the above result is the following: a higher price increases the faction's profit, which in turn attracts more citizens; such an effect dominates if the government is not able

to react and overcome it. In addition, an important remark on proposition 7 is that it presents only sufficient conditions.

Proposition 8. *Suppose that the better the economic performance of the region the lower the marginal effect of crackdowns on citizens' "labor supply", that is,* $\frac{\partial \bar{L}'(a)}{\partial \gamma} < 0$. *Then, better economic conditions make the government increase the repression against drug factions. If, instead,* $\frac{\partial \bar{L}'(a)}{\partial \gamma} > 0$, *the optimal level of crackdown is decreasing in* γ.

We must consider two cases related to proposition 8. First, suppose that $\bar{L}'(a) < 0$, such that the deterrence effect holds. Thus, the condition $\frac{\partial \bar{L}'(a)}{\partial \gamma} < 0$ means that the effect (its magnitude) is increasing in the economic performance of the region. Second, suppose that $\bar{L}'(a) > 0$, such that $\frac{\partial \bar{L}'(a)}{\partial \gamma} < 0$ means that, although there is the encouragement effect, it is decreasing in γ. In other words, in both cases, the government becomes more effective in discouraging citizens – or at least decreasing their incentives – from becoming traffickers as the economy improves. Whenever this is the case, it is optimal for the government to become more aggressive because the impacts on citizens' lives are lower when there is a booming economy (the formal labor market is more attractive). If better economic conditions have an opposite effect, namely making the marginal impact of crackdowns on labor supply lower, then the government must become softer in fighting trafficking.

5 CONCLUSION

In this work, we developed a game-theoretical model of the domestic war on drugs, grounded in the Brazilian experiences, especially in Rio de Janeiro and São Paulo. For that, we presented ethnographic literature and empirical evidence of those cities, to create a solid foundation for the model. We showed how the drug market is organized for factions (gangs) in those cities that actuate in impoverish neighborhoods such as favelas. This location allows them to keep their criminal activity discrete for the regular citizen but impact directly the citizens of the community.

As shown, those neighborhoods are not very assisted by the State, receiving less support of public goods than other regions. Notwithstanding, the government's appearance in favelas is usually provoked for confronts with the criminal faction, which results in a perverse impact on the lives of innocent citizens. We argue throughout our model that this interaction may create resentment in the community, encouraging its individuals to adhere to the faction. This is the strategical relation captured by our model.

We developed a sequential game of perfect information between the drug faction, the government, and the community citizens. This game contributed to the understanding that government crackdown on factions may have the reverse effect expected, increasing the faction's profit and the incentives of the citizens to join it. We also provide some comparative statics considering an increase in the drug price and the improvement of the economic environment.

This work provided a new perspective on fighting drug trafficking and approximated this subject with the field of game theory and modeling. We hope this could bring insights to policymakers and raise the debate about the war on drugs, especially for consumer countries such as Brazil and the United States of America.

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APÊNDICE A – PROPOSITION'S PROOFS

THE FACTION'S PROFIT MAXIMIZATION (PROPOSITIONS 1 AND 2)

We must start by solving the faction's cost minimization problem:

$$\min_K wL + g(a)K$$
$$\text{subject to } L = \bar{L} \text{ and } L^\alpha K^\beta = Q.$$

By substituting $L = \bar{L}$ into the production function we obtain the demand for guns:

$$K(w, g(a), Q) = \left(\frac{Q}{\bar{L}^\alpha}\right)^{\frac{1}{\beta}}.$$

We can then substitute $K(w, g(a), Q)$ into the objective function of the minimization problem to obtain the cost function $C(w, g(a), Q) = w\bar{L} + g(a)\left(\frac{Q}{\bar{L}^\alpha}\right)^{\frac{1}{\beta}}$.

We must now to solve the faction's profit maximization problem:

$$\max_Q \pi = pQ - \left(w\bar{L} + g(a)\left(\frac{Q}{\bar{L}^\alpha}\right)^{\frac{1}{\beta}}\right),$$

whose solution gives us

$$Q\left(a, \bar{L}\right) = \left(\frac{\beta p \bar{L}^{\frac{\alpha}{\beta}}}{g(a)}\right)^{\frac{\beta}{1-\beta}}.$$

By substituting this expression into the faction's profit, we have

$$\pi\left(a, \bar{L}\right) = \left(\frac{\beta p \bar{L}^\alpha}{g(a)^\beta}\right)^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{\beta}\right) - w\bar{L}.$$

Proof of Proposition 1: It suffices to show that

$$\begin{aligned}
\frac{\partial Q\left(a, \bar{L}\right)}{\partial \bar{L}} &= \left(\frac{\beta p \bar{L}^{\frac{\alpha}{\beta}}}{g(a)}\right)^{\frac{\beta}{1-\beta}} \frac{\alpha}{1-\beta}\frac{1}{\bar{L}} > 0 \\
\frac{\partial Q\left(a, \bar{L}\right)}{\partial a} &= \left(\frac{\beta p \bar{L}^{\frac{\alpha}{\beta}}}{g(a)}\right)^{\frac{\beta}{1-\beta}} \frac{-\beta}{1-\beta}\frac{1}{g(a)} < 0.
\end{aligned}$$

Proof of Proposition 2: Observe that

$$\begin{aligned}
\frac{\partial \pi\left(a, \bar{L}\right)}{\partial \bar{L}} &= \left(\frac{\beta p \bar{L}^\alpha}{g(a)^\beta}\right)^{\frac{1}{1-\beta}} \frac{\alpha}{\beta}\frac{1}{\bar{L}} - w \\
\frac{\partial \pi\left(a, \bar{L}\right)}{\partial a} &= -g(a)\left(\frac{\beta p \bar{L}^\alpha}{g(a)}\right)^{\frac{1}{1-\beta}} < 0,
\end{aligned}$$

such that $\frac{\partial \pi\left(a, \bar{L}\right)}{\partial \bar{L}} > 0$ if and only if

$$\left(\frac{\beta p \bar{L}^\alpha}{g(a)^\beta}\right)^{\frac{1}{1-\beta}} \frac{\alpha}{\beta}\frac{1}{\bar{L}} > w. \tag{A.1}$$

Notice now that the marginal productivity of labor is given by $MPL = \alpha L^{\alpha-1}K^{\beta}$. By substituting $L = \bar{L}$, $K = K(w, g(a), Q)$ and $Q\left(a, \bar{L}\right)$ into MPL, we have

$$PML = \left(\frac{\beta p^{\beta}\bar{L}^{\alpha}}{g(a)^{\beta}}\right)^{\frac{1}{1-\beta}} \frac{\alpha}{\beta}\frac{1}{\bar{L}}.$$

Thus, given that we can rewrite (A.1) as

$$\begin{aligned} p\left(\frac{\beta p^{\beta}\bar{L}^{\alpha}}{g(a)^{\beta}}\right)^{\frac{1}{1-\beta}} \frac{\alpha}{\beta}\frac{1}{\bar{L}} &> w \\ PML &> \frac{w}{p}, \end{aligned}$$

we prove the proposition's statement.

THE CITIZEN'S PROBLEM (PROPOSITIONS 3 AND 4)

Proof of Proposition 3: We have already seen that the mass of citizens who chooses to join the faction is implicitly defined by (4.5). In order to show existence, let us rewrite (4.5) as

$$G\left(\bar{L}\right) = 1 - H\left((2-a^2)f(\gamma)\delta - \pi(a,\bar{L}) - u(a) - w\right) - \bar{L}, \tag{A.2}$$

which is clearly a continuous function. Then, observe that

$$\begin{aligned} G(0) &= 1 - H\left((2-a^2)f(\gamma)\delta - u(a) - w\right) > 0 \\ G(0) &= -H\left((2-a^2)f(\gamma)\delta - \pi(a,1) - u(a) - w\right) < 0, \end{aligned}$$

where we use the fact that $\pi(a,0) = 0$. Thus, the Theorem of the Intermediate Value applies and guarantees that there exists $\bar{L}(a)$ such that $G\left(\bar{L}(a)\right) = 0$.

To guarantee uniqueness, we need to show that $G(\cdot)$ is strictly decreasing. By calculating its derivative, we have

$$G'\left(\bar{L}\right) = h\left((2-a^2)f(\gamma)\delta - \pi(a,\bar{L}) - u(a) - w\right)\frac{\partial \pi}{\partial \bar{L}} - 1.$$

Therefore, $G' < 0$ if and only if $h(\cdot)\frac{\partial \pi}{\partial \bar{L}} < 1$, but this is precisely the assumption of the proposition.

Proof of Proposition 4: We are interested in $\bar{L}'(a)$, which can be calculated applying the Implicit Function Theorem in (A.2):

$$\bar{L}'(a) = -\frac{h\left((2-a^2)f(\gamma)\delta - \pi(a,\bar{L}) - u(a) - w\right)\left(2af(\gamma)\delta + \frac{\partial \pi}{\partial a} + u'(a)\right)}{G'\left(\bar{L}\right)}.$$

Given that $G'\left(\bar{L}\right) < 0$ and $h(\cdot) > 0$, we have that $sign\left(\bar{L}'(a)\right) = sign\left(2af(\gamma)\delta + \frac{\partial \pi}{\partial a} + u'(a)\right)$. This implies that $\bar{L}'(a) > 0$ if and only if $\left|\frac{\partial \pi}{\partial a}\right| < 2af(\gamma)\delta + u'(a)$. Instead, $\bar{L}'(a) < 0$ if and only if $\left|\frac{\partial \pi}{\partial a}\right| > 2af(\gamma)\delta + u'(a)$, which finishes the proof.

Finally, we can calculate the following limits:

$$\begin{aligned}
\lim_{a\to 0} L^{'}(a) &= -\lim_{a\to 0} \frac{h\left((2-a^2)f(\gamma)\delta - \pi(a,\bar{L}) - u(a) - w\right)}{G'\left(\bar{L}\right)} \lim_{a\to 0}\left(2af(\gamma)\delta + \frac{\partial \pi}{\partial a} + u^{'}(a)\right) \\
&= X_0 \lim_{a\to 0}\left(\frac{\partial \pi}{\partial a} + u^{'}(a)\right) = -\infty < 0 \\
\lim_{a\to 1} L^{'}(a) &= -\lim_{a\to 1} \frac{h\left((2-a^2)f(\gamma)\delta - \pi(a,\bar{L}) - u(a) - w\right)}{G'\left(\bar{L}\right)} \lim_{a\to 1}\left(2af(\gamma)\delta + \frac{\partial \pi}{\partial a} + u^{'}(a)\right) \\
&= X_1 \lim_{a\to 0}\left(2f(\gamma)\delta + \frac{\partial \pi}{\partial a} + u^{'}(a)\right) \\
&= X_1 2f(\gamma)\delta > 0,
\end{aligned}$$

where $X_0 = -\lim_{a\to 0} \frac{h(\cdot)}{G'(\bar{L})} > 0$, $X_1 = -\lim_{a\to 1} \frac{h(\cdot)}{G'(\bar{L})} > 0$ and we use fact that $\frac{\partial \pi(0,\bar{L})}{\partial a} \to -\infty$, $\frac{\partial \pi(1,\bar{L})}{\partial a} \to 0$, $u^{'}(0) \to -\infty$ and $u^{'}(1) \to 0$.

THE GOVERNMENT PROBLEM (PROPOSITION 5)

Proof of proposition 5: Rewrite (4.6) as

$$F(a) = -\left[\frac{\partial \pi}{\partial a} + \frac{\partial \pi}{\partial \bar{L}} L^{'}(a)\right] - c^{'}(a), \tag{A.3}$$

which is a continuous function. Notice that the concavity of U^G implies that $F^{'}(a) < 0$. Furthermore, recall that $\frac{\partial \pi\left(0,\bar{L}\right)}{\partial \bar{L}} > 0$ for all $a \in [0,1]$.

Observe now that

$$\lim_{a\to 0} F(a) = -\left[\lim_{a\to 0}\left(\frac{\partial \pi\left(a,\bar{L}\right)}{\partial a} + \frac{\partial \pi\left(0,\bar{L}\right)}{\partial \bar{L}} L^{'}(a)\right) + c^{'}(0)\right] = +\infty > 0$$

because $\lim_{a\to 0} \frac{\partial \pi\left(a,\bar{L}\right)}{\partial a} \to -\infty$ (recall that $g^{'}(a) \to \infty$ when $a \to 0$), $c^{'}(0) = 0$, $\frac{\partial \pi\left(0,\bar{L}\right)}{\partial \bar{L}} > 0$ is a constant and $\lim_{a\to 0} L^{'}(a) = -\infty$. In addition, we have

$$\lim_{a\to 1} F(a) = -\left[\lim_{a\to 1} \frac{\partial \pi\left(a,\bar{L}\right)}{\partial a} + \frac{\partial \pi\left(1,\bar{L}\right)}{\partial \bar{L}} L^{'}(1) + \lim_{a\to 1} c^{'}(a)\right] = -\infty < 0,$$

because $\lim_{a\to 1} \frac{\partial \pi\left(a,\bar{L}\right)}{\partial a} \to 0$ (recall that $g^{'}(a) \to 0$ when $a \to 1$), $\frac{\partial \pi\left(1,\bar{L}\right)}{\partial \bar{L}} L^{'}(1) > 0$ is a constant and $\lim_{a\to 1} c^{'}(a) = \infty$. We can therefore apply the Theorem of Intermediate Value and conclude that there exists $a(w,\alpha,\beta,p,\delta,\gamma) \in (0,1)$ such that $F(a(w,\alpha,\beta,p,\delta,\gamma)) = 0$. Furthermore, given that $F^{'} < 0$, we have uniqueness.

COMPARATIVE STATICS (PROPOSITIONS 7 AND 8)

Proposition 7: The Implicit Function Theorem gives us the following:

$$\frac{da}{dp} = -\frac{\frac{\partial F}{\partial p}}{F'(a)}.$$

We have already seen that $F^{'}(a) < 0$, such that $sign\left(\frac{da}{dp}\right) = sign\left(\frac{\partial F}{\partial p}\right)$. Thus,

$$\frac{\partial F}{\partial p} = -\left[\frac{\partial^2 \pi}{\partial a \partial p} + \frac{\partial^2 \pi}{\partial \bar{L} \partial p}\bar{L}^{'}(a) + \frac{\partial \pi}{\partial \bar{L}}\frac{\partial \bar{L}^{'}(a)}{\partial p}\right] > 0 \tag{A.4}$$

if $\bar{L}^{'}(a) < 0$ and $\frac{\partial \bar{L}^{'}(a)}{\partial p} < 0$, given that $\frac{\partial \pi}{\partial \bar{L}} > 0$ and

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial a \partial p} &= -g^{'}(a)\left(\frac{\beta p \bar{L}^{\alpha}}{g(a)}\right)^{\frac{1}{1-\beta}}\frac{1}{(1-\beta)p} < 0 \\
\frac{\partial^2 \pi}{\partial a \partial p} &= \frac{\alpha}{\beta}\left(\frac{\beta p \bar{L}^{\alpha}}{g(a)^{\beta}}\right)^{\frac{1}{1-\beta}}\frac{1}{\bar{L}(1-\beta)p} > 0.
\end{aligned}$$

It straightforward to see that, if $\bar{L}^{a} > 0$ is large enough, then the term between brackets in (A.4) positive and $\frac{\partial F}{\partial p} < 0$.

Proof of proposition 8: Given that $\frac{da}{d\gamma} = -\frac{(\partial F)/(\partial \gamma)}{F^{'}(a)}$, we are interested in

$$\begin{aligned}
\frac{\partial F}{\partial \gamma} &= -\left[\frac{\partial^2 \pi}{\partial a \partial \gamma} + \frac{\partial^2 \pi}{\partial \bar{L} \partial \gamma}\bar{L}^{'}(a) + \frac{\partial \pi}{\partial \bar{L}}\frac{\partial \bar{L}^{'}(a)}{\partial \gamma}\right] \\
&= -\frac{\partial \pi}{\partial \bar{L}}\frac{\partial \bar{L}^{'}(a)}{\partial \gamma}
\end{aligned}$$

because $\frac{\partial^2 \pi}{\partial a \partial \gamma} = \frac{\partial^2 \pi}{\partial \bar{L} \partial \gamma} = 0$. Thus, $\frac{da}{d\gamma} > 0$ if and only if $\frac{\partial \bar{L}^{'}(a)}{\partial \gamma} < 0$.