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THE IMPACT OF PRODUCTIVITY AND CAPITAL SHOCKS IN A MACROECONOMIC GENERAL EQUILIBRIUM FRAMEWORK

PORTO ALEGRE
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Dissertação submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Mestre em Economia, área de concentração: Economia do Desenvolvimento.

Orientador: Prof. Dr. Sérgio Marley Modesto Monteiro

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To all those who need to fight daily to be able to
receive the blessing of knowledge.

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“There are things known and there are things unknown, and in between are the doors of perception.”

(HUXLEY, 2010).

RESUMO

Esta pesquisa tem como objetivo analisar como os choques de produtividade e capital afetam uma economia competitiva. Visa a relacionar os resultados de políticas públicas que possuam o objetivo de aumentar a produtividade dos fatores econômicos ou o estoque de capital da economia sobre os principais agregados macroeconômicos. Para isto, simulações foram realizadas a partir um modelo de Ciclos Reais de Negócios (RBC) utilizando o software Dynare. Os resultados encontrados apontam para uma maior capacidade dos choques de produtividade influenciarem positivamente os agregados econômicos em relação aos choques de capital. Os choques de capital parecem ser menos efetivos em aumentar a renda, ao passo em que prejudicam a alocação eficiente de recursos ao distorcerem, de maneira persistente, os preços relativos da economia. Além disso, também foi concluído que a implementação de uma política pública deve levar em consideração a situação atual do ciclo econômico.

Palavras-chave: Políticas públicas. Simulações computacionais. Macroeconomia. Modelos RBC.

ABSTRACT

This research aims to analyze how productivity and capital shocks affect a competitive economy, aiming to relate the results found to a possible effect generated by public policies, that have the objective of increasing the productivity of economic factors or the capital stock of the economy, over the main macroeconomic aggregates. For this, simulations were performed using a Real Business Cycle (RBC) model using the Dynare software. The results found point to a greater capacity of productivity shocks to positively influence economic aggregates in relation to capital shocks. Capital shocks appear to be less effective in increasing income while undermining the efficient allocation of resources by distorting, persistently, the relative prices of the economy. In addition, it was also concluded that the implementation of public policy must take into account the current situation of the economic cycle.

Keywords: Public policy. Computational simulations. Macroeconomics. RBC models.

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LIST OF ACRONYMS

FOC First Order Conditions

RBC Business Cycle

LIST OF SYMBOLS

π Aggregate Profits
ρ Autoregressive parameter
δ Depreciation rate
β Discount factor
α Elasticity of level of production in relation to capital
$\mathbb{E}$ Expected Value
$\forall$ For every
φ Marginal disutility with regard to supply of labor
Relative risk aversion coeficiente
ε Stochastic error component

CONTENTS

1 INTRODUCTION

Since the seminal paper proposed by Kydland and Prescott (1982), the use of exogenous shocks in order to understand the cyclical behave of economic variables becomes the primary tool to understand the impact of different innovations over the macroeconomic aggregates. On the view of Christiano, Eichenbaum and Trabandt (2018), micro-founded models with exogenous shocks become the edge of research of modern macroeconomics, dealing with problems related to a sophisticated set of assumptions that historic or aggregative models cannot deal. This new generation of models used a set of optimization tools like dynamic programming (BELLMAN *et al.,* 1954; STOKEY; LUCAS; PRESCOTT, 1989) to improve the knowledge about business cycles and general economic phenomena.

The use of stochastic innovations (or shocks) also becomes a useful tool to analyze the effects of some exogenous change in models, for theoretical simulations or econometric approaches. Many fields that rely on complex relations behind the decisions of agents can be studied with the help of stochastic shocks. Gambetti and Musso (2017) studied the effects of a set of loan supply shocks over the business cycle in the Euro Area, the United Kingdom, and the United States from 1980 to 2010. Basu and Bundick (2017) used uncertainty shocks in order to understand the stock market volatility and the expectations of agents. Blankeau, Kose, and Yi (2001) tried to understand the effect of an interest rate shock over the business cycle. The use of stochastic shocks as "proxies" to understand the response of the economy to innovation is a primarily used tool in the production of theoretical and econometric models.

Following the idea that stochastic shocks can be a useful tool to understand the net results of some exogenous innovation without depending on complex structures, this research aims to verify the impact of productivity and capital shocks in a competitive economy fluctuations, relating the results found to possible public policy applications that have similar effects. The reason behind this proposition is the fact that public policies are commonly used to interfere in the economy, either by taking actions that increase the level of productivity in the economy (BUCCIROSSI *et al*., 2010; HAVRYLYSHYN, 1990) or by expanding the capital stock available (MUNNELL *et al*., 1990)[1]. The reasons behind the politics, law, or even more complex economic problems can be relieved in order to understand the net impacts by considering these policies (or events) as an exogenous innovation.

This research has its importance given the fact that the acceptance of the use of public policies has become relevant within economic science; however, there are differences regarding the type of applied policy. Analyzing the differences between policies that aim to increase the capital stock and the productivity of the economy can help the development of new policies in the future, as well as serve as a basis for the design of new models and studies in the academy.

The results will be obtained from simulations using a Real Business Cycle (RBC) model following Costa Júnior (2016). Shocks will directly affect capital stock and productivity. The results will be delivered in the form of deviation of the variables at time t concerning their steady-state values. This model tends to deliver considerable approximations at the level of a theoretical construction of general macroeconomic equilibrium, with micro-based equations of behavior. In this way, the results obtained will be directly connected to the robust specifications of the microeconomics regarding the decisions of the agents, which would be a great advantage of this type of model in relation to the purely aggregated models.

In order to solve the maximization problems of the model, the study will apply the Lagrange Multiplier Method following Sundaram *et al.* (1996). Handling and solving non-linear models are generally very arduous. A standard procedure is to log-linearize the model around its steady state. The conversion of the model into a linear approximation will be made using a perturbation method (VILLEMOT *et al.,* 2011), since linear models are often easy to solve, in the way that the solution found can help to understand the behavior of the system at it non-linear form[2].

The output of the model and the shocks will be achieved using the Dynare Software (ADJEMIAN *et al.,* 2011). This computational method can handle a complex

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[1] The role of public investment is commonly defended by economists, however, the impact of fiscal multipliers (BLANCHARD; LEIGH, 2013) and how public budget deficits can affect the agents' decisions (BARRO, 1989) can lead to different recommendations.

[2] It is important to note that, at first, such linearization only allows us to analyze the behavior of the model in a steady-state neighborhood, that is, when the shocks are small.

system of equations in order to solve and give the time path of each variable of the model. The results will be compared in order to achieve if there is a significant difference between the models. The results of productivity and capital shocks will be analyzed in terms of its initial impact and the return time of the variables to the steady-state.

Beyond this introduction, the first chapter will develop the theoretical ideas behind the use of RBC models and stochastic shocks. The second chapter will present the theoretical model following Costa Júnior (2016) specification and adding capital shock. The third chapter will discuss the results of the shocks in the model and the difference between then. Concluding remarks will be made in the end.

2 LITERATURE REVIEW

This section presents the theoretical basis that led to the creation, development, and consolidation of microfounded models in modern macroeconomics. In addition, the relationship between stochastic shocks and macroeconomics will also be presented, as well as their implications for public policy analysis.

2.1 ON RBC MODELS

After the seminal critique made by Lucas *et al.* (1976), the macroeconomic analysis turned out to be more focused on establishing micro-founded models. This paradigm shift was driven mainly by the idea that the prediction of effects of a change in economic policy by using only observed in historical data could lead to wrong conclusions given the presence of not structural (not indifferent to policy) parameters, that would necessarily change whenever policy changes.

In response to that criticism, Kydland and Prescott (1982) created a model to "predict the consequence of a particular policy rule upon the operating characteristics of the economy" (KYDLAND; PRESCOTT, 1982). The Real Business Cycles (RBC) model was the first attempt to solve the problem addressed by Lucas *et al.* (1976). In that model, exogenous shocks change the effectiveness of capital and labor, which, in turn, affects the decisions of agents, who then alter what buy and produce, and then affecting the economy's income.

RBC models play an essential role in helping policymakers understand, openly and transparently, the net effects of economic policy. This kind of model was designed as an optimization-based framework and further opened new possibilities to a different macroeconomic analysis. The transparency comes from the fact that this kind of model is organized in a set of mathematical expressions that give to the agents' specific behavior rules compatible with microeconomics' principles. In the vision of Christiano, Eichenbaum and Trabandt (2018), historical approach and reduced-form methods face some limitations, giving micro-founded models the workhorse status for policy analysis.

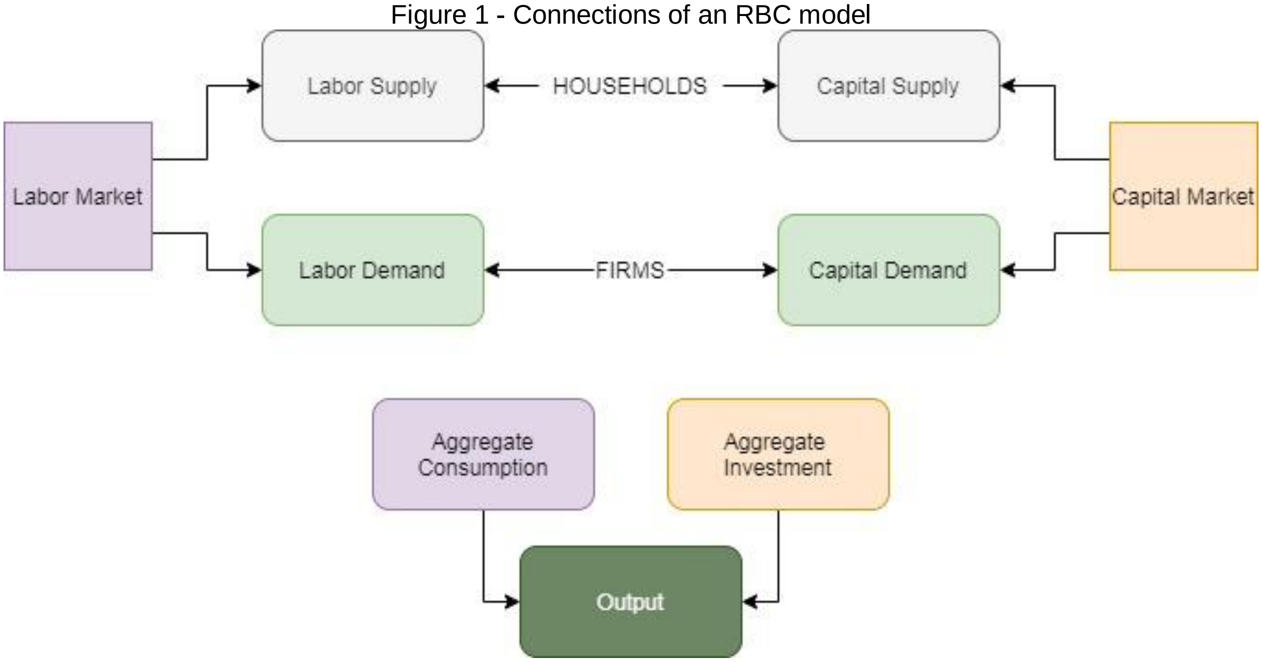
According to Galí (2008), DSGE and RBC models have a similar framework of agents optimizing their objective function subject to some restrictions and exogenous shocks that affect agent's decisions. Beyond that, some New Keynesian key elements distinguish most of DSGE models from the classical RBC models like monopolistic competition, nominal rigidity and short-run non-neutrality of monetary policy. These elements, for New Keynesian economics, resulting in a social welfare gap in the short run, i.e., the economy's response to shocks is generally inefficient. The non-neutrality of monetary policy gives space for a potentially welfare-enhancing intervention in the short run. In the long run, the model's steady-state is similar to an RBC model.

RBC models faced many critiques since the seminal work of Kydland and Prescott (1982) and that critiques lead to new developments like the Dynamic Stochastic General Equilibrium (DSGE) models (CHRISTIANO; EICHENBAUM; TRABANDT, 2018). As pointed out by Friedman (1968), the use of models in order to understand the real world is an efficient way to reduce complex relations to capture only the key variables of a given phenomenon. The use of reduced models should not be viewed as a "wrong explanation," but as an analysis method where humans, with their limited cognition, can understand complex problems of the real world. The use of a standard RBC model can be the right approach in the case that the research subject can be explored under the classical hypotheses of a competitive general equilibrium model. DSGE models feature some New-Keynesian specifications that lead to better match with real data. However, it does not lead to any superior conclusion in the case of purely theoretical analysis, but only reflects results under a set of different theoretical hypothesis.

As described by Stiglitz (2018), a good model must be simple in the form that its mechanisms could be understandable. Crescent complexity in the model can mean a crescent complexity to explain the results. This research will deal with a simple RBC model in order to understand how the economy deals with these shocks under a competitive general equilibrium framework. An example of the fundamental macroeconomic relationship in an RBC model follows two blocks: a representative household and a representative firm make decisions that lead to a connection between the labor and capital markets that generate the aggregated results for the goods markets where aggregate supply equals aggregate demand.

Figure 1 represents, in general, the connections between agents and markets in an RBC model. Families supply labor (when giving up leisure) and capital (when giving up consumption), making an intratemporal choice of their job offer and intertemporal when allocating their disposable income. Firms demand labor and capital from families to carry out the production process, given a specific production technology. The intersection of supply and demand for these inputs generates relationships in the form of a labor market and a capital market. In aggregate form, these relationships will determine what aggregate consumption and aggregate investment will be, which together will determine the final income of this economy.

Figure 1 - Connections of an RBC model



Source: author's own development.
Legend: This figure shows the connections between agents and how market relations are formed.

In the next chapter, these relationships will be further explored in the form of equations that will determine the behavior of agents and how their interactions generate a system in equilibrium. In order to understand the behavior of macroeconomic aggregates, a differential of this model will be to understand the optimal behavior of agents on a microeconomic scale, building a bridge between both approaches.

2.2 STOCHASTIC SHOCKS AND MACROECONOMIC MODELS

The economic relationship behind each variable is a complex problem given the fact that the number of connections between a significant number of variables (including phenomena beyond the economics' scope of analysis). Stochastic shocks can make an experiment viable by just assuming that some variables have a chance to assume a behave given some previously determined probability[3].

Shocks can be understood as different economic phenomena like a government increase in capital stock by using policy mechanisms or growth in productivity after a powerful scientific discovery. King *et al.* (1987) already show that part of the fluctuation and growth trend of the American economy can be explained by using stochastic productivity shocks. As much as the reason for explaining these shocks is not negligible, the ability to progress in the analysis of data and theoretical results without necessarily exploring or endogenizing shocks guarantees stochastic processes and exogenous shocks a position of a relevant scientific tool.

Different studies use the exogenous shock approach for the practical result of different complex phenomena. As previously mentioned, we can mention the work of Gambetti and Musso (2017) who studied the effects of a set of loan supply shocks over the business cycle in the Euro Area, the United Kingdom, and the United States from 1980 to 2010, Basu and Bundick (2017) who used uncertainty shocks in order to understand the stock market volatility and the expectations of agents, and Blankeau, Kose, and Yi (2001) who tried to understand the effect of an interest rate shock over the business cycle.

In a complex scenario where the government must decide to implement some policy, the results of a model with stochastic shocks can be useful in order to enlighten a

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3 Acemoglu (2012) points out several situations where the use of stochastic resources to assess exogenous shocks and regime changes can be used to understand phenomena related to economic growth.

decision of increasing the economy capital stock by the use of government spending or make an efficiency increase program by adopting a flexible position on international trade questions. Even not delivering full information on reasons behind some movement in economic variables, stochastic shocks have an essential role in helping central banks, governments, and firms in their decisions (CHRISTIANO; EICHENBAUM; TRABANDT, 2018).

For authors like Gabisch and Lorenz (2013), stochastic shocks are not a reasonable explanation for economic phenomena like fluctuations. In their view, theoretical constructions that deal with non-linearities can deliver results that are, in fact, explanations for the business fluctuations. On the other hand, the use of stochastic shocks plays a vital role in matching formal theoretical models with the available data.

For RBC models, stochastic shocks, for most cases, use a stationary process, i.e., a stochastic process whose particular joint probability distribution does not change when shifted in time. For this research, both shocks are designed to work with stationarity having mean 0 and constant variance. These specifications are useful in the sense that RBC models are interested in verifying phenomena related to fluctuations toward some trend (in this case, the steady-state of the equilibrium variables).

A random process $\{X_t, t \in j\}$ is stationary if its statistical properties do not change as time pass on. For example, for a stationary process, X_t and X_{t+n} have the same probability distributions:

$$FX_t(X) = FX_{t+n}(X), \forall\ t \mid t \in j$$

More generally, for a stationary process, the joint distribution of $X_{t+1}$and X_{t+2} is the same as the joint distribution of X_{t+n}. For example, if you have a stationary process X_t, then:

$$P\big((X_{t+1}), (X_{t+2}\big) \in A) = P((X_{t+n}) \in A)$$

for any set $A \in \mathbb{R}^2$. A random process is stationary if a time shift does not change its statistical properties.

2.3 SHOCKS AND POLICY IMPLICATIONS

The debate on the use of policies to increase productivity and the capital stock is classic within economics and persists to the present day. Since the seminal work done by Keynes (1936), the role of the State as an agent that can impact the economic cycle and growth is considered, but several interpretations lead economists to support certain types of policies to the detriment of others. As a way of comparing policies with the results of the theoretical model that will be developed in this research, two groups of public policy will be specified:

a) policies that aim to increase productivity: this set of policies aims to increase the productivity of economic factors. As an example of approaches that analyze this impact, we can include the work of Havrylyshyn (1990) that synthesized several results on productivity gains from the commercial opening of developing countries and the study by Buccirossi *et al.* (2010) that points to the existence of gains in productivity from competitiveness policies;
b) policies that aim to increase capital stock: these policies aim to expand the capital stock of the economy, increasing the need for labor and the production capacity of economies. Chang (1993) advocates that the State should act directly in the formation of the capital stock of an economy to guarantee economic growth and sound economic performance. For Kupfer *et al.* (2003), the performance of public policies in the formation of fixed capital to transform the current productive structure (towards a more industrialized economy) must promote the role of industrial policy that aims to increase the stock of productive capital.

Despite being separated into two groups for comparison purposes, public policies may occupy both positions, as is the case of Munnell *et al.* (1990), who shows the role of public investment in infrastructure to increase the productivity of the economy. A point to be made more evident is that the policies addressed tend to be separated in the sense that the productivity policies have this direct objective and the policies to increase the capital stock act directly on the amount of capital (machinery, equipment, and buildings) involved directly in the production process. Another critical point to be mentioned is that the analysis will focus on the impact within the cyclical fluctuation of the economy, given the structure of the model that maintains a constant steady state.

The use of stochastic shocks as a way of processing the impact of public policies in the literature tends to focus on the effects of monetary and fiscal policies. As mentioned above, some papers have a focus on variables like interest rate and the productivity of the factors. The work developed by Gourio (2012) - which was one of the primary sources of inspiration for this work - uses the impact of a fall in the capital stock as a way to simulate the impact of a disaster that affects the production capacity of the economy. Based on this idea, the design of capital shocks was done oppositely, while a productivity shock (more traditional in the literature) was also added for comparison.

The use of a capital shock as a proxy for a policy of increasing the capital stock, as well as the comparison with a productivity shock, inside an RBC model, constitutes the innovation of this work. However, it must be made clear that a policy of increasing the stock of productive capital and increasing productivity has a different implementation time, as well as the costs of implementation.

In the view of Chari, Kehoe and Mcgrattan (2009), some characteristics make a model useful for analyzing the macroeconomic impact of policies. The first, following Lucas *et al.* (1976) critique, points to the need for structural parameters that are invariable to politics (shocks). In this case, the model needs to have a list of parameters that are calibrated so as not to change in the presence of an exogenous innovation. The second characteristic is that the shocks are coherent and have a relevant interpretation. Besides, this work also follows the line proposed by Stiglitz (2018), where the vital point is to try to extract relevant information from a simple model since the presence of many variables and parameters precludes a precise analysis of the mechanisms that lead the shock input to the output of the selected endogenous variables.

3 THEORETICAL MODEL

In this section, a model will be developed in order to understand the impacts of a productive and capital shock in a scenario of a perfectly competitive economy. The model built will follow the specifications of Costa Júnior (2016). However, it will add a capital shock in order to understand the differences in the impacts of each shock[4]. The first step will be constructing the behavior equations, followed by the equilibrium conditions and the steady-state of the model[5]. Given the fact that this research aims to understand the theoretical aspects of each shock, the model construction should be made in order to understand how each equation of the model filters the shocks.

The economy of this model is one of perfect competition, with no wages or price rigidity. There is no government, financial, or external sector. Beyond that, this economy does not have a currency, and there are no adjustment costs.

3.1 HOUSEHOLDS

For this model, the consumption is intertemporally additively separable (no habit formation), population growth is ignored, and the labor market structure is one of perfect competition (no wage rigidity). The representative household Is formed by a unitary set of households indexed by $j \in [0,1]$ whose problem is to maximize an intertemporal welfare function that represents its utility. The utility function chosen can be separated into consumption and labor (with a positive and negative sign, respectively, thus signaling the fact that the function represents the utility of consumption and the disutility of labor. For this purpose, the utility function below will be used:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

[4] The original model can be found in Costa Júnior (2016) Cap. 2.
[5] The appendix at the end of this paper contains the entire resolution of the model.

where E_t is the expectations operator, β is the intertemporal discount factor, C is the consumption of goods, L is the number of hours worked, σ is the relative risk aversion coefficient, and φ is the marginal disutility in respect of labor supply.

This utility function has the first positive derivative in relation to consumption and negative in relation to labor, signaling that a marginal increase in consumption translates into an increase in utility, and the reverse is also true for work. Also, the second derivative is negative for both cases, where the next increment will be less than the previous one.

In this economy, resources are allocated by families that make an intertemporal choice, allocating their labor supply, and deciding between spending on consumer goods or capital goods. To this end, the utility function maximized by the representative household must be subject to a restriction given by a. intertemporal budget constraint. Households receive wages for labor and receive dividends from the capital stock. The equation below determines the equality that links aggregate demand (consumption and investment) with disposable income (wages, return on capital, and dividends):

$$P_t\left(C_{j,t} + I_{j,t}\right) = W_{j,t}J_t + R_tK_{j,t} + \Pi_t \quad (2)$$

where P is the general price level, I is level of investment, W is the level of wages, K is the capital stock, R is the return on capital, and Π is the firms' profit (dividends). Capital accumulation (over time) also needs a specification in the form of a law of motion. For this, the equation below will determine the capital stock in period t + 1:

$$K_{t+1} = (1-\delta)K_{j,t} + I_{j,t}Z_t \quad (3)$$

where δ is the depreciation rate of physical capital and Z_t is the value of capital shock at period t. Below is the stochastic process that governs the capital shock that has a normal distribution with mean 0 and constant variance $\varepsilon_t \sim N(0, \sigma Z)$:

$$\log Z_t = (1-\gamma Z)\log Z_{ss} + \gamma Z \log Z_{t-1} + \varepsilon_t \quad (4)$$

γZ is the autoregressive parameter of productivity[6] and Z_{ss} is the value of exogenous investment at the steady-state.

3.2 FIRMS

The model also has a representative firm that maximizes profit in a perfect competition structure. This means that the profit will be 0 for all periods ($\Pi_t = 0 \; \forall \; t$) and that the marginal cost will always be equal to the marginal revenue ($MgC = MgR \; \forall \; t$). The representative firm will be responsible for producing all the income from the economy using a Cobb-Douglas technology[7]:

$$Y_{j,t} = \mathrm{A}_t K_{j,t}^{\alpha} L_{j,t}^{(1-\alpha)} \tag{5}$$

For this end, $Y_{j,t}$ is the product, and α is the elasticity of the level of production with respect to capital. As in the previous case for the capital shock, the productivity shock follows a similar specification for an autoregressive stochastic process:

$$\log A_t = (1 - \gamma A) \log A_{ss} + \gamma A \log A_{t-1} + \varepsilon_t \tag{6}$$

Where A_{ss} is the value of productivity at the steady state, and γA is the autoregressive parameter of productivity.

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6 The autoregressive parameter of productivity needs an absolute value less than the unit ($|\gamma Z|<1$) to ensure that the process is stationary. The discussion about stationarity was covered in the previous chapter.

7 The use of a production function that has elasticity of substitution between its factors equal to 1 has problems as pointed out by Antrás (2004) and Cantore *et al.* (2015), however, for the purpose of an exercise based on classical hypotheses, the production function of Cobb-Douglas type will facilitate the reader's understanding of the model's mechanisms and facilitate a possible repetition of the exercise by third parties. The existence of alternative ways to specify the production within a model is well known to the author, but it is a simulation exercise for a specific situation, which can be further expanded.

3.3 EQUILIBRIUM CONDITIONS

Competitive equilibrium consists of finding a sequence of endogenous variables in the model such that the conditions that define equilibrium are satisfied. In short, this economy's model is defined by a set of equations that matches the macroeconomic equilibrium condition ($Y_t = C_t + I_t$) given the agents' microeconomic behavior listed in section 2.2.1 and 2.2.2. Table 1 synthesizes the model equilibrium conditions:

Table 1 – Equilibrium conditions of the model

Equation	Definition
$C_{j,t}^{\sigma} L_{j,t}^{\varphi} = \frac{W_t}{P_t}$	Labor Supply
$\left(\frac{\mathbb{E}_t C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta \left[(1-\delta) + \mathbb{E}_t \left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$	Euler Equation
$K_{t+1} = (1-\delta) K_{j,t} + I_{j,t} Z_t$	Law of Motion of Capital
$Y_{j,t} = \mathrm{A}_t K_{j,t}^{\alpha} L_{j,t}^{(1-\alpha)}$	Production Function
$K_t = \alpha \left(\frac{Y_t}{\frac{R_t}{P_t}}\right)$	Demand for Capital
$L_t = (1-\alpha) \left(\frac{Y_t}{\frac{W_t}{P_t}}\right)$	Demand for Labor
$P_t = \frac{1}{A} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t}{\alpha}\right)^{\alpha}$	Price Level
$Y_t = C_t + I_t$	Equilibrium Condition
$\log A_t = (1-\gamma A) \log A_{ss} + \gamma A \log A_{t-1} + \varepsilon_t$	Productivity Shock
$\log Z_t = (1-\gamma Z) \log Z_{ss} + \gamma Z \log Z_{t-1} + \varepsilon_t$	Capital Shock

Source: modified from Costa Júnior (2016).

Model's equilibrium consists of the following blocks: a) a price system W_t, R_t, and P_t; b) an endowment of values for goods and inputs Y_t, C_t, I_t, L_t, and K_t; and c) a production-possibility frontier that connects aggregate supply and aggregate demand.

The next step will be to find the steady-state values, i.e., the value of the variables that are maintained over time and where shocks make the endogenous variables fluctuates around for some t periods.

3.4 STEADY-STATES

Now that the equilibrium has been determined, it will also be necessary to determine the steady-state of the variables, i.e., the value that the variables will assume in the long run. These values, also treated as initials in the simulation process, show the trajectory (in this case, constant) of the variables in the absence of shocks. The steady-states will be a reference to understand how the shock impacts the variables through their deviations from their respective steady-state.

Some variables have their steady-state previously determined given an imposition of some factor that does not allow to find, within the model itself, an equation or value that represents its long-run behavior. The literature is generally assigning the steady-state of productivity as unity ($A_{ss} = 1$) given the fact that it is not possible to know its value at the steady-state (COSTA JÚNIOR, 2016).

Walras' law will be an important step to guarantee the behavior of price variables in steady-state. The next step will be to present Walras' law and specify how it will affect the variables in their respective steady-states. By using Costa Júnior (2016) proof, the proposition 1 (Walras' Law) and his proof can be presented:

Proposition 1 (Walras' Law). *For any price vector p, has $pz(p) \equiv 0$; i.e., the demand excess value is identically zero.*

Proof. In simple terms, the definition of excess demand is written and multiplied by p:

$$pz(p) = p\left[\sum_{i=1}^{n} x_i(p, p\, w_i) - \sum_{i=1}^{n} w_i\right] = \sum_{i=1}^{n} [x_i(p, p\, w_i) - pw_i] = 0$$

since $x_i(p, p\, w_i)$ satisfies the budget constraint $p\, x_i = p\, w_i$ for each individual $i = 1, \dots, n$.

■

In summary, if n-1 markets are in balance, then the n-th market will also be in balance. Walras' law guarantees that, if one market has an excess supply, another market will have an excess demand to balance. In the aggregate, we have that supply equals demand; thus, the aggregate price at steady state is equal to 1 ($P_{ss} = 1$). The system of relative prices can be summarized as:

$$P_{ss} = 1 \tag{7}$$

$$R_{ss} = P_{ss}\left[\left(\frac{1}{\beta}\right) - (1-\delta)\right] \tag{8}$$

$$W_{ss} = (1-\alpha) P_{ss}^{\frac{1}{(1-\alpha)}} \left(\frac{\alpha}{R_{ss}}\right)^{\frac{\alpha}{(1-\alpha)}} \tag{9}$$

To ensure that there is a condition of macroeconomic equilibrium where income equals consumption plus investment, a second proposition is necessary. Also, by using Costa Júnior (2016) proof, we can describe and prove the second proposition:

Proposition 2 (Market Adjustment). *Given k markets, if demand is equal to supply in $k-1$ markets and $pk > 0$, then demand must equal supply in the k^{th} market.*

Proof. If not, **proposition 1** is violated.

■

Starting from the equilibrium conditions found previously, it will also be possible to find steady-state values for consumption, investment and income:

$$C_{ss} = \frac{1}{Y^{\frac{\varphi}{\sigma}}} \left\{ \frac{W_{ss}}{P_{ss}} \left[\frac{\frac{W_{ss}}{P_{ss}}}{(1-\alpha)} \right]^{\varphi} \right\}^{\frac{1}{\sigma}} \tag{10}$$

$$I_{ss} = \left(\frac{\delta\alpha}{R_{ss}}\right) Y_{ss} \tag{11}$$

$$Y_{ss} = \left(\frac{R_{ss}}{R_{ss} - \delta\alpha}\right)^{\frac{\sigma}{\sigma+\varphi}} \left\{ \frac{W_{ss}}{P_{ss}} \left[\frac{\frac{W_{ss}}{P_{ss}}}{(1-\alpha)} \right]^{\varphi} \right\}^{\frac{1}{\sigma}} \tag{12}$$

It is also necessary that the factor market is operating in equilibrium, therefore, it is also important to find the equilibrium values for the supply of capital and labor:

$$K_{ss} = \alpha \frac{Y_{ss}}{\frac{R_{ss}}{P_{ss}}} \tag{13}$$

$$L_{ss} = (1-\alpha) \frac{Y_{ss}}{\frac{W_{ss}}{P_{ss}}} \tag{14}$$

These equations deliver the values for the steady-state given the calibration of the parameters for the simulation. Given the fact that this is a cycle model, these equations will exhibit constant comportment, i.e., there is no growth of these steady-state values. The primary purpose will be to understand how shocks affect the value of equilibrium equations (varying in time t) towards its steady-state values until the shock effects disappear, and the equilibrium equation value equals its steady-state value.

4 SIMULATIONS

Dynare processed all the results available in this research[8]. Dynare is free software that its source code is freely available and that it can be used for both non-profit and profitable purposes. It is available for the Windows, Mac, and Linux platforms and is fully documented by a user guide and reference manual. All the experiments of this research can be easily replicated since the free status of the software used and the detailed model specification in the previous section.

4.1 CALIBRATION AND SIMULATION SETTINGS

The first step will be calibrating the parameters of the model. The list of parameters and their calibration values can be checked in the table 2[9]:

Table 2 – Parameters calibration

Parameter	Parameter Meaning	Value
σ	Relative risk aversion	2
φ	Marginal disutility with regard to supply of labor	1.5
α	Elasticity of level of production in relation to capital	0.35
β	Discount factor	0.985
δ	Depreciation rate	0.025
ρA	Autoregressive parameter - productivity	0.95
σA	Standard deviation of productivity	0.01
ρZ	Autoregressive parameter - capital	0.95
σZ	Standard deviation of capital	0.01

Source: modified by the author from Costa Júnior (2016).

After that, the behavior equations and steady-states will be inputted in the Dynare file to be processed. The software will also make the steady-state values and the linearization process. Given the selected settings, Dynare will linearize the model by

[8] For more information visit Dynare website at http://dynare.org.
[9] All the parameter values follow Costa Júnior (2016) and its pointed as regular values for RBC models. The capital shock follows the same specification as the productivity shock in order to achieve comparable results.

using a perturbation method of order 1. These methods are based on Taylor's expansions and the implicit function theorem[10].

The shocks follow an AR (1) autoregressive process with normal distribution, mean 0 and constant variance $\varepsilon_t \sim N(0, \sigma)$:

$$\log Z_t = (1 - \gamma Z) \log Z_{ss} + \gamma Z \log Z_{t-1} + \varepsilon Z_t$$
$$\log A_t = (1 - \gamma A) \log A_{ss} + \gamma A \log A_{t-1} + \varepsilon A_t$$

The model will be analyzed in order to understand the behavior of 4 key variables: output, consumption, labor, and capital. The purpose is to understand the behavior of those key macroeconomic aggregates under the presence of both shocks. Price components will not be analyzed in these simulations[11]. The model will be simulated in its logarithmic form. Thus, it will be possible to analyze the variables in the form of a percentage deviation from their steady-state.

4.2 RESULTS

The Blanchard-Kahn condition gives a necessary condition for the stability of the system in the neighborhood of steady-state. Following Blanchard and Kahn (1980), a model in which agents are endowed with rational expectation has a unique and stable solution if the number of eigenvalues in the coefficient matrix A (when the model is in its state-space form) is equal to the number of forward-looking variables in the model. When the number of eigenvalues greater than 1 in modulus is less than the number of forward-looking variables, the model has an explosive trajectory and has no solution. For the inverse case, where the number of eigenvalues greater than 1 in modulus is higher than the number of forward-looking variables, the model will have multiple equilibria. Table 3 shows these results for the model presented in the previous chapter[12]:

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[10] For more information see Heer and Maussner (2009).

[11] Given the fact that this model operates under the hypothesis of a competitive economy, the value of price variables will not be the focus of this analysis.

[12] Results also provided by Dynare output.

Table 3 – Eigenvalues of A matrix

Modulus	Real	Imaginary
6.47e-16	6.47e-16	0
0.9614	0.9614	0
1.056	1.056	0
7.912e+16	-7.912e+16	0
0.95	0.95	0

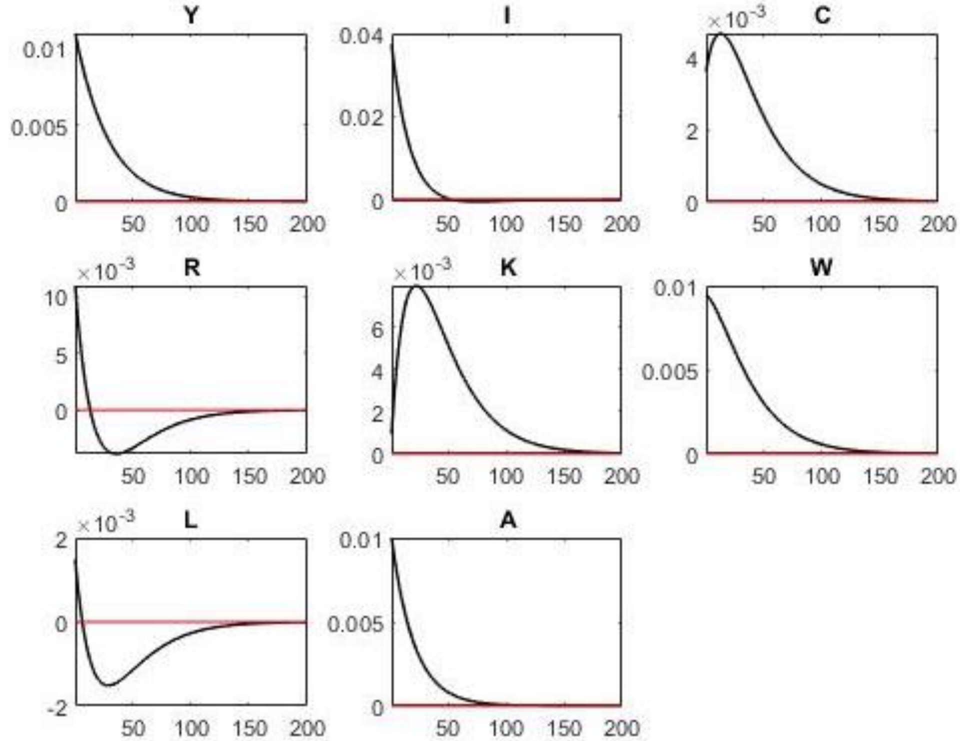
Source: author's own elaboration based on Dynare output.

The model has two eigenvalues larger than 1 in modulus for two forward-looking variables (K_{t+1}.and R_{t+1}). This way, the rank condition is verified, and the Blanchard-Kahn ensures the stability of the model. Beyond that, the shocks of the model present no autocorrelation and can be analyzed separately.

4.2.1 Productivity Shock

The first scenario will analyze the case of a productivity shock. This shock means an increase in Total Factors Productivity (TFP) and acts directly over the production function specification. Figure 2 synthesizes the results of the shock over selected variables:

Figure 2 - Model's output for a productivity shock



Source: Dynare 4.6.0
Legend: Dynare result where the response of endogenous variables is displayed in the presence of a productivity shock.

The first part of this analysis will focus on the model block that delivers the result of income and its components. The first notorious impact of a shock on total factor productivity is an increase in income of a similar magnitude. An increase in productivity makes the model deliver a higher output with the same number of factors of production.

An effect that happens at the same time is the increase in consumption and investment, derived from the increase in income that is now available for agents to consume or invest. Something interesting to note is that the increase in investment is less than the increase in consumption (this is due to the parameters that regulate the operation of the agents as the discount factor). It is essential to understand that the bell-shaped curve for capital and consumption is related to the fact that the capital stock changes one period ahead, given the specified law of motion of capital that changes the capital in period t+1 given a movement in the period t.

As for prices, wages, and return on capital, it is possible to notice an increase in both, but with a sharp drop in the return of capital to a level below the steady-state, followed by a recovery until reaching its long-term value balance. Wages, on the other hand, will fall straight until they reach their steady-state value. It is important to note that the capital stock will change in value with a period of difference, which leads to the effect of the sharp drop in the return on capital after its initial increase.

A plausible explanation for this phenomenon would be an increase in the supply of capital, while labor finds resistance to grow at the same pace in the presence of a substitution effect. This explanation manages to connect the fact that wages are falling more slowly to the steady-state while the return on capital drops sharply below the steady-state and then returns to its long-term value.

Finally, we can see that the behavior of the labor supply rises but falls rapidly below the steady-state level. This is since the substitution effect acts, decreasing the job supply of families. The level of wages does not fall to the same magnitude, given the technical components required for production, which is a structural parameter of the economy. The capital stock behaves with an increase after the shock period given the conditions that govern the behavior of investment in an intertemporal allocation, and the relationship between investment and capital stock that are also not contemporary (current investment is directly linked to the formation of capital stock for the following period).

This result is consistent with the results obtained by Costa Júnior (2016), as well as the productivity shock specifications in RBC type models such as Rios-Rull and Santaeulalia-Llopis (2010) and Lucke (1998)[13]. Being a traditional result within real cycle models, the results of this type of shock end up being useful in the sense of comparing shocks within a similar model.

4.2.2 Capital Shock

Now, in a different case, the model will suffer a disturbance from an investment shock, affecting the capital's law of motion. This shock will directly affect the capital

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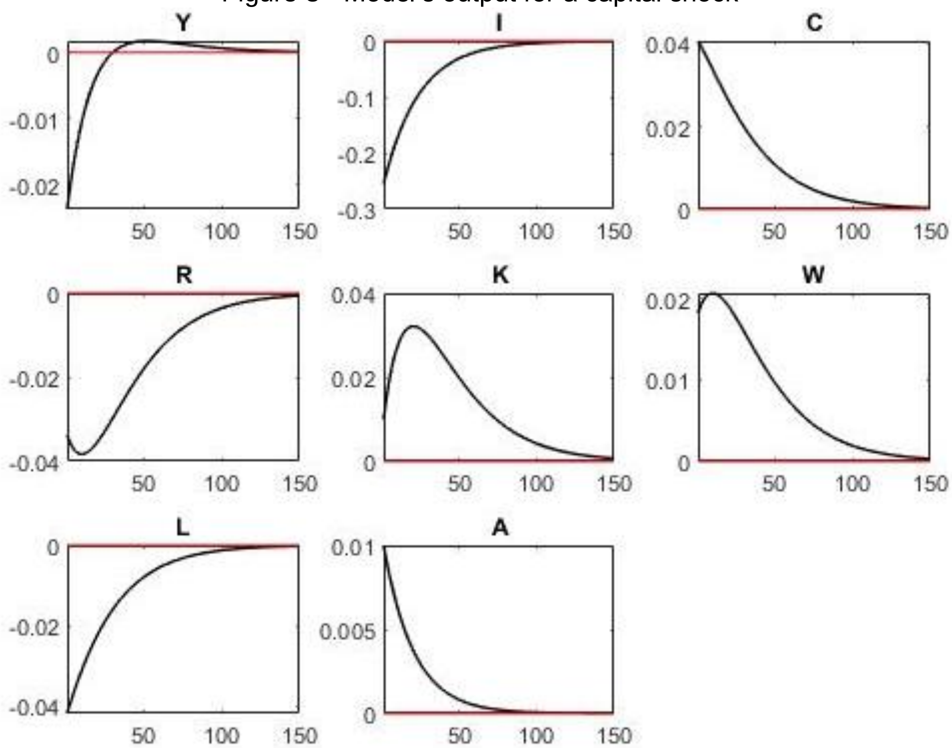
13 Dedola and Neri (2007) also found similar results in an empirical paper for the US economy.

stock in the period t+1. Figure 3 summarizes the results of a capital shock in the main variables of the model:

Figure 3 - Model's output for a capital shock



Source: Dynare 4.6.0
Legend: Dynare result where the response of endogenous variables is displayed in the presence of a capital shock.

The first fact to note is the initial drop in income, followed by a recovery after some periods with the return to steady-state. The shock impacts the agents' income immediately, but the increase in production will only happen in the later period. The presence of the shock leads to a fall in investment, since that, given the intertemporal allocation of resources, an allocation with a substantial drop in return on capital is not feasible (a phenomenon explained by the growth in capital supply). Consumption becomes greater, given the growth in wages since the discrepancy between the amount of capital and labor requires that labor have a higher price to supply the needs of production.

The return on capital falls dramatically, given an abrupt increase in the current capital stock. A capital shock appears to exhibit the behavior of disturbing the prices of factors of production so that wages have a substantial increase while the return on capital will fall sharply. The fall in return on capital is even more pronounced after the shock given the mismatch between investment and the capital generated by the shock. Capital has its upward effect delayed by the effective inflow of capital generated by the shock only in the period t+1. The amount of work falls quite initially due to the drop in production and will only recover later.

It must be taken into accounts, such as the delay in diffusing the effects of the shock in the other variables of the model and the compound effect on relative prices. A capital shock, by changing the amount of capital in the economy will have a severe effect on the return on capital already installed, will cause a drop in the expected return on investment in the future period, and depending on the technical structure of the economy will also change the demand for work drastically.

This analysis, restricted to this model, depends on the values of the parameters previously established. However, it is essential to realize that the result found here, for a competitive economy already signals the potential unbalancing effects that a capital shock has on the relative prices of the economy.

An important fact to be highlighted is that in the presence of hypotheses that move the model away from a competitive structure or insert other hypotheses such as the absence of perfect substitution between the production factors (in the case of a CES production function[14]) or possible variations in the use of installed capacity can generate even more complex results on relative prices.

4.2.3 Comparing Results

Given the simulation for both shocks, the table below summarizes the results for the initial impact of the shocks. These results can be viewed as the numerical result of policy functions in table 4:

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[14] For more information see Arrow *et al*. (1961).

Table 4 – Policy functions

Endogenous Variables	Productivity Shock	Capital Shock
Output (Y)	1.097082	-2.379229
Consumption (C)	0.361844	4.058903
Investment (I)	3.742127	-25.540629
Capital (K)	0.093553	1.000000
Labor (L)	0.149358	-4.198814
Wages (W)	0.947725	1.819585
Capital Return (R)	1.097082	-3.379229

Source: Dynare output.

The most notorious result among those presented is the severely negative impact of the capital shock on investment. As mentioned earlier, a shock to the capital stock has a significant impact on relative prices and the expected return on production factors. On the other hand, by raising the return on both factors, a shock of productivity increases investment. The impact on the increase in income is slightly higher than the original impact of the productivity shock. In contrast, for capital shock, this initial result is negative, given the condition imposed by the time difference between investment and the realization of capital that only happens in the next period.

Consumption, on the other hand, has an impact at a similar level for the productivity shock given the increase in income from the immediate increase in the model's output, while in the case of the capital shock this higher result is due to the loss of future return of the capital, which makes agents choose to consume more and invest less.

Wages raises in the presence of a productivity shock due to a higher demand for factors, for the case of the capital shock, the result is similar given the technical composition of production that will require labor participation. Given the nature of the shock, the increase is more considerable for a capital shock. On the other hand, the result of a capital shock to the return on capital, as mentioned earlier (and which appears to be a pivotal result to explain various movements of the variables in response to the shock), causes a drastic drop in the return on capital, which severely compromises relative prices. For the case of the productivity shock, as well as the wage, the return on capital has an increase due to a higher demand for the capital factor.

The behavior of the capital stock, for both shocks, is as expected: increased by the demand for productivity and increased by the shock itself when impacting the allocation in the current period (this result increases in the following periods). As for labor, the result is consistent in terms of the productivity shock that follows an increase followed by a fall due to the substitution effect but more sophisticated concerning the capital shock. This is a sharp drop with a slow return to the level of equilibrium, even with an increase in wages.

This movement can be explained by the intratemporal allocation given an intertemporal response from the agents, i.e., by having disposable income for a higher level of consumption (and therefore a lower return on this consumption in the form of utility), the agents decide to decrease the work, by while decreasing investment (drastically as previously described) as it is not a rationally viable choice. This result could be changed in the presence of different specifications for the parameters that shape the agents' behavior.

Table 5 below shows the correlation of shocks with the model's endogenous variables, explaining the degree of connection between the shock and the movement of the variables:

Table 5 – Correlations (Shocks – Endogenous Variables)

Variable	Productivity Shock	Capital Shock
Y	0.9837	-0.9165
C	0.8448	0.9486
K	0.6757	0.6757
L	-0.3148	-0.9780
I	0.9788	-0.9963
R	0.2633	-0.8459
W	0.9342	0.8459

Source: Dynare 4.6.0

The degree of correlation seems to follow the explanation given earlier on how the variables are communicating and generating the results in the presence of shocks. Special attention can be given when observing the correlation values for both shocks in the case of the investment and return on capital variables, purposefully placed close together in the table. As explained earlier, the capital shock dramatically, and negatively affects the behavior of investment and return on capital, impairing the functioning of relative prices.

A final factor that must be observed is how each variable feedback its results from its autocorrelation coefficients. Tables 6 and 7 summarizes the autocorrelation of the variables in the presence of shocks in up to 5 orders:

Table 6 – Coefficients of Autocorrelation – Productivity Shock

	1	2	3	4	5
Y	0.9656	0.9322	0.9000	0.8688	0.8387
I	0.9333	0.8705	0.8115	0.7560	0.7039
C	0.9921	0.9830	0.9727	0.9615	0.9494
R	0.9388	0.8811	0.8266	0.7753	0.7270
K	0.9990	0.9961	0.9916	0.9856	0.9781
W	0.9801	0.9600	0.9398	0.9196	0.8993
L	0.9859	0.9712	0.9559	0.9400	0.9237
A	0.9500	0.9025	0.8574	0.8145	0.7738

Source: Dynare 4.6.0

Table 7 – Coefficients of Autocorrelation – Capital Shock

	1	2	3	4	5
Y	0.9203	0.8458	0.7761	0.7109	0.6500
I	0.9574	0.9167	0.8777	0.8404	0.8047
C	0.9769	0.9540	0.9312	0.9086	0.8861
R	0.9920	0.9828	0.9725	0.9612	0.9490
K	0.9990	0.9961	0.9916	0.9856	0.9781
W	0.9920	0.9828	0.9725	0.9612	0.9490
L	0.9680	0.9369	0.9067	0.8774	0.8489
Z	0.9500	0.9025	0.8574	0.8145	0.7738

Source: Dynare 4.6.0.

By comparing tables 7 and 8 results, income has a faster return to its steady-state in relation to the displacement of the same variable under a productivity shock, that is, the effect of a productivity shock over aggregate income is more persistent. Consumption is also more persistent for productivity shock. The investment is more lasting in the event of a capital shock. However, the effect on it is negative. It is possible to observe an evident persistence of the relative prices, R and W, in returning to their equilibrium values in the presence of a capital shock, while capital K returns more slowly to its equilibrium level while labor L has a faster return, in this case, unbalanced. The capital stock behaves similarly for both shocks, as does labor. However, it is also important to remember the signal that the variables assume in the presence of each shock, that is, the persistent effects have a much more significant initial negative impact for the capital shock.

5 CONCLUDING REMARKS

This research evaluates the impact of productivity and capital shocks in a competitive economy using a simple RBC model. The shocks were used as a comparison parameter for public policies that try to increase the productivity of factors or to increase the level of capital stock in the economy.

At first, a theoretical review of RBC-type models, the role of stochastic shocks in macroeconomic models, and the use of shocks to assess public policies were presented. In the second step, the theoretical model used to perform the simulations was presented. In a third moment, the results of the simulations were presented, showing the shocks individually and then comparing them.

As intended, the results demonstrated how the different shocks impact the other economic variables in a competitive economy, also managing to choose which shock, as public policy, seems to have the highest potential to trigger positive effects on an economy's business cycle. The results found point to a higher capacity for productivity shocks (also directly related to public policy cases that aim to increase the total productivity of factors) in improving the results of macroeconomic variables in a competitive economy. In contrast, a capital shock it has lesser effects on income and consumption while managing to create severe distortions on relative prices, hampering the allocation of resources in the economy.

This research also concluded that the timing of implementation of public policies matters and that each policy can impact the economic cycle differently. Therefore, it is crucial that the design of public policies takes into account the current moment of the business cycle and how the chosen policy will interact with the current state of the economy.

It is important to realize that this exercise was a theoretical analysis with calibrated parameters, so the effectiveness of the results can be questioned in the presence of different settings. A refinement of this work could capture the effects of shocks in the presence of different parameters, observing the behavior of shocks given the changes.

Another point is the absence of friction in the markets. The DSGE models (which were the evolution of the RBC models) have price and wage rigidity in order to corroborate the empirical evidence. The main point of excluding this artifice in this exercise is the fact that this inclusion, as is usually done on DSGE models, does not have a solid micro-foundation, It is only an *ad hoc* way of including New-Keynesian rigidity within a model that tries to express itself as a general equilibrium structure[15]. Advancing towards the inclusion can provide interesting results, but this would not be the original proposal of this work.

A profound criticism of these results is pointed out by the works that report the role of aggregation in macroeconomic models, as mentioned by Sonnenschein (1972) and Debreu (1974). Thus, the results provided here can, in some way, be questioned from a theoretical point of view within the general equilibrium theory. Furthermore, as pointed out by Landini, Gallegati and Rosser (2018), there are consistency problems as indicated by Gödel-Rosser theorems. Thus, an analysis of this type of object could generate more plausible results by considering heterogeneous interacting agents and scaling laws (GATTI *et al*., 2005).

It is the role of economics as a science to study economic phenomena using the most varied methods, always pointing out possible problems in the methodologies used. In this sense, future work should follow two distinct lines: a first centered on incorporating the frictions offered by modern DSGE models (prices, wages, financial market); and a second approach centered on models that observe the economy as a complex system in constant evolution.

[15] Following the spirit of Rogers (2018), essential theoretical results cannot be effectively addressed by frictions added to the model. The substitution of the model, in the face of difficulties in framing the results with the data, can be a more effective way to explain the empirical data.

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APPENDIX A – MODEL'S SOLUTION

By using the Lagrange Multiplier method (SUNDARAM *et al*., 1996), the solution of the household's problem will be given by solving:

$$L = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right] - \lambda_{j,t} \left[P_t C_{j,t} + P_t K_{j,t+1} - P_t (1-\delta) K_{j,t} - W_t L_{j,t} - R_t K_{j,t} - \Pi_t \right] \right\}$$

Where λ is the Lagrange multiplier. First Order Conditions (FOC) will be:

$$\frac{\partial L}{\partial C_{j,t}} = C_{j,t}^{-\sigma} - \lambda_{j,t} P_t = 0$$

$$\frac{\partial L}{\partial L_{j,t}} = -L_{j,t}^{\varphi} + \lambda_{j,t} W_t = 0$$

$$\frac{\partial L}{\partial K_{j,t+1}} = -\lambda_{j,t} P_t + \beta \mathbb{E}_t \lambda_{j,t+1} [(1-\delta) E_t P_{t+1} + \mathbb{E}_t R_{t+1}] = 0$$

By solving the FOC obtained previously for λ_t, the household labor supply equation can be found:

$$C_{j,t}^{\sigma} L_{j,t}^{\varphi} = \frac{W_t}{P_t}$$

Euler's equation will be given by:

$$\left(\frac{\mathbb{E}_t C_{j,t+1}}{C_{j,t}} \right)^{\sigma} = \beta \left[(1-\delta) + \mathbb{E}_t \left(\frac{R_{t+1}}{P_{t+1}} \right) \right]$$

The representative firm also solves an optimization problem:

$$\max_{L_{j,t}K_{j,t}} \Pi_{j,t} = \left[A_t K_{j,t}^{\alpha} L_{j,t}^{(1-\alpha)}\right] P_{j,t} - W_t L_{j,t} - R_t K_{j,t}$$

With the FOC:

$$\frac{d\Pi}{dK_{j,t}} = \alpha A_t K_{j,t}^{\alpha-1} L_{j,t}^{(1-\alpha)} P_{j,t} - R_t = 0$$

$$\frac{d\Pi}{dL_{j,t}} = (1-\alpha) A_t K_{j,t}^{\alpha} L_{j,t}^{-\alpha} P_{j,t} - W_t = 0$$

By reordering, we find the demand for inputs in which marginal costs are equal to marginal products:

$$\frac{R_t}{P_{j,t}} = \alpha \frac{Y_{j,t}}{K_{j,t}}$$

$$\frac{W_t}{P_t} = (1-\alpha) \frac{Y_{j,t}}{W_{j,t}}$$

As the model follows the RBC approach, the price level must be equal to marginal cost. In order to obtain the marginal cost, the previous equations must be combined to find the equilibrium for both conditions:

$$\frac{W_t}{R_t} = \frac{(1-\alpha) K_{j,t}}{\alpha L_{j,t}}$$

The left-hand side is the economic rate of substitution, which measures the rate at which labor can be replaced by capital while maintaining the same cost. By rearranging previous expression for labor:

$$L_{j,t} = \left(\frac{1-\alpha}{\alpha}\right)\frac{R_t}{W_t}K_{j,t}$$

Then substituting in the production function and finding the capital stock in time t:

$$Y_{j,t} = \mathrm{A}_t K_{j,t}^{\alpha}\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{R_t}{W_t}K_t\right]^{(1-\alpha)}$$

$$K_{j,t} = \frac{Y_{j,t}}{A_t}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_t}\right]^{(1-\alpha)}$$

And substituting in the labor equation once more:

$$L_{j,t} = \frac{Y_{j,t}}{A_t}\left(\frac{1-\alpha}{\alpha}\right)\frac{R_t}{W_t}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_t}\right]^{(1-\alpha)}$$

$$L_{j,t} = \frac{A_t}{Y_{j,t}}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_t}\right]^{-\alpha}$$

total cost (TC) is represented by:

$$TC_{j,t} = W_t L_{j,t} + R_t K_{j,t}$$

By substituting $L_{j,t}$ and $K_{j,t}$:

$$TC_{j,t} = W_t\frac{Y_{j,t}}{A_t}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_t}\right]^{-\alpha)} + R_t\frac{Y_{j,t}}{A_t}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_t}\right]^{(1-\alpha)}$$

$$TC_{j,t} = \frac{Y_{j,t}}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$

Marginal cost is derived from total cost:

$$MC_{j,t} = \frac{\partial TC_{j,t}}{\partial Y_{j,t}} = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$

Given the fact that this model follows a competitive economy hypothesis, all firms are under the same production conditions, in this way, the general price level equals the marginal cost:

$$P_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$