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MARKOV SWITCHING GAS COPULA MODEL

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Dissertação submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Mestre em Economia.

Orientador: Prof. Dr. Flávio Augusto Ziegelmann

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“No! Try not! Do or do not, there is no try”

(Master Yoda)

RESUMO

Análise da dependência entre ativos e suas aplicações a gerência de risco têm ganhado muita tração na pesquisa em finanças empíricas. Nesta dissertação propomos a nova abordagem para capturar mudança de regime na dependencia. nós mostramos com experimentos numéricos de Monte carlo que nossa abordagem é robusta a má especificação além de superar os outros modelos presentes na literatura. Além disso utilizamos nosso modelo para analisar dados de índices europeus, especificamente os índices de mercado FTSE100(UK), DAX(GER), CAC40(FRA) e BEL20(BEL). Nós mostramos que há evidencia de mudança significativa após 2016, diminuindo a dependencia de cauda inferior entre alguns pares de ativos podendo indicar um exito do BREXIT, ao menos do ponto de vista de expectativas, distanciar o Reino Unido de choques negativos europeus.

Palavras-chaves:Mudança de Regime. GAS. Copulas. Dependência.

ABSTRACT

Dependence modelling and its applications to risk management have recently gained much ground as a field of research in empirical finance. In this paper we propose a new approach for capturing dependence regime switching via copulas, and show through numerical Monte Carlo simulation exercises that our approach is quite robust to model mispecification. Furthermore we employ our model in the analysis of European financial data, specifically the European market indexes FTSE100(UK), DAX(GER), CAC40(FRA) and BEL20(BEL). We show that there is evidence of a significant change in early 2016, when there is a diminished lower tail dependence which could be associated with the BREXIT referendum.

Keywords:Markov-Switching. GAS. Copulas. Dependence.

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1 INTRODUÇÃO

A enciclopédia de filosofia de Stanford apresenta diversas definições de risco e entre outras coisas descreve o conceito de risco da forma: “ The relationship between the two concepts of “risk” and “uncertainty” seems to be in part analagous to that between “truth” and “belief”.” Esta analogia é especialmente interessante quando lidamos com risco no âmbito de mercados financeiros. De forma mais precisa esta analogia é interessante pois ela induz o leitor a percepção de que medidas de risco, como um conceito, devem ser quantificáveis de alguma forma.

Uma das aplicações mais comuns do conceito de risco quantificável foi apresentada por Markowitz (MARKOWITZ, 1952) onde uma forma estruturada de alocar ativos foi descrita. Esta forma nada mais é que um balanceamento entre risco e retorno esperado. Em seu trabalho, Markowitz utiliza do desvio padrão do retorno dos ativos como uma aproximação do nível associado de risco e com isso permitiu o desenvolvimento de um novo campo de pensamento em finanças empíricas que lida com otimização de portfolios. Além disso este trabalho também inicia a avaliação da dependência entre ativos e o risco do portfolio. A estrutura apresentada permitiu uma explicação matemática da percepção empírica de que a diversificação em investimentos possuía um efeito positivo e era portanto desejada. Além disso permitiu uma perspectiva com mais nuance das condições necessárias para que a diversificação tenha de fato efeito positivo.

Desta forma o conceito de dependência entre ativos foi introduzido na conversa de balanceamento do trade-off entre risco e retorno. A dependência pode portanto ser vista como uma fonte de risco ou, por outro lado, uma oportunidade de diversificação. Esta percepção incentivou a criação de diversas medidas de dependência permitindo, com cada uma, o melhor entendimento da intuição que segue o conceito de risco.

Além disso, devido a explosão tecnológica dos últimos 50 anos mercados financeiros tem se tornado cada vez mais integrados e seu comportamento mais ativo no sentido de responder a notícias e efeitos de mercado. Este aumento implicou na necessidade de uma forma dinâmica de observar a dinâmica da dependência. O principal desafio deste tipo de modelo reside na característica da dependência ser naturalmente não observável. A Solução para este problema foi, até certo ponto, redescoberta por Patton (PATTON, 2006) que adaptou o modelo matemático proposto por Sklar(SKLAR, 1959) para um cenário dinâmico.

A solução apresentada residia em utilizar funções chamadas Cópulas, a partir das quais consegue-se construir uma função de verossimilhança para modelagem dos parâmetros de dependência e além disso estas funções ainda efetuam a união de distribuições univariadas em distribuições multivariadas. Esta contribuição possuiu um efeito ainda maior devido a crescente literatura de medidas de risco, isto pois não só medidas práticas de risco mas a maior parte da construção axiomática de análise de risco, veja Föllmer et al.(FÖLLMER; SCHIED, 2002), se baseia no uso das funções de distribuição para definir medidas de risco. Portanto o aspecto, de forma ingênua, acidental do uso das funções de copula conseguiu mais uma vez trazer a fronteira do pensamento de risco e dependência para uma mesma análise.

Mais formalmente Patton (PATTON, 2006) introduziu o uso de cópulas para modelar a dinâmica da dependência entre taxas de câmbio. O apelo do uso de modelos com copulas tem a ver com a simplicidade de seu uso. Elas permitem separar distribuições marginais de sua estrutura de dependência permitindo uma estimação por máxima verossimilhança relativamente simples. Outro fato importante é que, ao contrário de outros modelos multivariados, o uso de copulas permite modelar dependência de ordem superior, permitindo comportamento dinâmico para medidas como dependência de cauda que podem, para analistas de risco, ser muito mais informativas.

Apesar desta grande contribuição feita por Patton, o entendimento do comportamento dinâmico da dependência ainda é um desafio no âmbito acadêmico. Diversos modelos têm sido apresentados para capturar comportamentos diferentes deste componente. Chollete, Heinen and Valdesogo (CHOLLETE; HEINEN; VALDESOGO, 2009) usam uma mudança de regime markoviano mudando a estrutura da copula entre os regimes para um grande número de índices financeiros. Creal, Koopman and Lucas (CREAL; KOOPMAN; LUCAS, 2013) sugerem o uso de um componente de score auto regressivo (Generalized Autoregressive Score) para ser o determinante da dinâmica da dependência, este modelo foi ainda descrito em exaustão em Patton (PATTON et al., 2012) e posteriormente desenvolvido em um modelo para alta frequência em Salvatierra and Patton(PATTON et al., 2012) que usa da correlação realizada intradiária. Silva Filho, Ziegelmann and Dueker (FILHO; ZIEGELMANN; DUEKER, 2012) combinam a especificação dinâmica de Patton (PATTON, 2006) com uma mudança de regime markoviano para o intercepto da equação de dinamica para a dependência, e Bartels and Ziegelmann (BARTELS; ZIEGELMANN, 2016) utilizam a estrutura GAS em portfolios de alta dimensão para obter diversas medidas de risco.

Desta forma é fácil perceber que o campo de pesquisa que desenvolve este tipo de modelos é repleto de inovações e tem crescido muito nos últimos 10 anos. Portanto a proposta deste trabalho é de contribuir ao presente corpo de pesquisa de duas formas, a primeira contribuição é na estrutura do modelo, nosso modelo propõe combinar o GAS de Creal, Koopman and Lucas (CREAL; KOOPMAN; LUCAS, 2013) como motor a dependência com a abordagem de mudança de regime na dependência proposta por Silva Filho, Ziegelmann and Dueker (FILHO; ZIEGELMANN; DUEKER, 2012). Como Creal et al. demonstrou a utilização dos modelos GAS permite melhor aderência que o modelo ad hoc proposto por Patton (PATTON, 2006) que é o mesmo que Silva Filho, Ziegelmann and Dueker (FILHO; ZIEGELMANN; DUEKER, 2012) utilizaram para sua dinâmica. Portanto esperamos que nosso modelo seja um bom substituto não só por ter melhor desempenho mas também por possuir maior apelo intuitivo. A segunda contribuição deste trabalho será apresentada num exercício empírico onde o modelo será utilizado para analisar três conjuntos de dados distintos. Além disto a metodologia utilizada é descrita de forma detalhada no capitulo 2 que apresenta um artigo completo com a introdução, descrição detalhada do modelo proposto juntamente dos resultados obtidos.

2 MARKOV SWITCHING GAS COPULA MODEL

Stanford encyclopedia of philosophy presents us several useful definitions of risk and, among other things, it states that the relationship between the two concepts of " risk" and " uncertainty " seems to be in part analogous to that between " truth" and "belief". This analogy is specially informative when dealing with the financial interpretation of risk, as it speaks to the fact that risk must be observable or quantifiable while on the other hand uncertainty has a broader interpretation and does not require any quantification, uncertainty is simply present in the understanding of economic behaviour.

One of the most lasting applications of the concept of quantifiable risk is present in Markowitz (MARKOWITZ, 1952) where a structured way of balancing risk and reward was devised. In his seminal paper Markowitz used the standard deviation of returns as a quantifiable proxy for risk and managed to introduce a new field of financial thought regarding portfolio optimization. What was also present in his paper was the relationship between assets dependence and the portfolio risk. His theory managed to circumscribe the empirical perception that investment diversification was a positive thing, and also helped determine in which instances diversification led to further benefits.

This work helped introduce the concept of dependence between assets into the conversation of risk reward trade-offs. Dependence can therefore be seen as a source of risk or, in some other sense, as an opportunity for diversification. This awareness of the importance of dependence between assets provided an additional incentive to the creation of several different measures of dependence carrying with them different intuitions into the meaning of financial risk.

However, due to the explosive technological increase in the last 50 years, markets have become more and more integrated, and their behavior even more responsive to sudden changes in perception. This increase also led to the necessity of an understanding of the way dependence dynamics behave. The issue was that of modeling the dynamics of a purely unobervable component. Patton (PATTON, 2006) proposed then one attractive solution to this problem by adapting Sklar's (SKLAR, 1959) copula framework to a dynamic setting.

More formally Patton (PATTON, 2006) introduced the use of copulas to model the dynamics of asymmetric dependence structures between exchange rates. The appeal of modeling dependence with copulas has much to do with the fact that their use is somewhat straightforward and by separating marginal distributions from dependence structure they allow for estimation using maximum likelihood. It is also worth noting that unlike other usual models of multivariate series, copulas allow for the presence of dependence of a higher order. Therefore these models not only capture linear correlation but also tail dependence which is a deeper measure that risk analysts take as much more informative, see Embrechts et al. (EMBRECHTS; MCNEIL; STRAUMANN, 2002).

Even with the additional generality brought in the dynamic behavior of the dependence is still a challenge to be reckoned with. Many different models have been proposed by the current literature in order to capture different potential behavior. Patton (PATTON, 2006) incorporates some Ad Hoc drivers to the dependence dynamics acknowledging in his work that there was still a lack of precise meaning to these driving forces. Chollete, Heinen and Valdesogo (CHOLLETE; HEINEN; VALDESOGO, 2009) used a Markov switching regime change to define two different regimes for normal copulas allowing for fluctuation in the dependence structure in a large group of financial indexes. Creal, Koopman and Lucas (CREAL; KOOPMAN; LUCAS, 2013) suggest the use of a Generalized autoregressive score (GAS) in order to incorporate some dynamics to the dependence parameter, this model is then exhaustively described in Patton (PATTON et al., 2012) and later developed in Salvatierra and Patton (SALVATIERRA; PATTON, 2015) into a high frequency model using realized correlation. Silva Filho, Ziegelmann and Dueker (FILHO; ZIEGELMANN; DUEKER, 2012) combines the dependence dynamic specification of Patton (PATTON, 2006) with a markov switching process for the intercept of the parameter dynamics equation. Bartels and Ziegelmann (BARTELS; ZIEGELMANN, 2016) uses the GAS structure for the dynamics dependence in high dimension portfolios to obtain various measures of risk.

Our objective is then to combine some of the strengths of the previously offered models. Creal, Koopman and Lucas (CREAL; KOOPMAN; LUCAS, 2013) model is by far the more intuitive in terms of the dynamic driver, so in our proposed model we combine this specification with a markov switching parameter that allow us to capture discrete swifts in the dependence structure. We show that our model outperforms the model presented in Silva Filho, Ziegelmann and Dueker (FILHO; ZIEGELMANN; DUEKER, 2012) in all instances. When compared to non regime changing models of Creal et al.(CREAL; KOOPMAN; LUCAS, 2013) and Patton (PATTON, 2006) our model works better when there is regime change, as expected, and in the absence of regime change our model collapses to Creal's and their performance follow the same pattern. Finally we use this model to show that there was a change in regime associated with the Brexit referendum, in contrast to the results presented by Aristeidis (ARISTEIDIS; ELIAS, 2017).

The paper is then organized in three additional sections. One describing all of the econometric tools used in the construction of our model, namely the econometric model section. One section presenting the hypothesis and results of the simulation exercise, containing the main results. One section with an empirical application of our model to European financial indexes with an analysis of the potential implications of the regime change making for some interesting results as well. In the last section we present the concluding remarks.

2.1 Econometric Model

This section will be divided in two parts, the first goes through some of the basics of copula theory setting the tools and framework we will use henceforth. In the second part we present our proposed approach, discuss some of it's expected properties and put it in perspective by showing it's relation to other models in the literature.

2.1.1 Copula basics

Before explaining how Copula theory works with dependence modelling we must first define what these so called “copulas"are. Copulas are mathematical functions that can connect univariate marginal distributions in order to obtain a multivariate distribution. One way to look at it is that any multivariate distribution can be decomposed as a copula function applied to the specified marginal functions. This result, known as Sklar's Theorem(SKLAR, 1959) can be formally stated as

$$\begin{aligned} Y &\equiv [Y_1, \dots, Y_n]' \sim F, \quad Y_i \sim F_i \\ \exists\, C &: [0,1]^n \to [0,1] \\ F(y) &= C(F_1(y_1), \dots, F_n(y_n)) \quad \forall y \in \Re^n \end{aligned} \tag{2.1}$$

where Y is a n-dimensional random variable, with cumulative distribution function denoted by F.Then, for $i = 1, \dots, n$, F_i is the cumulative distribution function of the univariate variable Y_i, the lower case y_i represents the realization of the variable Y_i and C denotes the copula function. As one can see, a copula function can be interpreted as a map connecting the univariate marginal distributions F_i to the multivariate distribution F.

If we define U_i as the probability integral transform, that is $U_i = F_i(y_i)$,then $U_i \sim Uniform(0,1)$. From this we can reinterpret equation one for only two variables as:

$$\begin{aligned} Y &\equiv [Y_1, Y_2]' \sim F, \quad Y_i \sim F_i \\ \exists C &: [0,1]^2 \to [0,1] \\ F(y) &= C(U_1, U_2) \quad \forall y \in \Re^2. \end{aligned} \tag{2.2}$$

The definition in Sklar's theorem also implies that there should be an analogous version for the marginal densities, described as

$$\begin{aligned} f(y_1, \dots, y_n) &= c(F_1(y_1), \dots, F_n(y_n)) \times \textstyle\prod_{i=1}^n f_i(y_i) \\ c(u_1, \dots, u_n) &= \frac{\partial^n C(u_1, \dots, u_n)}{\partial u_1 \dots \partial u_n}, \end{aligned} \tag{2.3}$$

where, for $i = 1,\dots,n$, f_i is the marginal probability distribution function for the variable y_i. Bringing some intuition to this, if we were to fix $c(F_1(y_1),\dots,F_n(y_n)) = 1$ we would obtain the structure of independent marginal distributions that can be separated as a product of the marginals. The copula structure then enters this framework as an additional term multiplying the separated marginals. From this structure it is also natural to expect, as we will detail later on, that the structure of the marginals and copula could be separated to make for a more tractable estimation structure.

Now for our purposes we will specify the functional form of the copula we will use. With the aim of being parsimonious our choice is to use the Symmetrized Joe Clayton copula. Before describing it wee must first describe the Joe Clayton copula, given as

$$
\begin{aligned}
&C_{jc}(u,v;\tau^U,\tau^L) = 1-(1-[(1-u^\theta)^{-\delta}+(1-v^\theta)^{-\delta}-1]^{-\frac{1}{\delta}})^{\frac{1}{\theta}}\\
&\theta = \frac{1}{\log_2(2-\tau^U)}\\
&\delta = \frac{-1}{\log_2(\tau^L)},\ and\ \tau^U,\tau^L \in (0,1),
\end{aligned}
$$

the Symmetrized Joe Clayton is obtained from a static mixture of this Joe Clayton Copula, given as

$$
C_{sjc}(u,v;\tau^U,\tau^L) = 0.5(C_{jc}(u,v;\tau^U,\tau^L)+C_{jc}(1-u,1-v;\tau^U,\tau^L)+u+v-1).
$$

This form has both upper and lower tail dependence given by the parameters τ^U and τ^L, and when the parameters are equal it has a symmetric dependence structure. These tail dependence parameters are a interesting measure in term of risk management since the lower(upper) tail dependence represents the probability of having simultaneously low(high) values in terms of the quantiles. More formally the tail dependences can be described in the copula context as:

- Lower tail
  $\lim_{\varepsilon\to 0} Pr[U \le \varepsilon | V \le \varepsilon] = \lim_{\varepsilon\to 0} Pr[V \le \varepsilon | U \le \varepsilon] = \lim_{\varepsilon\to 0} \frac{C(\varepsilon,\varepsilon)}{\varepsilon} = \tau^L$
  The copula presents lower tail dependence if the above limits exist and the lower tail dependence is denoted as τ^L
- Upper Tail
  $\lim_{\varepsilon\to 1} Pr[U > \varepsilon | V > \varepsilon] = \lim_{\varepsilon\to 1} Pr[V > \varepsilon | U > \varepsilon] = \lim_{\varepsilon\to 1} \frac{1-2\varepsilon+C(\varepsilon,\varepsilon)}{1-\varepsilon} = \tau^L$
  The copula presents upper tail dependence if the above limits exist and the upper tail dependence is denoted as τ^U

The use of this structure is therefore very convenient insofar as it allows for a greater variety of dependence structures. With this information we are able to piece together the framework to look at dependence parameters in a dynamic setting. The following section will elucidate the transition from static to dynamic dependence and introduce our proposed model. n

2.1.2 Dynamic copulas: Markov Switching Gas Proposal

It is clear that the previous representation dealt with an exclusively static framework, there is no time index in any variable. Interestingly from this structure since we can construct the multivariate marginal densities, then Equation 3 could, in the right framework, provide a functional form for a likelihood function. Patton (PATTON, 2006) presents this framework adapting all of these previous theorems to a conditional form and elaborating on the conditions necessary for this process to work.

Joe (JOE, 2005) had already showed that the structure in Equation 3 in fact could also be seen as a opportunity for separating the marginal modelling and the dependence modelling, and proved in this paper that the 2-step process was asymptotically efficient. Therefore Joe's contribution paved the way so that Patton(PATTON, 2006) could isolate the two processes and model the marginal dynamics as is usual in financial theory. By having this structure in place we are now able to model the evolution of the dependence and use standard likelihood maximization to get the parameters for the evolution's equation.

The only caveat is that the dynamic structure for the dependence has to be specified in order for maximum likelihood estimation to take place. This raises the question of which model to use to describe the time dynamics of a parameter that is, by its very nature, unobservable. Much of the literature that followed from this work tried to explore different specifications and argued why their models managed to capture certain stylized facts. We will now present our way to do so and subsequently we will make note of the differences and parallels between our proposed specification and those previously used in economic modelling literature.

Our proposed dynamic behavior is described by

$$\lambda_t = \Lambda(w_{m_t} + \phi s_t + \alpha \lambda_{t-1}), \tag{2.4}$$

where Λ is an adequate transformation, w_{m_t} is a level parameter that follows an underlying hidden markov chain, s_t is the score function, and λ_t is the dependence parameter in question. To make sense of this notation we will describe all of these components below, only we will show them in a reverse order.

The third element is λ_{t-1}, it incorporates an auto regressive dynamic to the dependence, where α is an auto regressive coefficient and λ_{t-1} is the dependence parameter in the previous instant. This particular component was used in Patton (PATTON, 2006),Creal et al. (CREAL; KOOPMAN; LUCAS, 2013) and Silva Filho et al. (FILHO; ZIEGELMANN; DUEKER, 2012) and in many other empirical papers. The auto regressive structure should be stationary which would make the α bounded in $(-1,1)$. The trait we intend to capture in using an autoregressive component is that the dependence should be somewhat dependent in its past values and that shocks to it should have an effect on future values and this effect should decrease as time goes on.

The second element is s_t, it is a component proposed by Creal, Koopman and Lucas (CREAL; KOOPMAN; LUCAS, 2013) where s_t is called a Score component. This score is a useful interpretation of the point wise derivative of the likelihood function. Formally it can be described as

$$s_t = S_t \bigtriangledown_t, \quad \bigtriangledown_t = \frac{\partial lnf(y_t|\lambda_{t-1}, \mathscr{F}_{t-1}; \theta)}{\partial \lambda_{t-1}}, \quad S_t = S(t, \lambda_{t-1}, \mathscr{F}_{t-1}; \theta), \tag{2.5}$$

and for our purposes S_t is the Identity matrix of order k where k is the number of dependence parameters, that is,

$$S_t = \mathscr{I}_k. \tag{2.6}$$

Intuitively this mechanism makes use of the interpretation of the model's pdf as a likelihood function, and the score itself is the pointwise derivative of the likelihood conditioned on the information at $t-1$. Interpreting the likelihood function along with the meaning of a derivative we can intuit that the score term gives the direction the likelihood would go if the parameter was changed by a small positive amount. Therefore having a positive score value would mean that a slightly higher dependence better might be better, pointwise, using this mechanism as a driver means that in each step we are updating our variable in the direction that would bring the likelihood closer to a maximum.

From this brief discussion it is easy to see how more intuitive this driver is. Furthermore, it is also worth noting that since the score is the derivative of the pdf, this driver uses all of the information imbued in the copula structure. As a contrast we can look at Patton (PATTON, 2006) where different drivers are assumed for each different dependence parameter in a way that only somewhat relates to the parameters themselves and does not make direct use of the information provided in the copula structure. The advantage in using the Score lies in making use of the copula structure in a more imperative way.

The first element is w_{m_t}, this element is variable and can assume one of two values in each point in time, more formally

$$w_{m_t} := \begin{matrix} w_0 & if & m_t = 0 \\ w_1 & if & m_t = 1 \end{matrix}.$$

In this structure m_t follows a first order markovian process with two distinct states and is called the state variable.To capture this last component we make use of the filter proposed by Kim(KIM, 1994). Here we can see the potential of discrete jumps between two states and capturing such behavior is extremely important in financial markets because new government policies, financial crisis or even technological innovation can often be viewed in this framework. This supports the argument that states are not permanent from a financial perspective therefore we should not think of assets dynamics as being set in stone.

This structure comes with some challenges in implementation. The main challenge lies in making markov switching stucture operational since dependence is by its very nature unobservable. When we conseider the traditional filter by Kim (KIM, 1994) we know that the smoothing part of the filter relies on taking the last observed value and using it to refit the dynamics going backwards and therefore minimizing in-sample missteps. From the unobservable characteristic, it is clear that this second step could hardly be performed, and therefore some accommodations had to be made. To make the filter operable we create distinct states that follow our proposed dynamic completely described by input parameters, then we make use of these two virtual states and apply the filter to obtain a likelihood value for the entire dynamic structure, from there it is natural to perform a maximum likelihood estimation of the optimal parameters. And furthermore, since we make no constraints forcing parameters to be distinct our model has shown empirically that it can easily entertain the notion no changes in regime.

In summary, our construction incorporates a more parsimonious view into the dynamic behavior of the dependence, we allow it to fluctuate between two different states and in doing so we can perhaps gain insight regarding economic conditions each state is associated with. This type of change was first proposed by Silva Filho, Ziegelmann and Dueker (FILHO; ZIEGELMANN; DUEKER, 2012), but in their structure the main driver is given as in the Ad Hoc measures Patton (PATTON, 2006) proposed.

In this description we have described all of the econometric structure used henceforth can now properly evaluate our model. The results will be presented in two sections. The first is a detailed analysis of our model's performance when compared to other dependence models in the literature. In order to do so we use the Models depicted in Oswaldo Filho, Ziegelmann and Dueker (FILHO; ZIEGELMANN; DUEKER, 2012), along with some non regime switching counterparts as described in Creal et al.(CREAL; KOOPMAN; LUCAS, 2013) and in Patton (PATTON, 2006). The second is an empirical application where we use our model to analyse European markets and see whether we can find any evidence of regime change in the dependence between the pairs FTSE100xDAX, FTSE100xCAC40 and FTSE100xBEL20.

2.2 Simulation Results

In this section we will consider our simulated data to evaluate the performance of our model. In the this effort we have made four different generating models, each associated with a distinct model in the previously discussed literature. For making this analysis as thorough as possible we have used the Symmetrized Joe Clayton Copula as our main engine, this means that the model with regime change, have it on both tails each following a slightly different dynamic but with the same underlying unobservable component driven by a two state Markov Switching process.

In this experiment we use the sample size $T = 2000$ and make 1000 replicas using the same underlying dynamic. For the simulation we will then create time series with a Monte Carlo experiment using four different structures. First we will only name the dependence dynamics structures and relate them to the paper they were drawn from.

- ARMA(1,10)
  This is the dependence structure as proposed by Patton (PATTON, 2006).
- MS - ARMA(1,10)
  This is the dependence structure as proposed by Silva Filho, Ziegelmann and Dueker (FILHO; ZIEGELMANN; DUEKER, 2012).
- GAS
  This is the dependence structure as proposed by Creal, Koopman and Lucas (CREAL; KOOPMAN; LUCAS, 2013).
- MS-GAS This is the dependence structure we are proposing

We have generated data with all the four models and used all four to evaluate their relative performance. The full dynamic structure is described below

- MS GAS

$$\tau_t^u(m_t) = \Lambda(-0.87(m_t) + 1.05(1-m_t) - 0.5\tau_{t-1}^u + 0.5s_t^u)$$
$$\tau_t^l(m_t) = \Lambda(-0.81(m_t) + 1.2(1-m_t) - 0.8\tau_{t-1}^l + 0.4s_t^l)$$

  where s_t^i is the score function in relation to the corresponding tail i.

- MS ARMA(1,10)

$$\tau_t^u(m_t) = \Lambda(-0.87(m_t) + .2(1-m_t) - 0.79\tau_{t-1}^u + 2\frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|)$$

$$\tau_t^l(m_t) = \Lambda(-0.81(m_t) + .2(1-m_t) - 0.9\tau_{t-1}^l + 3\frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|)$$

- GAS

$$\tau_t^u = \Lambda(-0.87 - 0.5\tau_{t-1}^u + 0.5s_t^u)$$
$$\tau_t^l = \Lambda(-0.81 - 0.8\tau_{t-1}^l + 0.4s_t^l)$$

- ARMA(1,10)

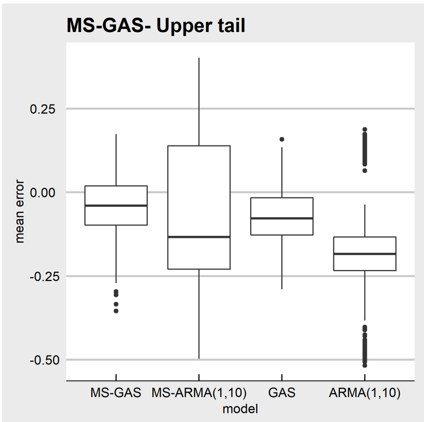
$$\tau_t^u = \Lambda(-0.87 - 0.79\tau_{t-1}^u + 2\frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|)$$

$$\tau_t^l = \Lambda(-0.81 - 0.9\tau_{t-1}^l + 3\frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|)$$

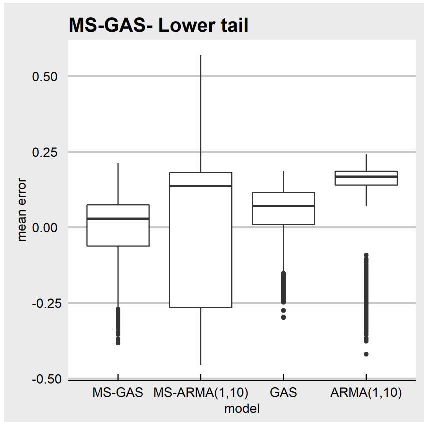
With the simulated data we use these same specifications, to estimate and evaluate the models, determine whether they are numerically consistent and if they can be accurately estimated. We analyse our results with two different measures, we take the mean error of all estimated models for all samples and use them to get boxplots, this first measure tells us whether our estimated dependence is biased, our second measure is the mean squared error for each sample to form boxplots.

Figura 1 – Markov Switching Gas Generating Model

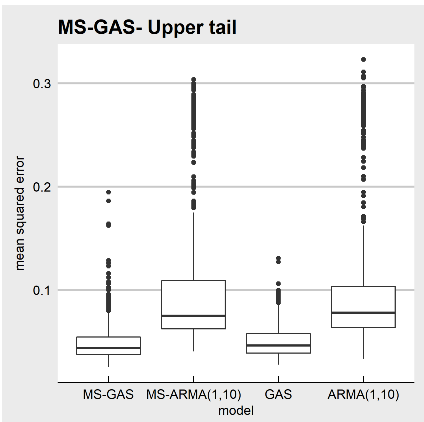
(a) Upper Tail - Bias



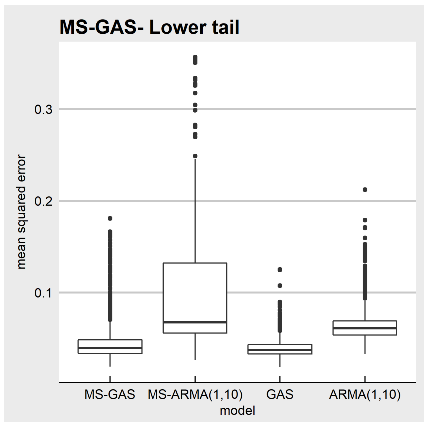
(b) Lower Tail - Bias



(c) Upper Tail - Error Variance



(d) Lower Tail - Error Variance

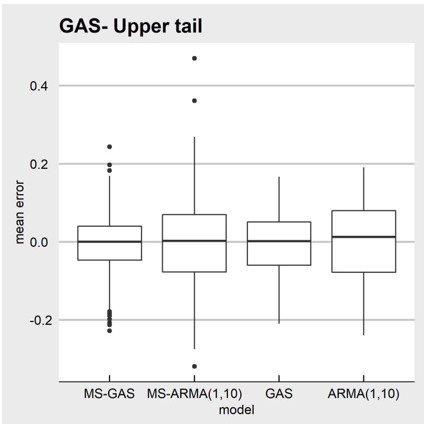


Fonte: Elaboração do Autor

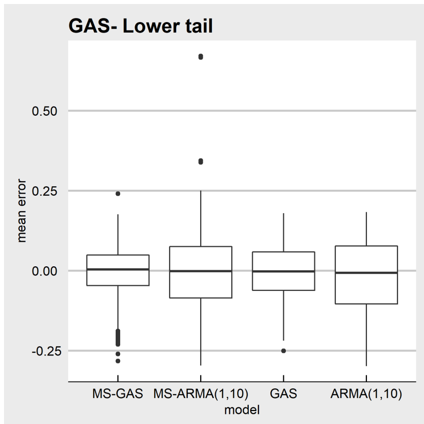
In Figure 1 we can see that in term of mean squared error the model with gas driver and one regime comes close to our proposed one, but it is clearly far more biased. This is consistent with the fact that it only captures one of the two regimes. The other two models perform significantly worse than our model both in bias and in mean squared error. In Figure 2 we see that our model only performs slightly worse than the real generating model, wich once again is the one regime GAS model. The other two performs significantly worse, which is also expected. In Figure 3 we see that our model outperforms even the generating model, both in bias and in variance and it is the best performing one. Lastly in Figure 4 is the only case which our model is outperformed, but only by the true generating model, its main contender the 2 regimes ARMA(1,10) proposed by Silva Filho and Ziegelmann (FILHO; ZIEGELMANN; DUEKER, 2012) is therefore outperformed in every sample.

Figura 2 – Gas Generating Model

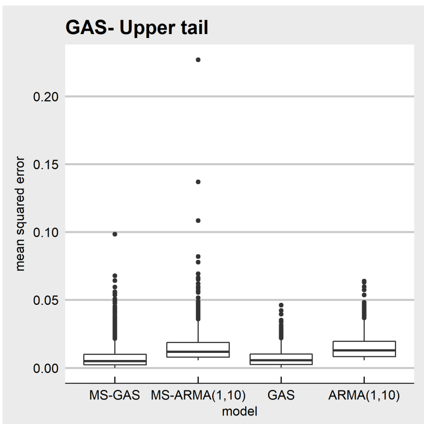
(a) Upper Tail - Bias



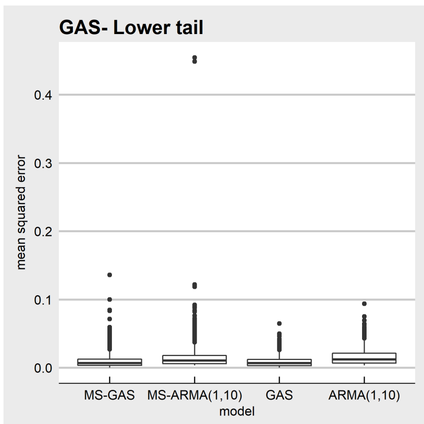
(b) Lower Tail - Bias



(c) Upper Tail - Error Variance



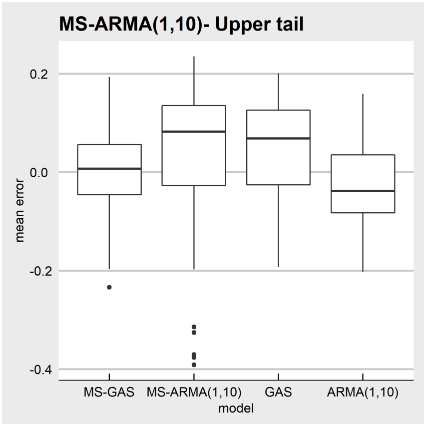
(d) Lower Tail - Error Variance



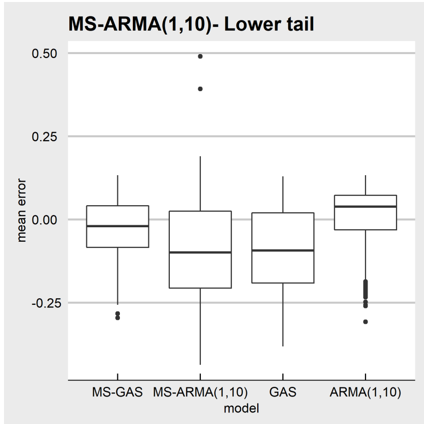
Fonte: Elaboração do Autor

Figura 3 – Markov Switching ARMA(1,10) Generating Model

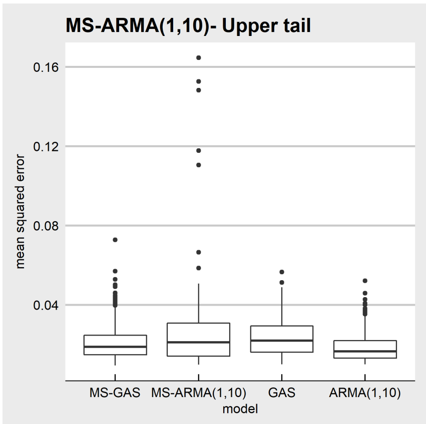
(a) Upper Tail - Bias



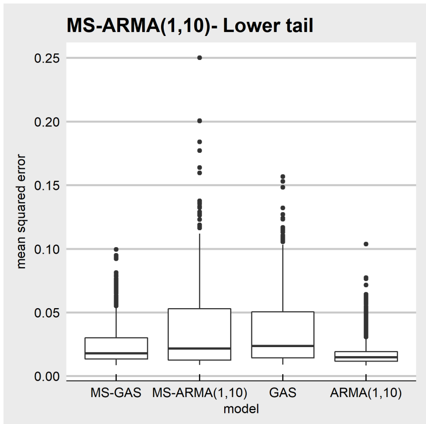
(b) Lower Tail - Bias



(c) Upper Tail - Error Variance



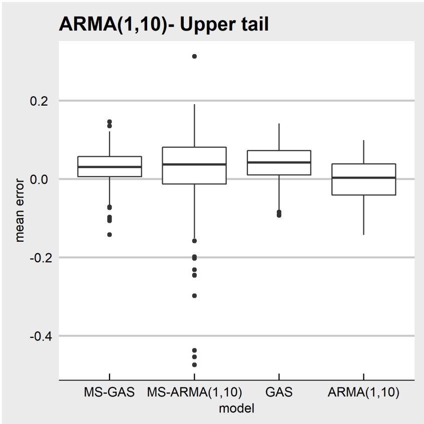
(d) Lower Tail - Error Variance



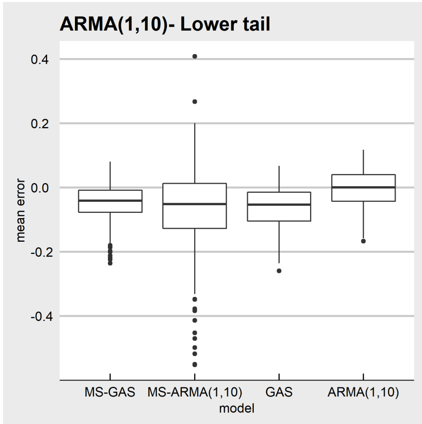
Fonte: Elaboração do Autor

Figura 4 – ARMA(1,10) Generating Model

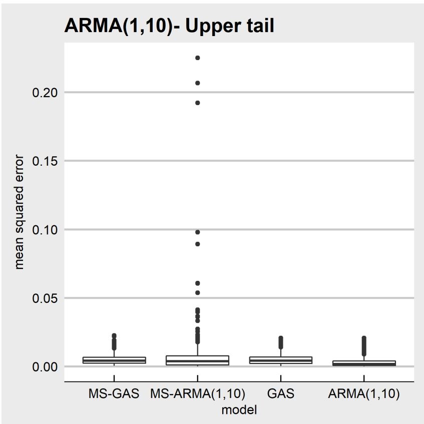
(a) Upper Tail - Bias



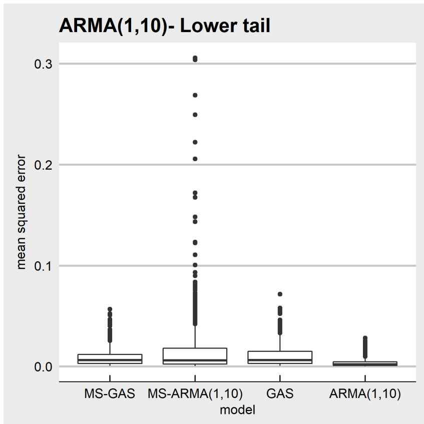
(b) Lower Tail - Bias



(c) Upper Tail - Error Variance



(d) Lower Tail - Error Variance



Fonte: Elaboração do Autor

2.3 Empirical Results

In this section we will perform our empirical analysis to investigate if there were any significant dependence shifts in European markets. This work was motivated by the recent Brexit referendum that took place in 23 June 2016 deciding whether the United Kingdom would continue to be a part of the European Union. We were led by the question of whether this change could have had a significant effect for the Financial Markets of the countries involved. We will first present some intuitive appeal informing our choice of data the statistical description and .

As mentioned the motivation for this analysisis to look into the possible effects of the BREXIT referendum, we know that Aristeidis (ARISTEIDIS; ELIAS, 2017) has looked into this, using the model in Silva Filho et al (FILHO; ZIEGELMANN; DUEKER, 2012) and using intraday data to try and capture moment changes in regime. In contrast to this application we select a different data structure and our results contradict the findings in Aristeidis.

We use daily closing prices for the FTSE100, DAX, CAC40 and BEL20 indexes. These

Tabela 1 – Descriptive Statistics

	FTSE100	DAX	CAC40	BEL20
nobs	2058.000	2058.000	2058.000	2058.000
Mean	0.012	0.033	0.012	0.020
Median	0.034	0.080	0.035	0.033
Variance	0.935	1.555	1.683	1.229
Stdev	0.967	1.247	1.297	1.108
Skewness	-0.187	-0.285	-0.146	-0.063
Excess Kurtosis	2.497	2.655	3.934	4.513

Fonte: Elaboração do Autor

represent the three largest indexes in Europe (FTSE100, DAX, CAC40) and one additional medium sized index. This choice seems natural since in empirical finance literature indexes are commonly used as representatives of the entire market. So in our analysis each of these indexes is supposed to represent a different European country, FTSE100, DAX, CAC40 and BEL20 represent United Kingdom, Germany, France and Belgium, respectively. The first three are not only the largest indexes but are also representative of the three largest economies in the European common market being commonly used as measures of economic activity in Europe.

The choice of Belgium might be more contentious, because the argument can be made as to how representative is this small to medium sized economy is in the bigger European structure this argument is, however, the very thought behind our choice we want to look at the disparity in behavior when we pair UK with a relatively small economy.

After all, in comparing Germany and France with the UK we are investigating dependence structures between the largest players, in contrast when we look at the dependence between UK and Belgium there might be other effects in play. Perhaps the Belgium market is not as well diversified meaning there could still be structural inefficiencies making it harder to perceive any dependence shifts. Otherwise the Belgium economy might be too correlated with the economic environment in its big neighbours from trade balances also making dependence fluctuations harder to capture. Arguable as it is we keep this choice because we want to try to perceive any differences in pairing big markets and pairing one big and one small market.

The data we used goes from January 4 of 2010 up to April 3 of 2018. This period covers the emergence of the Greek Sovereign debt crisis along with the following waves of European debt crisis, and goes all the way to almost two years after the decision of the United Kingdom to leave the European Common Market. One compelling argument for analysing this incident is that the economic implications of the british decision are yet to make themselves clear.

We transform these closing prices in daily returns as is usual in empirical finance literature and present the descriptive statistics of the returns on Table 1, with this information we gather that all the series have evidence of non normal marginal distributions, they all have negative

Tabela 2 – Marginal models

Parameter	FTSE100	DAX	CAC40	BEL20
ar1	-1.0720	-0.6698	-0.7519	-0.7780
	(0.0212)	(0.0215)	(0.0069)	(0.0222)
ar2	-0.4922			
	(0.0201)			
ma1	1.0687	0.7037	0.7611	0.8225
	(0.0184)	(0.0213)	(0.0065)	(0.0204)
ma2	0.45225			
	(0.0188)			
omega	0.0342	0.0382	0.0370	0.0465
	(0.0089)	(0.0107)	(0.0097)	(0.0150)
alpha1	0.0979	0.0908	0.1046	0.1029
	(0.0141)	(0.0142)	(0.0121)	(0.0177)
beta1	0.8874	0.9026	0.8916	0.8783
	(0.0187)	(0.0173)	(0.0151)	(0.0254)
eta11	1	1	1	1
	(0.1421)	(0.1216)	(0.0817)	(0.1262)
skew	0.8765	0.9031	0.9061	0.9170
	(0.0236)	(0.0202)	(0.0247)	(0.0280)
shape	9.6937	6.5530	7.0721	7.8567
	(1.7170)	(0.9735)	(1.0976)	(1.2415)
Log likelihood	-2521.1	-3059.78	-3100.7	-2808.2
Akaike	2.4598	2.9813	3.0211	2.7369
Bayes	2.4871	3.0032	3.043	2.7588
Shibata	2.4597	2.9813	3.021	2.7368
ARCH Lag[7] p-value	0.9992	0.9777	0.7284	0.8678
Anderson Darling p-value	0.956	0.306	0.998	0.971
Komolgorov Smirnov p-value	0.805	0.311	0.995	0.906
Cramer von mises p-value	0.92	0.246	0.997	0.949

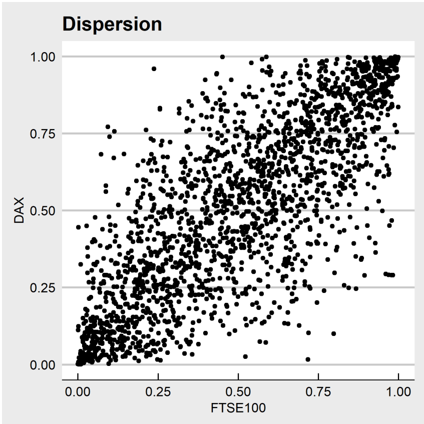
Fonte: Elaboração do Autor

Skewness, albeit quite low for the Belgium index, and they all have significant excess kurtosis. There is also presence of significant serial correlation both for the level and for the volatility of of all indexes. From this we proceed to model the marginal distributions using ARMA-GARCH models with non normal distributions for the errors(skewed t distribution), we tested ARMA models up to ARMA(5,5) and volatility models of the GARCH family up to GARCH(2,2) with three different volatility specifications(GARCH (BOLLERSLEV, 1986), TGARCH(ZAKOIAN, 1994) and GJR-GARCH (GLOSTEN; JAGANNATHAN; RUNKLE, 1993)).

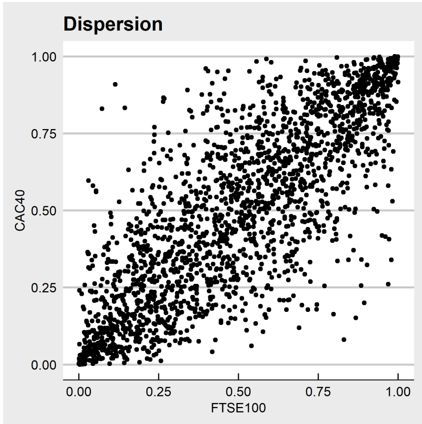
We present the marginal models and estimated parameters in Table 2. Also in Table 2 we present several tests to check whether the marginal models are well specified, all the models pass the tests and we can conclude that the transforms of the marginal distributions can be $Uniform(0,1)$ allowing us to proceed to copula estimation. We abstained from describing

Figura 5 – Residuals dispersion- Evidence of Tail Dependence

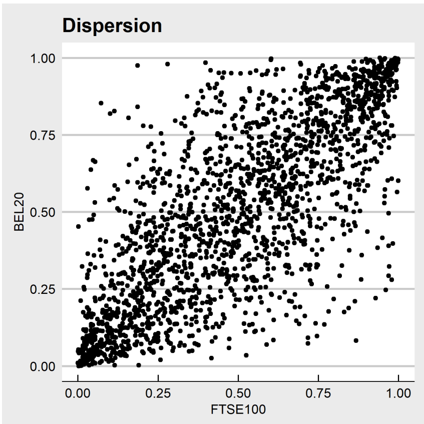
(a) FTSE100xDAX



(b) FTSE100xCAC40



(c) FTSE100xBEL20



Fonte: Elaboração do Autor

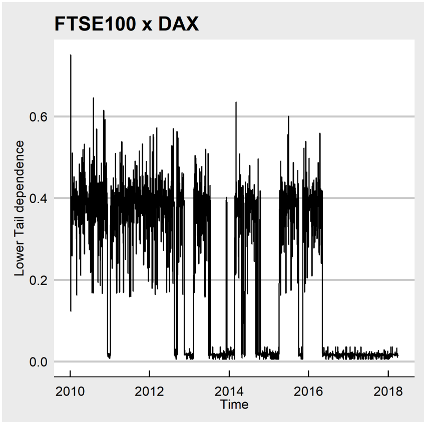
the marginal models in the econometric section, but for more detail see Zakoian (ZAKOIAN, 1994), Bollersev (BOLLERSLEV, 1986) and Glosten, Jagannathan and Runkle (GLOSTEN; JAGANNATHAN; RUNKLE, 1993) .

For a more intuitive appeal we show the dispersion graphics of each pair of indexes in Figure 5. with this in sight it is clear that the series probably present tail dependence given that we can see clustering in the upper right hand corner, as well as in the lower left hand corner in all the figures. This overlook informs our decision of which copula to use, leading us in the choice of the Symmetrized Joe Clayton Copula since it has both upper and lower tail dependence parameters and it can lead to symmetric distributions or not. This is one of the most versatile copulas allowing to capture many different stylized fact of financial assets. In Table 3 we present the estimated parameters for the Symmetrized Joe Clayton copula.

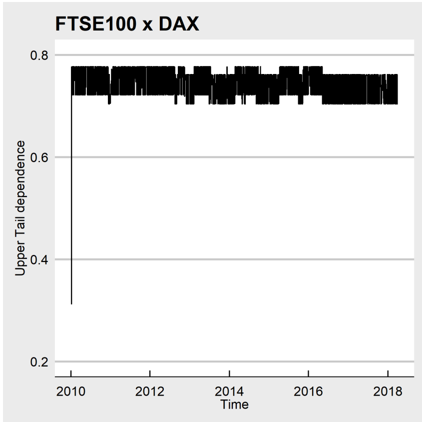
In analysing Figure 6 we see that the Lower tail dependence did suffer some changes in regime, primarily around 2013 and more persistently after 2016 for both FTSE100xDAX and FTSE100xCAC40. When we look at Figure 6a we see that there were some inklings of a regime

Figura 6 – Symmetric Joe Clayton Copula - Dependence Dynamics

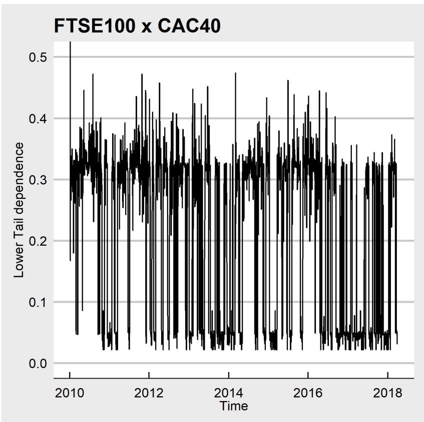
(a) Lower Tail - FTSE100xDax



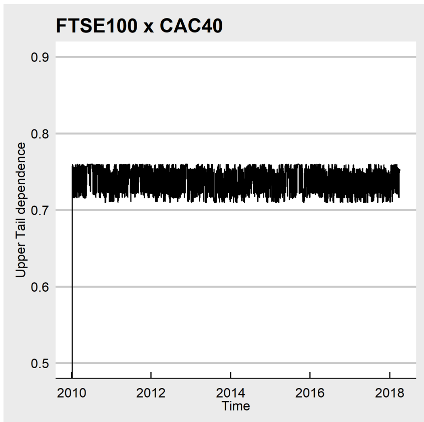
(b) Upper Tail - FTSE100xDax



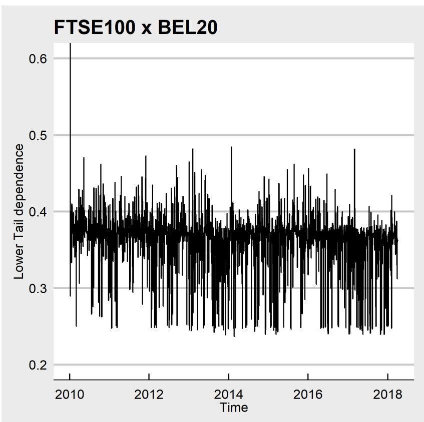
(c) Lower Tail - FTSE100xCAC40



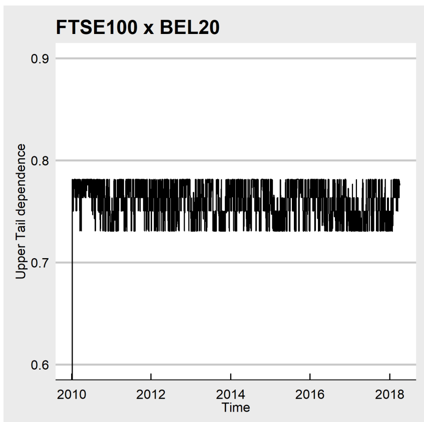
(d) Upper Tail - FTSE100xCAC40



(e) Lower Tail - FTSE100xBEL20



(f) Upper Tail - FTSE100xBEL20



Fonte: Elaboração do Autor

Tabela 3 – Dependence Parameters

	FTSE100xDAX	FTSE100xCAC40	FTSE100xBEL20
w_u^0	0.8758966	1.07667349	0.3943487
w_l^0	-1.2271633	-0.52834201	-1.7148759
w_u^1	0.9063016	1.25213919	0.4548315
w_l^1	-0.5469227	-0.86258818	-0.89898
ϕ_u	0.2208016	0.05114637	0.1245811
ϕ_l	0.6815526	0.62243721	0.8127119
α_u	0.1456375	0.01752216	0.8060495
α_l	0.1343687	-0.16185481	-0.3072627
$p00$	0.6701717	0.57112243	0.8131067
$p11$	0.5321212	0.43949405	0.8179543

Fonte: Elaboração do Autor

change around 2013 going from a 0.3 lower tail to a very low value of about 0.05, then it returns to the initial regime and after 2016 the change seems to be more persistent. From Figure 6c we see that throughout the sample there were relatively frequent changes from one regime to the other in the years from 2012 to 2016 but for the most part the dependence stayed in the high level of 0.35, however after 2016 the regime change seems to have occurred less often and the lower level for the dependence of about 0.06 is more predominant. For the pair FTSE100xBEL20 we found no evidence of changes in the level of dependence.

With this result we can infer that after 2016 the British index seems to have isolated itself from shocks that affect the other two large european markets, this result could be of great importance politically as it suggests that exiting the european union might have had the desired result in sense in increasing isolation and maybe even stepping aside from the repercussions of the, lately frequent, sovereign debt crisis. Furthermore we can observe that the upper tail dependence has not suffered any significant changes in regime. These results for positive shocks could mean that in terms of growth, the underlying factors that boost these economies might not be directly related to the existence of the common market one other possible underlying reason could be a significant diversification of all these markets, making positive technology shocks more frequently happen in unison.

2.4 Concluding Remarks

In this paper we have proposed a new model to deal with regime change in dependence modelling with copulas. We have proved with a monte carlo experiment that our model outperforms the model proposed in Silva Filho and Ziegelmann (FILHO; ZIEGELMANN; DUEKER, 2012). When compared with some of the other models in the literature, specifically Creal, et al. (CREAL; KOOPMAN; LUCAS, 2013) and Patton (PATTON, 2006), our model's performance is robust and shows mostly improvements, being always at least as good as the other models.

In the empirical section we have applied our model to european markets indexes and showed that, contrary to the findings in Aristeidis (ARISTEIDIS; ELIAS, 2017), there was a change in the tail dependence between some markets. Namely the lower tail dependence between FTSE100 and DAX , and between FTSE100 and CAC40 have shown evidence of changes in regime. The disparity in our results from Aristeidis's might be atributed to distinct data collection choices and also to our superior model.

The empirical findings are especially interesting for allowing a glimpse into the real effects of BREXIT. From a political perspective , if nothing else, the UK seems to have achieved its goal a increasing isolation to european shocks. Also interesting is the fact that the separation has led to changes only in relation to negative shocks, making the net effect potentially positive insofar as positive shocks are still received.

3 CONSIDERAÇÕES FINAIS

Nesta dissertação propomos uma forma inovadora de modelar dependência entre ativos. Em relação a seu competidor direto, proposto por Silva Filho Ziegelmann e Dueker (FILHO; ZIEGELMANN; DUEKER, 2012), nosso modelo se mostra claramente superior. Além disso quando comparado com sistemas mais simples como em Creal, et al. (CREAL; KOOPMAN; LUCAS, 2013) e em Patton (PATTON, 2006) sua performance se mantém robusta.

Na abordagem empirica identificamos mudança de regime na cauda inferior da dependencia entre FTSE100 e CAC40, assim como na dependencia entre FTSE100 e DAX. Este resultado vai em desacordo com o encontrado por Aristeidis (ARISTEIDIS; ELIAS, 2017), entretanto as distinções entre os dados utilizados na análise pode ser apontada como potencial causa deste contraste.

O resultado empírico é especialmente interessante pois permite um vislumbre dos efeitos reais da separação do Reino Unido da União Europeia. Da perpectiva politica a decisão parece ter, se nada mais, alcançado o objetivo de aumentar o grau de separação do Reino Unido. Mais interessante ainda é que a separação se mostra clara apenas se tratando de choques negativos, indicando que seu efeito pode ser visto como primariamente positivo ao isolar o reino unido de choques negativos ao mercado europeu.

REFERÊNCIAS

ARISTEIDIS, S.; ELIAS, K. Empirical analysis of market reactions to the uk's referendum results–how strong will brexit be? Journal of International Financial Markets, Institutions and Money, Elsevier, 2017.

BARTELS, M.; ZIEGELMANN, F. A. Market risk forecasting for high dimensional portfolios via factor copulas with gas dynamics. Insurance: Mathematics and Economics, Elsevier, v. 70, p. 66–79, 2016.

BOLLERSLEV, T. Generalized autoregressive conditional heteroskedasticity. Journal of econometrics, Elsevier, v. 31, n. 3, p. 307–327, 1986.

CHOLLETE, L.; HEINEN, A.; VALDESOGO, A. Modeling international financial returns with a multivariate regime-switching copula. Journal of financial econometrics, Oxford University Press, v. 7, n. 4, p. 437–480, 2009.

CREAL, D.; KOOPMAN, S. J.; LUCAS, A. Generalized autoregressive score models with applications. Journal of Applied Econometrics, Wiley Online Library, v. 28, n. 5, p. 777–795, 2013.

EMBRECHTS, P.; MCNEIL, A.; STRAUMANN, D. Correlation and dependence in risk management: properties and pitfalls. Risk management: value at risk and beyond, v. 176223, 2002.

FILHO, O. C. da S.; ZIEGELMANN, F. A.; DUEKER, M. J. Modeling dependence dynamics through copulas with regime switching. Insurance: Mathematics and Economics, Elsevier, v. 50, n. 3, p. 346–356, 2012.

FÖLLMER, H.; SCHIED, A. Convex measures of risk and trading constraints. Finance and stochastics, Springer, v. 6, n. 4, p. 429–447, 2002.

GLOSTEN, L. R.; JAGANNATHAN, R.; RUNKLE, D. E. On the relation between the expected value and the volatility of the nominal excess return on stocks. The journal of finance, Wiley Online Library, v. 48, n. 5, p. 1779–1801, 1993.

JOE, H. Asymptotic efficiency of the two-stage estimation method for copula-based models. Journal of Multivariate Analysis, Elsevier, v. 94, n. 2, p. 401–419, 2005.

KIM, C.-J. Dynamic linear models with markov-switching. Journal of Econometrics, Elsevier, v. 60, n. 1-2, p. 1–22, 1994.

MARKOWITZ, H. Portfolio selection. The journal of finance, Wiley Online Library, v. 7, n. 1, p. 77–91, 1952.

PATTON, A. J. Modelling asymmetric exchange rate dependence. International economic review, Wiley Online Library, v. 47, n. 2, p. 527–556, 2006.

PATTON, A. J. et al. Copula methods for forecasting multivariate time series. Handbook of economic forecasting, Elsevier Oxford, v. 2, p. 899–960, 2012.

SALVATIERRA, I. D. L.; PATTON, A. J. Dynamic copula models and high frequency data. Journal of Empirical Finance, Elsevier, v. 30, p. 120–135, 2015.

SKLAR, M. Fonctions de répartition à n dimensions et leurs marges. [S.l.]: Université Paris 8, 1959.

ZAKOIAN, J.-M. Threshold heteroskedastic models. Journal of Economic Dynamics and control, Elsevier, v. 18, n. 5, p. 931–955, 1994.