Ao Carlos com um forte abraço
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AN AUSUBELIAN APPROACH TO PHYSICS INSTRUCTION: AN EXPERIMENT IN AN INTRODUCTORY COLLEGE COURSE IN ELECTROMAGNETISM

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AN AUSUBELIAN APPROACH TO PHYSICS INSTRUCTION: AN EXPERIMENT IN AN INTRODUCTORY COLLEGE COURSE IN ELECTROMAGNETISM

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Based on the learning theory of David Ausubel, an experimental introductory college physics course in electromagnetism was organized and taught to students of science and engineering. Four equivalent groups of students, forming two pairs of experimental-control groups participated in the study. One of these pairs was taught under a self-paced format and the other under a traditional lecture approach.

According to Ausubel's theory, the most inclusive, most general ideas, phenomena, and concepts should be presented early in instruction to serve as conceptual "anchorage" for subsequent learning. Following this principle, Maxwell's Equations and inclusive concepts such as electromagnetic force and electromagnetic field were introduced, in the experimental groups, at the beginning of the course in order to serve as basis for subsequent presentation of electric and magnetic phenomena. The control groups followed a traditional content organization found in most textbooks on the subject, which starts with electricity, followed by magnetism, and ends with electromagnetic phenomena and the Maxwell Equations.

Traditional achievement measures and concept association tests were used to search for differences, that could arise from the two different organizational approaches, in terms of the student's ability to apply, relate, differentiate and hierarchically organize electromagnetic concepts.

No significant differences in achievement were found in terms of traditional measures such as unit-tests, quizzes and exams. However, in terms of concept learning there was evidence that the Ausubelian approach fostered concept differentiation, relatedness and meaningful hierarchical organization to a greater extent than the traditional approach, specially in the self-paced program comparisons.

These results are both consistent with and supportive of Ausubel's theory and provide evidence that this theory is useful for physics instruction and for research in physics education.

BIOGRAPHICAL SKETCH

The author was born in São Leopoldo, RS, Brazil, on May 25, 1942. His secondary education was completed in 1961 at Colégio Sinodal, in São Leopoldo. In 1962, he was admitted to the Faculty of Philosophy, Sciences and Letters of the Federal University of Rio Grande do Sul (UFRGS), Brazil, where he graduated in Physics in 1965. From 1964 to 1966 he taught physics and mathematics in Brazilian public secondary schools. From 1967 to 1969 he was an instructor of physics at the Institute of Physics and at the School of Engineering of UFRGS. In 1970 he became a full-time instructor at the Institute of Physics and was appointed Head of the Teaching Division of this Institute. He received his M.Sc. degree in Physics from this institution in December 1971 and was promoted to the position of Assistant Professor.

In 1972 he came to Cornell, for the first time, as a visiting fellow in the Physics Department to participate in physics education projects and seminars. (At that time, the author became acquainted with Science Education projects that were being carried out in the Department of Education under the leadership of Professor Joseph D. Novak. This acquaintance had a strong influence in his decision to pursue a Ph.D. degree at Cornell three years later.)

Back to Brazil, in 1973, the author resumed his position at the Institute of Physics of UFRGS. In July 1973 he was elected Teaching Secretary of the Brazilian Society of Physics for a two-year term. In

1974 he was elected Head of the Physics Department of the Institute of Physics for a one-year term.

In January, 1975 the author returned to Cornell in order to meet the formal requirements of credit and residence for the Ph.D. degree in Science Education, which were completed in the spring of 1977.

The author is a member of the Brazilian Society of Physics, of the Brazilian Society for Advancement of Science, of the American Association of Physics Teachers, of the National Science Teachers Association, and of the American Educational Research Association. He is currently a Regional Editor of the Brazilian Journal of Physics and a member of the International Commission on Physics Education of the International Union of Pure and Applied Physics.

He is married to the former Marli Merker and has four children, Simone, Suzane, Stefanie and Fabio.

To my parents.

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Chapter I

INTRODUCTION

This study was conducted in Brazil during the first semester of 1976 in a college physics course. This course, Physics II, is the second one of a sequence of three one-semester courses in General Physics offered by the Department of Physics of the Federal University of Rio Grande do Sul, Brazil, to students of science and engineering. The content is electricity and magnetism at the level of "Physics" by D. Halliday and R. Resnick (1966) and the prerequisites are one semester of calculus (derivatives and integrals) and one semester of physics (mechanics). Enrollment ranges from 300 to 500 students each semester.

We have been involved with this course since 1967, either as a teacher or coordinator. During this time several experiments, some controlled and some not, were carried out in the search of better solutions to the big teaching-learning problems involved in such a course. To some extent, this study is both a continuation and a result of such efforts.

Although we have always faced the usual logistic and administrative difficulties found in a large course like this, they can hardly be used as a major reason for the teaching-learning problems observed. As a matter of fact, we believe that even if these difficulties were completely overcome, we would still have problems as long as other more important aspects of the course were not given attention. Thus, our first efforts were directed to what we thought was the major problem: the teaching method.

In 1967, the basic teaching format was lectures for large groups (120 students) and problem solving and laboratory sessions for small groups (20 students). The articulation between the lectures and the small sessions was difficult; attendance at the lectures was low; achievement indices were low and students' complaints were high. Under such circumstances, we changed to a new teaching mode where the same teacher was in charge of all classes (lectures, problem solving and laboratory) of a group of 40 to 50 students. This approach represented some improvement over the previous one, but it brought new problems, e.g., different achievement criteria among teachers, and did not solve most of the old difficulties.

The next trial, in 1969, was a "small-group-guided-study" method (Moreira, 1975). In this method, in each class a group of 40 to 50 students was divided into small groups of 4 to 5 students, and each group received a study-guide containing indications about sections of the textbook to be read, questions to be answered and problems to be solved by the group. Sometimes the group work was preceded by a brief introduction made by the teacher. During the class, the teacher acted mostly as a facilitator. At the end of the class, each group presented a written report which was evaluated and returned to the group with an answer sheet in the next class. Laboratory experiments were performed under the same scheme. Course evaluation was carried out through individual quizzes and the daily work of the group. This method was first tried in a small scale and, later on, progressively extended to all sections of the Physics II course with greater success than the previous methods (Moreira and Costa, 1971; Moreira, 1972). The number of students passing the course increased and the achievement level was at least equal to previous semesters. Students' complaints were reduced to a minimum, and the method was eventually extended to other disciplines where it is still being used today with slight modifications. In the Physics II course, it was used until 1972, but, due to the initial successes, almost all other instructional practices were abandoned in favor of this new method. As a consequence, it became monotonous and boring. Furthermore, instructional materials were not truly modified from one semester to another and students started to use answer sheets from previous semesters to do their work in each class. In addition to this, our dissatisfaction with student understanding of physical phenomena and concepts reappeared after a brief period of enthusiasm with this approach.

Considering that the small group method was some sort of half-way between group and individualized instruction, the next natural method to try would be a self-paced method. Fortunately, in 1972 we had the opportunity to have contact with the Audio-Tutorial Approach to Learning (Postlethwait, Novak and Murray, 1972) at Cornell University, where it was being used in a large introductory physics course. We also had, in February 1973, the opportunity to attend a workshop on the Personalized System of Instruction (PSI) (Keller, 1968) held at the University of Brasilia, Brazil. During this workshop, we prepared the written materials of our first PSI course.

The course was taught in the first semester of 1973 to a group of 48 students of the Physics II course. The success in terms of students, proctors and teacher satisfaction was remarkable (Moreira, 1973). Almost at the same time, we started a small scale experiment with the Audio-Tutorial Approach to Learning (AT) which was also very successful (Moreira and Levandowski, 1974). As a consequence of these good results, both the AT and PSI self-paced formats were progressively extended to a larger number of students with emphasis on the last one because it did not depend so heavily on equipment and special facilities.

With individualized instruction, students' attitudes toward the course were highly satisfactory; the passing index was larger than in previous courses; and a controlled experiment (Dionisio and Moreira, 1975) indicated that, in terms of achievement, students under PSI were doing at least as well as those in non-PSI sections of the course.

By the second semester of 1974, we were operating full scale: almost 300 students were under PSI and an AT learning center with 15 carrels was available as an additional instructional resource. However, in such a large scale, besides logistic difficulties, we also faced other problems such as shortage of good proctors, cheating and rote memorization of test answers (Moreira, 1976). Consequently, the number of students under PSI was decreased in the following semesters. Today, almost four years after the first experiment with PSI, it is being used with only two sections (approximately 100 students) of the Physics II course. In a small scale, PSI is certainly the best instructional method that we've tried in this course so far. Both PSI and AT were also used in other courses in our Department with very good results in a small scale.

Several studies concerning these two methods were conducted in the Department (e.g., Levandowski, 1975; Buchweitz, 1975; Dionisio, 1975). In general, these studies tend to favor individualized instruction in comparison to group instruction, but none of them provide sufficient evidence to claim a superiority in terms of concept learning in physics. Studies conducted elsewhere also fail to provide such an evidence (Moreira, 1973).

One of those experiments, however, has motivated the present study, more than the others. In the first semester of 1974 we carried out a retention study (Moreira and Dionisio, 1975) comparing PSI and group instruction. A retention test was administered to two matched groups three and six months after they had taken the Physics II course, either under group or individualized instruction. We expected that PSI would result in greater retention than in group instruction but no significant differences were found.

Having discarded possible explanations for the lack of significant differences such as mortality, instrumental or history effects, and given our acquaintance with Ausubel's Learning Theory (Ausubel, 1968) we preferred to interpret the results in terms of this theory: in spite of well defined differences in the instructional approaches, the content sequence was exactly the same, not emphasizing concept learning, not fostering concept differentiation and not attempting to match new learning to students' existing cognitive structure.[1]

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1. The term cognitive structure represents, in Ausubel's theory, a framework of hierarchically organized concepts, where concepts are the individual's representation of sensory experience (Novak, 1977).

In such a case, it would be unlikely that PSI, just because the content was divided into small units and positive reinforcement was given, would result in a higher degree of content retention.

Thus, we decided to carry out a study to investigate the effect of an experimental approach to the content organization of the Physics II course based on Ausubel's learning theory in comparison to the conventional approach based on the textbook (Halliday and Resnick, 1966). As we still believe that PSI is a valuable approach, it was chosen as the instructional method for both experimental and control groups. In addition, two lecture groups were also used as experimental and control for further investigation on the effect of the new content organization. Basically, this "experimental" content organization consisted in starting the course with the more general phenomena, concepts and equations of electromagnetism and progressively differentiating them as the course developed. Thus, concepts like electromagnetic force and electromagnetic field as well as Maxwell's Equations were introduced at the very beginning of the course. Consequently, since the beginning electricity and magnetism were seen as particular instances of electromagnetism.

As this content organization is based on Ausubel's theory, we believe that, before going into further design details, some essential features of this theory, which is the main theoretical framework of this study, must be introduced.

Chapter II

THE LEARNING THEORY OF DAVID AUSUBEL

It would be an ambitious task to attempt to describe Ausubel's theory (1968) in a single chapter and the result would probably be only a rough approximation, unfair to the relevance of this theory. However, as this theory plays a central role in this study and also because we think that it is potentially relevant for physics instruction in general, we feel that at least some basic features of this theory should be discussed here. Thus, what follows is neither a complete description nor a summary of Ausubel's theory, but rather a partial description of this theory. A comprehensive interpretation of such a theory can be found in "A Theory of Education" by J. D. Novak (1977).

II-1 Meaningful and Rote Learning

Ausubel distinguishes between meaningful and rote learning. Meaningful learning is a process in which new information is related to an existing relevant aspect of an individual's knowledge structure. That is, this process involves the linkage of new information with a specific knowledge structure, which Ausubel defines as subsuming concepts or subsumers, existing in the individual's cognitive structure. In physics, for example, if the concepts of force and field already exist in the learner's cognitive structur they serve as subsumers for new information concerning a certain type of force and field, e.g., the electromagnetic force and field. Thus, during meaningful learning, new information is associated with existing relevant subsumers in cognitive structure. This association, in turn, results in further growth and modification of the existing subsumer. Thus, subsumers can be relatively large and well developed or they may be limited and poorly developed depending on the frequency that meaningful learning occurs in conjunction with a given subsumer. In our example, an intuitive idea of force and field would serve as subsumer for new information concerning the gravitational, electromagnetic and nuclear forces and fields. However, if these specific concepts are meaningfully learned in association with the existing general concepts of force and field, the result is growth and modification of these latter concepts in the sense that they are now more elaborated, more inclusive, more capable of subsuming new information. In such a case they will be even more efficient as subsumers for any other specific type of force and field or related concepts.

Rote learning occurs when relevant concepts, or subsuming concepts do not exist in the individual's cognitive structure. In such a case, new information must be arbitrarily stored in the cognitive structure, that is, it is not linked with existing concepts or, in other words, it is rotely learned. This does not mean that rote learning tasks are mastered in a cognitive vacuum. They are relatable to cognitive structure but only in an arbitrary fashion that does not result in acquisition of meanings. (Meaningful learning, on the other hand, refers to the process of acquiring meanings from the potential meanings, presented in the learning task.)

An obvious example of rote learning in physics, is the rote memorization of formulas, but our previous example can be used again: if the general ideas of force and field were not available in the learner's cognitive structure, he could only rotely learn that an electric charge generates an electric field that exerts a force on a second charge placed in such a field. He could memorize this, but, according to Ausubel, it would not result in acquisition of meanings.

The distinction between rote and meaningful learning, however, is not to be confused with the distinction between reception and discovery learning. According to Ausubel, in reception learning the entire content of what is to be learned is presented to the learner in final form, whereas, in discovery learning, the principal content of what is to be learned is not given, but must be discovered by the learner. However, after discovery learning itself is completed, the discovered content is made meaningful in much the same way that presented content is made meaningful in reception learning. Thus, each distinction (rote versus meaningful and reception versus discovery) is an independent dimension of learning as illustrated in Figure II-1, and, contrarily to the widespread belief that reception learning is invariably rote and that discovery learning is necessarily meaningful, both reception and discovery learning can be either rote or meaningful, depending on the conditions under which learning occurs. In both instances meaningful learning takes place if the learning task can be related, in nonarbitrary fashion, to what the learner already knows, and if the learner adopts a corresponding learning set to do so. Rote learning, on the other hand, occurs if the learning task consists of

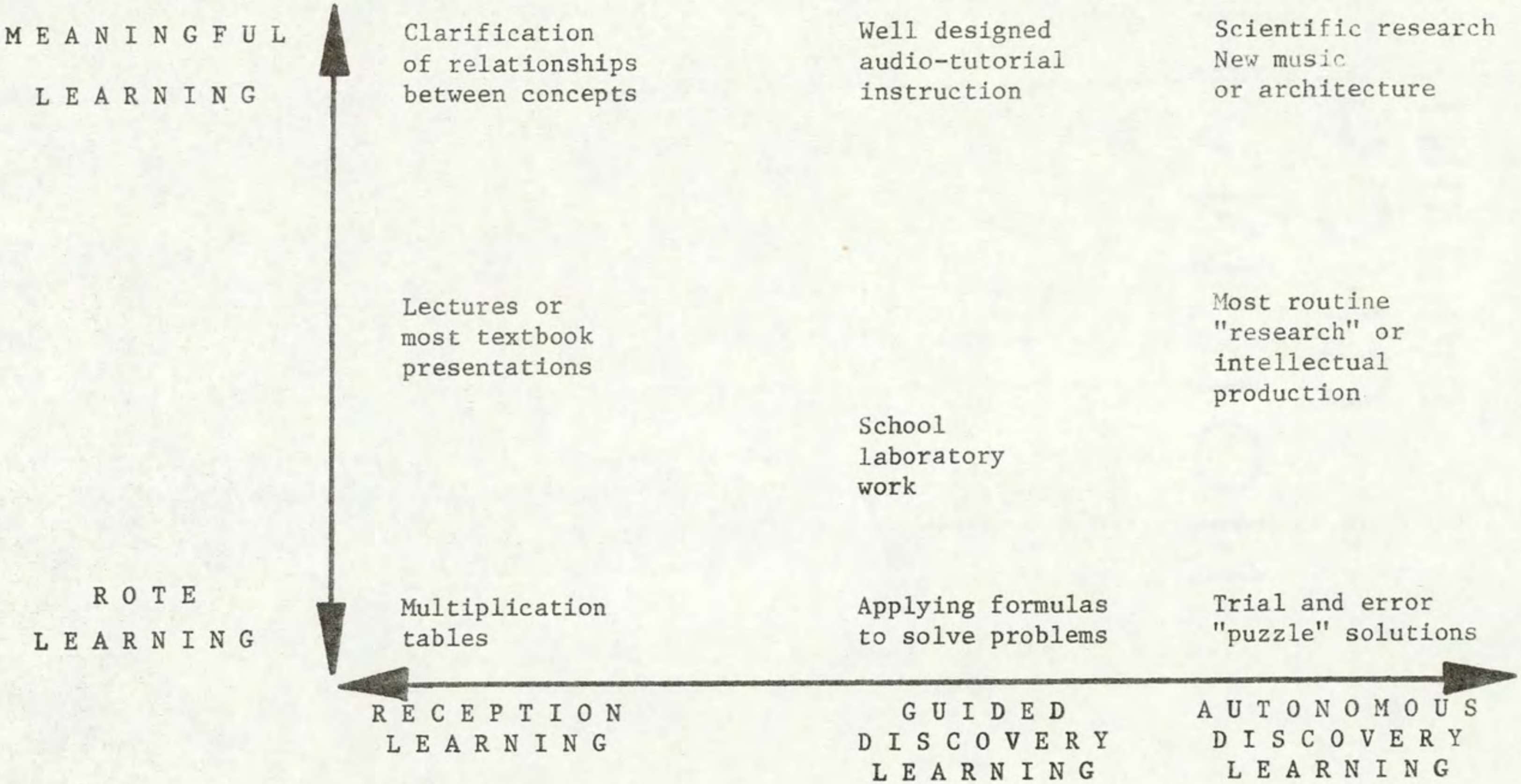


Figure II-1. Reception and discovery learning are on a continuum distinct from rote learning and meaningful learning. Typical forms of learning are shown to illustrate representative different "positions" in the matrix (Novak, 1977).

purely arbitrary associations, if the learner lacks the relevant subsumers, i.e., the relevant prior knowledge necessary for making the learning task potentially meaningful, or if the learner is merely predisposed to internalize it in an arbitrary fashion.

From the above discussion, we can see that meaningful learning which is the most central idea in Ausubel's theory presupposes both that the learner manifests a disposition to relate the new material non-arbitrarily to his cognitive structure, and that the material is potentially meaningful to him, i.e., relatable to his cognitive structure. Supposing then, that the learner manifests such a disposition but lacks the relevant subsumers (the material in such a case is not potentially meaningful to him), how could meaningful learning be made possible? This question could also be phrased as: How are subsumers generated?

According to Novak (1977), rote learning is always necessary when an individual acquires new information in a knowledge area completely unrelated to what he already knows. That is, rote learning must occur until some elements of knowledge, relevant to new learning on a given subject, exist in the cognitive structure to provide potential for meaningful learning. Ausubel would recommend the use of <u>advance organizers</u> that would serve to anchor new learning and lead to the development of a subsuming concept which could facilitate subsequent learning.

II-2 Advance Organizers

The use of organizers is a major strategy advocated by Ausubel in his book for deliberately manipulating cognitive structure to facilitate meaningful learning. Organizers are appropriately relevant and inclusive introductory materials that are introduced in advance of the learning material itself. Contrarily to summaries and overviews, which are ordinarily presented at the same level of abstraction, generality and inclusiveness simply emphasizing the salient points of the material, organizers are presented at a higher level of abstraction, generality, and inclusiveness. Organizers are selected on the basis of their appropriateness for explaining, integrating, and interrelating the material they precede.

The following excerpt from Ausubel's book (1968, pp. 148-149) will make more clear the function of organizers:

> In short, the principal function of the organizer is to bridge the gap between what the learner already knows and what he needs to know before he can successfully learn the task at hand. The function of the organizer is to provide ideational scaffolding for stable incorporation and retention of the more detailed and differentiated material that follows in the learning passage, as well as to increase discriminability between the latter material and similar or ostensibly conflicting ideas in cognitive structure. In the case of completely unfamiliar material, an 'expository' organizer is used to provide relevant proximate subsumers. These subsumers, which bear a superordinate relationship to the new learning material, primarily furnish ideational anchorage in terms that are already familiar to the learner. In the case of relatively familiar learning material, a 'comparative' organizer is used both to integrate new ideas with basically similar concepts in cognitive structure, as well as to increase discriminability between new and existing ideas which are essentially different but confusably similar. . . .

In physics, for example, when introducing the learner to a new subdiscipline like electromagnetism, a general discussion about physics, situating this new area in the whole context of the discipline, would serve as an advance organizer for the new information. In the same way, a general discussion about the concepts of force and field emphasizing the instances of such concepts already existing in the learner's cognitive structure (e.g., gravitational force and field) would be an organizer (somewhat "comparative") for the concepts of electromagnetic force and field. It should be emphasized, however, that in these examples, the organizers would function as cognitive bridges between some relevant subsumers already existing in the learner's cognitive structure and the new information. The point is that organizers would facilitate only the learning of potentially meaningful material, i.e., relatable to the learner's cognitive structure.

II-3 Assimilation and Obliterative Assimilation

In order to account more explicitly for the acquisition, retention, and organization of meanings in cognitive structure, Ausubel introduces what he calls the principle of assimilation, which is represented symbolically as:

New, potentially meaningful idea	→	Related to and assimilated by	→	Established idea in cognitive structure	→	Interaction Product
a				A		A'a'

Thus, assimilation is a process that takes place when a potentially meaningful concept or proposition a is subsumed under a more inclusive established idea or concept in cognitive structure as an example, extension, elaboration, or qualification of the established idea or concept. As suggested in the above diagram, the new meaning a' that emerges when a is related to and interacts with A in this way is the interactional product a'A'. That is, not only the new potentially meaningful idea a but also the established idea A, to which it is related, is changed by the interaction. Furthermore, both interactional products a' and A' remain in relationship with each other as linked co-members of a new composite ideational unit (i.e., a modified subsumer) a'A'. This is the fundamental basis of Ausubel's assimilation theory.

For example, if the new potentially meaningful concept of nuclear force should be learned by a student who already has the concept of force well established in his cognitive structure, this new concept would be subsumed under the more inclusive concept of force. However, given that this type of force is a short-range force (in opposition to other types which are long-range), not only the concept of nuclear force would acquire meaning for the student, but also the established concept of force would be modified by increasing its inclusiveness (i.e., now the concept of force would include also short-range forces).

Ausubel suggests that the assimilation or anchoring process probably has a general facilitating effect on retention. However, to explain how newly assimilated meanings actually become available during the retention period, he assumes that, for a variable period of time, they are dissociable from their anchoring ideas, and, hence, reproducible as individually identifiable entities. This is schematically represented as:

$$A'a' \rightleftarrows A' + a'$$

That is, the newly learned and assimilated meaning a', which is the less stable co-member of the interactional product A'a', is initially dissociable from the anchoring idea A', i.e., the interactional product A'a' dissociates into A' and a'.

Ausubel also contends that the assimilation process besides accounting for the superior retention of meaningfully learned ideas, also implies a plausible mechanism for the subsequent forgetting of these ideas, i.e., the gradual reduction of their meanings to the meanings of the corresponding anchoring ideas (1968, pp. 93-94):

> Thus, although the retention of newly-learned meanings is enhanced by anchorage to relevant established ideas in the learner's cognitive structure, such knowledge is still subject to erosive influence of the general reductionist trend in cognitive organization. Because it is more economical and less burdensome merely to retain the more stable and established anchoring concepts and propositions than to remember the new ideas that are assimilated in relation to them, the meaning of the new ideas tends to be assimilated or reduced, over the course of time, to the more stable meanings of the established anchoring ideas. Immediately after learning, therefore, when this second or obliterative stage of assimilation begins, the new ideas become spontaneously and progressively less dissociable from their anchoring ideas as entities in their own right, until they are no longer available and are said to be forgotten. When the dissociability strength of a' falls below a certain critical level (the threshold of availability), it is no longer effectively dissociable from A'a' (in other words, is no longer retrievable). Eventually zero dissociability is reached, and A'a' is further reduced to A', the original idea. . . .
>
> Forgetting is thus a continuation or later temporal phase of the same assimilative process underlying the availability of newly learned ideas. And the same nonarbitrary relatability to a relevant established idea in cognitive structure that is necessary for meaningful learning of a new idea, and that leads to its enhanced retention through the process of anchoring the emergent meaning to that of the established idea, provides the mechanism for most later forgetting. . . .
>
> Unfortunately, however, the advantages of obliterative assimilation for cognitive functioning are gained at the expense of losing the differentiated body of detailed propositions and specific information that constitute the flesh, if not the skeleton,

> of any body of knowledge. The main problem of acquiring the content of an academic discipline, therefore, is counteracting the inevitable process of obliterative assimilation that characterizes all meaningful learning. . . .

It should be pointed out, however, that the occurrence of obliterative assimilation as a natural continuation of the assimilation process itself does not mean that the subsumer returns to its original form prior to the subsumption: the residue of the obliterative subsumption is A', the more stable co-member of A'a', which is the subsumer modified as a consequence of the interaction. Another important remark concerning the process of assimilation is that to describe this process in terms of a single interaction Aa' is an over-simplification. As pointed out by Ausubel, to a lesser extent, the new learning item also forms additional interactional products with other ideas, the amount of assimilation in each case being roughly proportional to a gradient of relevance.

II-4 Progressive Differentiation and Integrative Reconciliation

As meaningful learning occurs the subsuming concepts are developed, elaborated and differentiated as a consequence of the successive interactions. However, concept development proceeds best when the most general, most inclusive elements of a concept are introduced first, and then the concept is progressively differentiated in terms of detail and specificity (Novak, 1977). According to Ausubel, this progressive differentiation should be used when programming subject matter, i.e., the most general and inclusive ideas of the discipline should be presented first, and, then, progressively differentiated in terms of detail and specificity.

However, programming of subject matter must not only provide for progressive differentiation, but must also explicitly explore relationships between propositions and concepts, to point out significant similarities and differences, and to reconcile real or apparent inconsistencies. This must be done to achieve what Ausubel calls <u>integrative reconciliation</u>.

As we said at the beginning, this chapter is just a partial description of Ausubel's theory, but we trust that at least some basic features were highlighted. Further references to his theory in the following chapters will, hopefully, add breadth and depth to this description. For example, progressive differentiation and integrative reconciliation will be discussed in detail in Section III-2 where these principles are relevant in understanding the differences between the two content approaches used in this study.

Chapter III

DESIGN

This experiment was designed to be carried out in a real classroom situation, in a real course, that is, without artificial experimental conditions.

Four groups of students, which were four regular sections of the Physics II course, participated in the experiment. These groups formed two pairs of experimental-control groups, one under the PSI mode of instruction, and the other under a lecture approach. We were in charge of the PSI groups and a colleague of ours who is the best lecturer of our group, was in charge of the lecture groups. Thus, in each of these pairs, the teaching method and the teacher were the same. In addition, the evaluation practices and the grading system, which were the usual ones of the course, were also the same for each pair, as well as the content of the course. The main difference was the conceptual organization and sequencing of the content.

The experiment was carried out during one regular semester, and association tests were used as instruments to look for differences in the way students relate and organize the basic concepts of electricity and magnetism as a result of the conceptual approach of the course. Word association, numerical concept association and graphical concept association tests were administered to the four groups at the beginning, at the middle and at the end of the course. A different test was used to look for differences in students' laboratory performance. (All these tests will be discussed in Sections III-5 and III-6.)

III-1 The Samples

Samples were not randomly selected. Students are free to enroll in any of the eight sections of Physics II, but, once they are enrolled, no changes can be made. They are not aware of which teacher will be in charge of a certain section, nor of which method will be used in this section. The major factor determining preference for a certain section seems to be the convenience of the schedule for the student. For example, students who have a job tend to prefer evening schedules.

Given this schedule preference, care was taken to have each pair of experimental-control groups under the same schedule range. The PSI groups had afternoon classes and the lecture groups had evening classes. In addition, a physics pretest was administered and data was collected concerning several variables, which had been shown to be relevant in previous studies, to assess to what extent the samples in each pair were equivalent or different. These variables are: a) if the student is repeating the course, i.e., if the student failed in a previous semester and is now retaking the course; b) if the student is working besides studying or not; c) the number of hours of work per week for working students; d) the number of courses that students are taking besides Physics II; and e) the year the student entered in the university.

The statistical analysis of the data collected concerning these variables (not necessarily ranked) for each pair of experimental-control groups, i.e., not across pairs, shows the following:

Physics pretest: A 24-item physics pretest on the content of mechanics (the prerequisite) was administered to all four groups before any instruction. The split-half reliability of this test is .74. Means and standard deviations are presented in Table III-1; no significant differences were found at the .05 level of significance.

TABLE III-1

PHYSICS PRETEST RESULTS

Group	Number of Students (N)	Mean (M)	Standard Deviation (SD)	F ratio (F)	t Score (t)
E1 (PSI)	38	13.34	4.24		
C1 (PSI)	36	12.81	4.13	1.06*	.54*
E2 (Lecture)	38	11.24	3.28		
C2 (Lecture)	27	10.96	4.00	1.49*	.31*

*$p > .05$, two-tailed hypothesis.

Average number of courses: This was a point of concern because our previous experience indicated that this variable has an effect on student's performance. For example, students taking too many courses, at the same time, tend to do only the minimum necessary to pass the courses. In spite of being advised, some students take as many as 9 or 10 courses per semester. This fact is reflected in the relatively high averages presented in Table III-2. This table presents the average number of courses, including Physics II, taken by the students involved in the study. No significant difference was found at the .05 level.

TABLE III-2

AVERAGE NUMBER OF COURSES
TAKEN BY THE STUDENTS

Group	N	M	SD	F	t
E1 (PSI)	38	6.74	1.35		
C1 (PSI)	36	6.19	1.49	1.22*	1.67*
E2 (Lecture)	38	5.74	1.95		
C2 (Lecture)	27	5.30	1.41	1.92*	1.00*

*$p > .05$, two-tailed.

<u>Number of students repeating the course</u>: It is obvious that different proportions of students repeating the course in the samples could have an effect. Students retaking the course because they failed in the previous semester, at least have some familiarity with the content of the course, the method, the evaluation procedures, and so on. Tables III-3 and -4 are 2x2 contingency tables showing the frequencies of repeating and non-repeating students in each sample. Given the small frequencies found in Table III-3, the "Fisher exact probability test" (Spiegel, 1956) was used in this case. A chi-square test was used in Table III-4. In both cases no significant differences were found.

TABLE III-3

NUMBER OF STUDENTS REPEATING AND NOT REPEATING THE COURSE IN THE PSI GROUPS

	Repeating Physics II	Not Repeating Physics II		
Group E1	4	34	38	$p = .52^*$
Group C1	3	33	36	
	7	67	74	

*Fisher exact, one-tailed.

TABLE III-4

NUMBER OF STUDENTS REPEATING AND NOT REPEATING THE COURSE IN THE LECTURE GROUPS

	Repeating Physics II	Not Repeating Physics II		
Group E2	10	28	38	$\chi^2 = .06^*$
Group C2	7	20	27	
	17	48	65	

*$p > .05$, two-tailed.

Number of students working besides studying: This is also an important factor in determining student's performance in the course. One obvious reason being the fact that a student who has a job has less time available to study. Some of them barely have time to attend the classes.

Tables III-5 and -6 show the frequencies of working and non-working students in each sample and Table III-7 presents the average number of hours of work per week among working students. No significant differences were found within a given pair.

TABLE III-5

WORKING AND NON-WORKING STUDENTS
IN THE PSI GROUPS

	Working	Not Working		
Group E1	11	27	38	$\chi^2 = .01^*$
Group C1	9	27	36	
	20	54	74	

$^*p > .05$, two-tailed.

TABLE III-6

WORKING AND NON-WORKING STUDENTS
IN THE LECTURE GROUPS

	Working	Not Working		
Group E2	20	18	38	$\chi^2 = .77^*$
Group C2	18	9	27	
	38	27	65	

$^*p > .05$, two-tailed.

TABLE III-7

AVERAGE NUMBER OF HOURS OF WORK PER WEEK AMONG WORKING STUDENTS

Group	N	M	SD	F	t
E1	11	26.36	10.98		
C1	9	25.00	12.09	1.21*	.26*
E2	20	32.50	10.31		
C2	18	30.61	8.51	1.47*	.61*

*p > .05, two-tailed.

Year of admission to the University: The study was carried out in the first semester of 1976, in a course designed for second year (or third semester) students of engineering and science. Consequently, students enrolled in the course should have been admitted to the University in 1975. As previously observed, students who fall behind, for one reason or another, tend to have a poor performance in the course. Thus, if the samples had different proportions in the number of students admitted in 1975 or in previous years, this would be a point to consider. The frequencies of "1975-students" and "before-1975" are presented in Tables III-8 and -9. No significant differences were found.

TABLE III-8

YEAR OF ADMISSION TO THE UNIVERSITY; PSI GROUPS

	Admitted in 1975	Admitted before 1975		
Group E1	31	7	38	$\chi^2 = .03^*$
Group C1	29	7	36	
	60	14	74	

*p > .05, two-tailed.

TABLE III-9

YEAR OF ADMISSION TO THE UNIVERSITY; LECTURE GROUPS

	Admitted in 1975	Admitted before 1975		
Group E2	18	20	38	$\chi^2 = .98^*$
Group C2	17	10	27	
	35	30	65	

*p > .05, two-tailed.

Summing up, four samples of students of science and engineering, forming two pairs of experimental-control groups, were used in the study. One of these pairs was formed by students enrolled in afternoon classes, and the other in evening classes. The samples were not randomly selected, but the groups forming each pair were equivalent in terms of several variables that we considered relevant in the sense that they could cause an initial bias. However, the pairs cannot be considered equivalent. Both our experience and results shown in Tables III-1 to -9 indicate that there are differences between the pairs. For example, the evening pair had lower scores in the pretest; lower average number of courses; more students repeating the course; more working students; higher average number of hours of work among working students; and more students that were admitted to the University before 1975. (These differences were not checked statistically because we are sure that there is a practical significant difference among the population of evening students and the population of afternoon students, and because the main purpose of the study was to look for differences within pairs and not between pairs.) In addition, evening students are usually older and many of them already have family responsibilities.

Thus, the study was conducted under the assumption that the samples were equivalent within pairs, but not between pairs. This assumption was strongly supported later on by the results of the association tests administered at the beginning of the course.

III-2 The Two Content Approaches

As mentioned before, the organizational approach used with the control groups was the usual one based on the course textbook, whereas the organizational approach used with the experimental groups was based on Ausubel's learning theory. It was also noted previously that this was the key difference in treatment. The purpose of this section is to make clear differences in the two approaches, but, in order to achieve this goal, let us first quote some of Ausubel's statements about principles involved in programmatic organization of subject matter (1968, pp. 152-160):

> Once the substantive organizational problem (identifying the basic organizing concepts in a given discipline) is solved, attention can be directed to the programmatic organizational problems involved in the presentation and sequential arrangement of component units. Here, it is hypothesized, various principles concerned with the efficient programming of content are applicable, irrespective of the subject-matter field. These principles naturally include and reflect the influence of the previously listed cognitive structure variables--the availability of a relevant anchoring idea, its stability and clarity, and its discriminability from the learning material.
>
> Progressive Differentiation
>
> When subject matter is programmed in accordance with the principle of progressive differentiation, the most general and inclusive ideas of the discipline are presented first, and are then progressively differentiated in terms of detail and specificity. This order of presentation presumably corresponds to the natural sequence of acquiring cognitive awareness and sophistication when human beings are spontaneously exposed either to an entirely unfamiliar field of knowledge or to an unfamiliar branch of knowledge. It also corresponds to the postulated way in which this knowledge is represented, organized, and stored in the human cognitive system. The two assumptions we are making here, in other words, are that: (a) It is less difficult for human beings to grasp the differentiated aspects of a previously-learned, more inclusive whole than to formulate the inclusive whole from its previously-learned differentiated parts, and (b) An individual's organization of the content of a particular subject-matter discipline in his own mind consists of a hierarchical structure in which the most inclusive ideas occupy a position at the apex of the structure and subsume progressively less inclusive and more highly differentiated propositions, concepts, and factual data.
>
> Now if the human nervous system as a data processing and storing mechanism is so constructed that both the acquisition of new knowledge and its organization in cognitive structure conform <u>naturally</u> to the principle of progressive differentiation it seems reasonable to suppose that optimal learning and retention occur

> when teachers deliberately order the organization and sequential arrangement of subject matter along similar lines. . . .
>
> But even though this principle seems rather self-evident it is rarely followed in actual teaching procedures or in the organization of most textbooks. The more typical practice is to segregate topically homogeneous materials into separate chapters and subchapters and to order the arrangement of topics and subtopics (and the material within each) solely on the basis of topical relatedness without regard to their relative level of abstraction, generality, and inclusiveness. This practice is both incompatible with the actual structure of most disciplines and incongruous with the postulated process whereby meaningful learning occurs, with the hierarchical organization of cognitive structure in terms of progressive gradations of inclusiveness, and with the mechanism of accretion through a process of progressive differentiation of an undifferentiated field. Thus, in most instances, students are required to learn the details of new and unfamiliar disciplines before they have acquired an adequate body of relevant subsumers at an appropriate level of inclusiveness.
>
> As a result of this latter practice, students and teachers are coerced into treating potentially meaningful materials as if they were rote in character and consequently experience unnecessary difficulty and little success in both learning and retention. The teaching of mathematics and science, for example, still relies heavily on rote learning of formulas and procedural steps, on rote recognition of stereotyped 'type problems', and on mechanical manipulation of symbols. In the absence of clear and stable ideas which can serve as anchoring points and organizing foci for the incorporation of new logically meaningful material, students are trapped in a morass of confusion and have little choice but rotely to memorize learning tasks for examination purposes. . . .
>
> Progressive differentiation in the programming of subject matter is accomplished by using a hierarchical series of organizers (in descending order of inclusiveness), each organizer preceding its corresponding unit of detailed, differentiated material, and by sequencing the material within each unit in descending order of inclusiveness. In this way not only is an appropriately relevant and inclusive subsumer made available to provide scaffolding for each component unit of differentiated subject matter, but the ideas within each unit as well as the various units in relation to each other, are also progressively differentiated-organized in descending order of inclusiveness. The initial organizers, therefore, furnish anchorage at a global level before the learner is confronted with <u>any</u> of the new material. . . .

As we will explain later in this section, the principle of progressive differentiation guided the organization and sequencing of the content for the experimental groups. The whole course started with general and inclusive ideas about physics and electromagnetism, which were then progressively differentiated. In addition, the same order of presentation was used in each unit. That is, each unit began with the most inclusive propositions and concepts embedded in the topic that was the object of the unit, then, these propositions and concepts were progressively differentiated.

Integrative Reconciliation

> The principle of integrative reconciliation in programming instructional material can be best described as antithetical in spirit and approach to the ubiquitous practice among textbook writers of compartmentalizing and segregating particular ideas or topics within their respective chapters or subchapters. Implicit in this latter practice is the assumption (perhaps logically valid, but certainly psychologically untenable) that pedagogic considerations are adequately served if overlapping topics are handled in self-contained fashion, so that each topic is presented in only <u>one</u> of the several possible places where treatment is relevant and warranted, the assumption that all necessary cross-referencing of related ideas can be satisfactorily performed, and customarily is, by students. Hence, little serious effort is made <u>explicitly</u> to explore relationships between these ideas, and to reconcile real or apparent inconsistencies. Some of the undesirable consequences of this approach are that multiple terms are used to represent concepts that are intrinsically equivalent except for contextual reference, thereby generating incalculable cognitive strain and confusion, as well as encouraging rote learning; that artificial barriers are erected between related topics, obscuring important common features, and thus rendering impossible the acquisition of insights dependent upon recognition of these commonalities; that adequate use is not made of relevant, previously learned ideas as a basis for subsuming and incorporating related new information; and that since significant differences between apparently similar concepts are often perceived and retained as identical. . . .
>
> Organizers may also be expressly designed to further the principle of integrative reconciliation. They do this by explicitly pointing out in what ways previously-learned, related ideas in cognitive structure are either basically similar to, or essentially different from, new ideas and information in the learning task. Hence, for one thing, organizers explicitly draw upon and mobilize all available concepts in cognitive structure that are relevant

> for and can play a subsuming role in relation to the new learning material. This maneuver effects great economy of learning effort, avoids the isolation of essentially similar concepts in separate, noncommunicable compartments, and discourages the confusing proliferation of multiple terms to represent ostensibly different but essentially equivalent ideas. In addition, organizers increase the discriminability of genuine differences between the new learning materials and seemingly analogous but often conflicting ideas in the learner's cognitive structure. . . .

This principle was also used as a guide in organizing and sequencing the experimental materials. For example, electricity and magnetism were not studied separately. Instead of this, they were seen as instances of electromagnetism and were studied together, whenever possible, explicitly attempting to explore relationships between electric and magnetic phenomena and concepts. In addition, in the experimental approach, effort was made to point out the similarities and differences between new concepts (e.g., electromagnetic force and field) and related concepts already existing in the learner's cognitive structure (e.g., gravitational force and field).

Sequential Organization

> The availability of relevant anchoring ideas for use in meaningful verbal learning and retention may obviously be maximized by taking advantage of natural sequential dependencies among the component divisions of a discipline--of the fact that the understanding of a given topic often logically presupposes the prior understanding of some related topic. Typically the necessary antecedent knowledge is more inclusive and general than the sequentially dependent material, but this is not always true (for example, superordinate learning). In any case, by arranging the order of topics in a given subject-matter field as far as possible in accordance with these sequential dependencies, the learning of each unit, in turn, not only becomes an achievement in its own right but also constitutes specifically relevant ideational scaffolding for the next item of the sequence.
>
> In sequential school learning, knowledge of earlier-appearing material in the sequence plays much the same role as an organizer in relation to later-appearing material in the sequence. . . . For maximally, effective learning, however, a separate organizer should

be provided for each unit of material. Thus, sequential organization of subject matter can be very effective, since each new increment of knowledge serves as an anchoring post for subsequent learning. This presupposes, of course, that the antecedent step is always thoroughly consolidated. . . .

Consolidation

By insisting on consolidation or mastery of ongoing lessons before new material is introduced, we make sure of continued subject-matter readiness and success in sequentially organized learning. This kind of learning presupposes, of course, that the preceding step is always clear, stable, and well-organized. If it is not, the learning of all subsequent steps is jeopardized. Thus, new material in the sequence should never be introduced until all previous steps are thoroughly mastered. This principle also applies to those kind of intra-task learning in which each component task (as well as entire bodies of subject-matter) tends to be compound in content and to manifest an internal organization of its own. Consolidation, of course, is achieved through confirmation, correction, clarification, differential practice, and review in the course of repeated exposure, with feedback, to learning material. . . .

The principles of sequential organization and consolidation were also guiding principles for the experimental approach, but the latter was really applied just in the PSI groups. The order of units was arranged in accordance with some natural sequential dependencies found in electromagnetism (which is not necessarily the sequence found in most textbooks), but in such a way that an antecedent unit was usually more inclusive and general than a sequentially dependent unit. However, mastery of the content of a certain unit as a condition to proceed to the next one was required only for the PSI groups. As a matter of fact, mastery (or consolidation) of ongoing materials before new material is introduced is a basic feature of PSI which can hardly be achieved in traditional group instruction.

Once provided this theoretical background, we can proceed with the description of the two different content approaches used in the experiment. Figure III-1 shows a schematic organization of content sequence for the control groups. This is a classical sequence for an introductory course in electricity and magnetism at college level. It is a linear "bottom to top" sequence that starts with electricity, then goes into magnetism and ends with electromagnetism. The concept of charge, which is a key concept but highly specific, is at the beginning of the sequence, and the Maxwell Equations, which are the general equations describing electromagnetic phenomena, are at the top. In a sense, this sequence is exactly opposed to Ausubel's position because it starts with the specific and ends with the general, whereas Ausubel argues that, in accordance with the principle of progressive differentiation, the most general and inclusive ideas should be presented first. This sequence is much more in accordance with the psychology of Robert Gagné (1965) than with Ausubel's theory:

> Gagné's theory, based on progressively larger units of S-R connections, leads him to postulate that learning is best when we move from mastery of the smallest conceptual units to the more general and inclusive, whereas Ausubel recommends that we proceed <u>from</u> the more general, more inclusive to specific subordinate concepts in the process of progressive differentiation of cognitive structure (Novak, 1977).

Another possible criticism of this sequence under an Ausubelian framework is that, except in the final units, it segregates electricity from magnetism. That is, the course begins with the study of electricity, then follows the study of magnetism, and finally they are put together in the study of electromagnetism. (Ausubel's criticism to this practice can be found at the beginning of this section.) Under an Ausubelian point of view, we should speak about electromagnetism since the beginning because this is a more general and inclusive idea that encompasses both electricity and magnetism.

MAXWELL EQUATIONS	
↑	
ELECTROMAGNETIC INDUCTION	(Faraday's Law, time varying fields, induced fields, . . .)
↑	
MAGNETIC FIELD	(Ampère's Law, Biot-Savart's Law, . . .)
↑	
MAGNETIC FORCE	(definition of $\vec{B}$, magnetic force on a current, . . .)
↑	
ELECTRIC CURRENT	(Ohm's Law, resistance, circuits, . . .)
↑	
ELECTRIC POTENTIAL	(electric potential energy, capacitors, . . .)
↑	
ELECTRIC FIELD	(Gauss's Law, . . .)
↑	
ELECTRIC FORCE	(Coulomb's Law, . . .)
↑	
ELECTRIC CHARGE	(conductors, insulators, conservation, quantization, . . .)

Fig. III-1. Content Sequence for the Control Groups.

As mentioned before, this is the sequence of the textbook of the course (Halliday and Resnick, 1966), which was the main reading assignment (if not the only one) of the control groups. All the lectures of the lecture control group and all the study-guides of the PSI control group were based on this book.

Before moving to the approach used with the experimental groups, a point must be made: the fact that the content approach used with the control groups can be criticized under an Ausubelian point of view should not serve as a bias to judge it a "bad appraoch." On the contrary, the content sequence is the classical one, the same one found in hundreds of books and courses, and the textbook is one of the most successful books in college physics and it is used world-wide in similar courses.

Figure III-2 shows a scheme of the organization of the content used for the experimental groups. This approach is an attempt to organize the content in accordance to the Ausubelian principles described at the beginning of this section. However, these principles were used as a frame of reference, not as a kind of "recipe." That is, the approach is Ausubelian in the whole and not necessarily in the details. For example, the use of organizers in the written materials was not rigorous or systematic as one could expect if the objective of the study were to investigate the specific effect of organizers.

The sequence starts at a very general level with a discussion about physics and its evolutionary nature; about what physicists do; about classical and modern physics; about the role of concepts in physics, and so on. This first unit ends with a general map of classical physics emphasizing key concepts and its use in different subdisciplines of physics. After this general view of physics and concepts in physics, the next unit is a little bit more specific because it focuses on two concepts only: forces and fields. However, it is also general because it deals with forces and fields describing the gravitational, electromagnetic and nuclear interactions emphasizing the concepts of force and field and not any particular instance of these concepts.

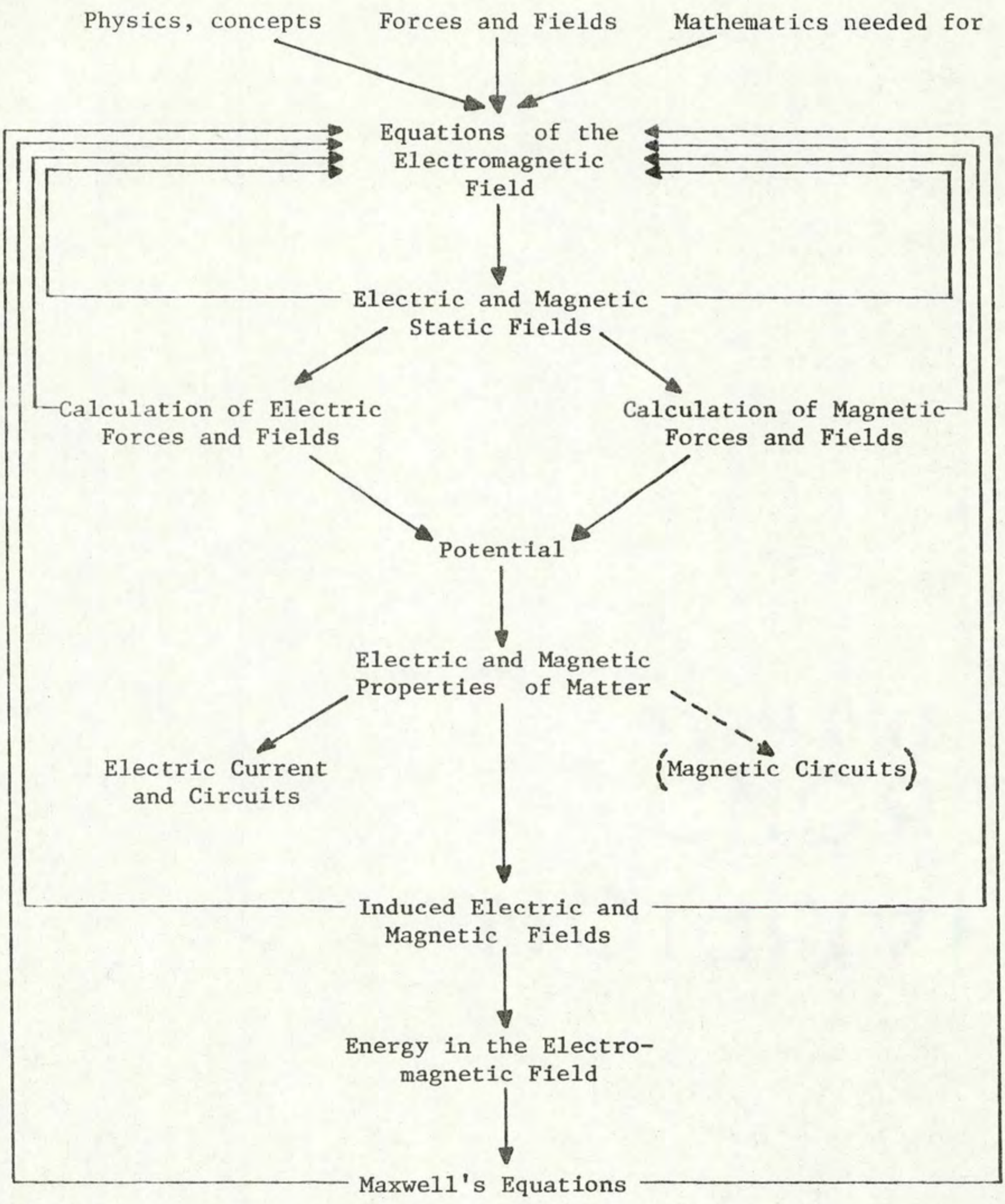


Fig. III-2. Content Organization for the Experimental Groups.

In accordance to the principle of progressive differentiation, the next unit of the sequence is more specific than the previous ones because it deals only with the electromagnetic interaction and the concepts of electromagnetic force and field. On the other hand, this unit is a general view of electromagnetism which is also an organizer for the entire course. All the basic electromagnetic phenomena and concepts are introduced in this unit in a very general and qualitative way. The four Maxwell Equations are also introduced as general equations describing electromagnetic phenomena. However, they are not used quantitatively, but rather discussed qualitatively trying to unpack their meaning. This unit ends with a general map of electromagnetism relating the key concepts and the Maxwell Equations.

In order to continue with the progressive differentiation of Maxwell Equations we should start to use Maxwell Equations analitically in the integral form. However, instead of just assuming that students have the needed mathematical concepts, a unit on mathematics was inserted in the sequence to review such concepts. (This was also done with the control groups.) After this, the units become more and more specific but keep electric and magnetic phenomena and concepts together whenever possible, emphasizing the analogies and pointing out the differences as well. For example, in the unit on potential some time was devoted to the scalar magnetic potential to keep the analogy, but it was remarked that this concept is not useful in the magnetic case due to the inexistence of the magnetic counterpart of the isolated electric charge. A brief discussion on magnetic circuits was also included (not as a whole unit) to keep the analogy between electric and magnetic phenomena, but, specially in this case, it emphasized that this was just an analogy.

After the study of induced electric and magnetic fields and the electromagnetic energy, the sequence ends with a discussion of Maxwell Equations and the general map of electromagnetism introduced in the third unit of the sequence. However, as indicated by the arrows in Fig. III-2, constant reference to the general equations and basic phenomena is made during the course. For example, when Ampère's Law must be used to calculate a magnetic field, it is pointed out that this law is a particular instance of a more general one which is one of the Maxwell Equations.

According to Novak (1977), to achieve integrative reconciliation more surely, we must organize instruction so that we move "up and down" the conceptual hierarchies as new information is presented. This is another basic difference between Gagné's and Ausubel's theory of progressive concept development: Gagné's hierarchy points only upward indicating the sequence in which learning must proceed (as in the sequence of the control groups), whereas Ausubel contends that "cycling" from more general to more specific concepts and "backing up" again is needed to achieve integrative reconciliation.

Not surprisingly, no book was found coherent with this content approach. Consequently, written materials were prepared for the experimental group under the form of "Notes." These "Notes" were written for all units of the course and, specially in the first units, they were the main reading assignment for the students. In later units, it was frequently possible to direct the students to selected sections of the same textbook used by the control groups. Further details of these "Notes" and the corresponding study-guides will be given in the following sections where the two methods used in the experiment will be described.

III-3 The Methods

III-3.1 The PSI method

PSI is an individualized system of instruction, based on principles of positive reinforcement, with the following basic features (Moreira, 1972 and 1975):

1) Self-pacing: the student proceeds through the material of the course with his own pace. He may spend as much time as he needs to master the content of a unit of study, and to achieve the objectives of such a unit.

2) Mastery-learning: the student is allowed to proceed to a certain unit of study only after he has mastered the content of the preceding one.

3) Emphasis on written materials: the use of this type of instructional materials is emphasized in the communication between teacher and students.

4) Lectures and demonstrations as vehicles of motivation: when used, lectures and demonstrations are vehicles of motivation rather than critical sources of information.

5) The use of proctors: proctors are students who have already taken the course and showed a high degree of mastery on the content of the course. They help the teacher in providing individualized attention to the students and to carry out the evaluation of unit-tests.

In practice, the PSI method was used, with one pair of experimental-control groups, as follows:

The content of the course was divided into 17 units, and, for each one, a study-guide and a set of five tests were prepared. Each study-guide contained an introduction relating the content of the unit with the content of the preceding and the following ones, a set of objectives, and a suggested procedure designed to lead the student to the achievement of these objectives. This procedure included a sequence of suggested readings (sections of the textbook or the special "Notes" prepared for the experimental group), questions, problems and additional references. (Answers to the suggested problems were also provided in the study-guides.) The five unit-tests were based on the objectives of the unit. They were equivalent and usually contained five or six items (essay questions and problems).

After receiving the study-guide of the first unit, the students could study the unit at their own pace, and present themselves for a test on the content of the unit when they felt prepared to take it. The tests were immediately evaluated by the proctors or by the teacher.

If the performance of a student on the unit was considered as evidence that he had mastered the content of the unit, he received the study-guide of the next unit and was allowed to proceed. If not, the student could take as many tests as necessary to achieve the objectives of the unit. There was no penalty for retaking unit tests.

Three two-hour classes were scheduled per week, but no lectures were given. Instead, these periods were used for individual assistance and testing. That is, during these periods, students could come to the classroom to ask for individual assistance or take unit-tests. The teacher and four proctors were always available during these "class-hours."

At the end of the semester, students who completed all the units were given a final grade of A. Students who did not complete all units but completed at least 75% of them, should take a final exam on the content of the remaining units. In such a case, the final grade was given as a function of the number of completed units and the result of the exam. Students who did not complete at least 75% of the units, received a failing grade.

All the methodological variables were the same for the PSI experimental and control groups except for the sequence of units, which was a natural consequence of the two different content approaches. The two sequences of units are presented in Figure III-3. The content of each unit can be found in Appendices I (experimental group) and II (control group).

III-3.2 The Lecture Method

The other pair of experimental-control groups also had three two-hour classes per week. But, in this case, the class periods were used in the traditional way: in general, a lecture was given during the first hour and the second one was used for discussion and problem-solving. The lectures followed the sequence of the textbook for the control group, and the sequence of the "Notes" for the experimental group and, most of the time, they were designed to help the students grasp the content of these written materials. In the discussion and problem-solving sessions the teacher usually proposed a question or a problem and provided some time for the students to try their own solutions. After this, in most cases, the teacher provided the right solutions.

Few laboratory sessions were scheduled during the semester and they usually included more demonstrations than experiments performed by the students. This is a result of the extensive program that must be covered by

UNIT	EXPERIMENTAL GROUP	CONTROL GROUP
I	Physics and Concepts	Electric Charge and Electric Force
II	Forces and Fields	Mathematics Review
III	The Electromagnetic Interaction	The Electric Field
IV	Mathematics Review	Gauss's Law
V	Electric and Magnetic Static Fields	Electric Potential
VI	Calculation of Electric Forces and Fields	Study of an Electrostatic Field (Lab)
VII	Calculation of Magnetic Forces and Fields	Capacitors and Dielectrics
VIII	Potential	Current - Resistance - Emf
IX	Study of an Electrostatic Field (Lab)	Linear and Nonlinear Resistors (Lab)
X	Electric and Magnetic Properties of Matter	RC Circuit (Lab)
XI	Electric Current and Circuits	The Magnetic Field
XII	Linear and Nonlinear Resistors (Lab)	Ampère's Law
XIII	RC Circuit (Lab)	Biot-Savart's Law
XIV	Electric and Magnetic Induced Fields	Faraday's Law
XV	Electromagnetic Induction (Lab)	Electromagnetic Induction (Lab)
XVI	Energy in the Electromagnetic Field	Inductance
XVII	Maxwell Equations	Mag. Prop. of Matter-Maxwell Equations

Fig. III-3. The Two Unit Sequences.

the teacher during the semester: no time is left for laboratory, which, although extremely important, is very time consuming. (Thus, the lack of emphasis on the laboratory is circumstantial and not a design feature.)

Course evaluation was carried out through five quizzes and a final examination. The quizzes were different for the two groups because the content sequence was different, but the final exam was the same.

III-4 The Written Materials

III-4.1 The study-guides

A general idea about the written materials used in the experiment was already given in the two previous sections where the content approaches and the teaching methods were discussed. This section, however, will focus on these materials in a more detailed way.

Let us start with the control groups: the written materials used by the control groups were the textbook and the study-guides of each unit. As a matter of fact, the study-guides were designed primarily for the PSI control group, but they were also used by the lecture control group as an additional instructional aid. (In this case the distribution of the study-guides was made after the corresponding lectures.) These study-guides, with minor modifications, were the same used in previous semesters with very satisfactory results. They were carefully designed to be used with the textbook. They contained an introduction, a set of behavioral objectives, and a detailed suggested procedure which the students were encouraged to follow. A sample of one of these study-guides is in Appendix III.

The experimental groups also used the textbook (but to a much lesser extent) and study-guides. The general format of the study-guides was the same, however, the introduction might have been more general; the objectives in many cases were not strictly behavioral; the suggested procedure sometimes was not so detailed; and, usually, study-questions and problems other than those of the textbook were used. These minor differences in the format of the study-guides were a natural consequence of the different approach used with the experimental groups. (By the way, the lecture experimental group also used the study-guides, but only as an additional instructional aid.) An example of one of the study-guides for the experimental groups can be found in Appendix IV.

In addition, the study-guides of the experimental groups referred the students to the "Notes" where the real emphasis on an Ausubelian approach was placed.

III-4.2 The "Notes"

As mentioned before, written materials in the form of "Notes", to be used only by the experimental groups, were prepared for all the seventeen units. (Four of them were concerned with laboratory experiments and will be discussed in Section III-6.) In the first four units, these "Notes" constituted the main reading materials used by the students. Because of their introductory and general character (they were some sort of advance organizer for the whole course), it was not possible to use the textbook or any other available book. However, starting in the fifth unit, the "Notes" referred the students to sections of the textbook whenever possible, and at a time appropriate for the experimental approach. These "Notes" usually started at a general level, reviewing some already known concepts, situating the student in the whole context of the course, focusing on some historical aspects of the subject, etc. Next, the subject or the basic concepts of the unit were discussed with more specificity before going into the calculations which represented the greatest degree of detail. The "Notes" usually ended with a diagram where concepts and, sometimes, phenomena, examples and equations were put together attempting to "integrate or organize" the content of the unit. That is, the "Notes" were written in accordance with the principles of progressive differentiation discussed in Section III-2. In addition, the first part of the "Notes" was designed to work as an advance organizer, i.e., to bridge the gap between what should be learned and what the learner already knew. Examples of "Notes" are in Appendices V, VI and VII. For example, "Notes II" (Appendix V), which is concerned with forces and fields, starts with a review of Newton's Laws and the definition of force through the second of these laws (which the learners had already studied in Physics I), then, the three basic forces of Nature are discussed under a general point of view. Following this, the electric and magnetic forces are discussed before the presentation of a general "map" of forces. Fields are discussed in this unit in a similar fashion. Another example is the unit on potential (Appendix VI): before focusing on the electric potential, the concept of potential is discussed and the gravitational potential (already studied in Physics I) is recalled. Then, the concept of electric potential is discussed before going into the details, i.e., the calculation of the electric potential. The magnetic potential is also discussed in these "Notes" which end with a general "map" for potential.

The principles of progressive differentiation and integrative reconciliation were used as guides not only within the "Notes" of each unit, but also in the whole set of "Notes." For example, "Notes II" (Appendix V) which corresponds to the beginning of the course was quite general; "Notes VIII" (Appendix VI) which corresponds to the middle of the course is more specific. On the other hand, "Notes XVII" (Appendix VII) which corresponds to the last unit attempts to integrate the whole course.

Perhaps other examples of "Notes" would provide a better picture of the nature of the written materials used by the experimental groups, but, most of them, were too extensive to be reproduced even as appendices.

As it can be inferred from the above discussion and from the examples provided in Appendices V, VI, and VII a special feature of the "Notes" was the use of "concept maps."

III-4.3 Concept Maps

Concept maps, as they were used in this study, are simply sets of key concepts put together in two-dimensional diagrams showing a hierarchical relationship between them. Concept maps may be drawn for a whole discipline, for a subdiscipline, for a specific piece of knowledge in a subdiscipline, and so on. There are many ways of drawing a concept map, i.e., there are different ways of showing a conceptual hierarchy in a diagram. In addition, concept maps drawn by different experts in a field probably will reflect minor differences in understanding and interpretation of the relationships between the key concepts of such fields. The point is that a concept map must always be seen as "<u>a</u> concept map" and not "<u>the</u> concept map" of whatever set of concepts it is representing. On the other hand, at least in a discipline with a well defined conceptual structure like physics, one should not expect large and contradictory differences in two maps for the same concepts drawn by two different experts.

We prefer the type of map shown in Figure III-4. In this map, the most inclusive and subsuming concept (or concepts) is at the top, whereas subordinate concepts are at an intermediate position between the broadest concept and the least inclusive concepts or examples. That is, this map has a vertical downward hierarchy indicating subsuming relationships between concepts. This map is coherent with Ausubel's principle of progressive differentiation or progressive concept development. It should be noted, however, that the lines connecting the concepts are not arrows pointing downward, they are just lines indicating a relationship between the concepts. The reason is that these arrows would indicate a unidirectionality not implied by Ausubel's principle of progressive differentiation. As a matter of fact, "cycling" from more general to more specific concepts and "backing up" again is needed to achieve integrative reconciliation of concepts (Novak, 1977). (Incidentally, a concept map following Gagné's model would have arrows pointing upward only.)

We have used concept maps in almost all "Notes," mostly at the end as an element of integration of the unit content. Most of our maps were similar to Figure III-4 which is obviously an oversimplification of a concept map, but in some cases we have added equations and laws to the maps as well. In other cases, when several subsuming concepts were presented in the same map (e.g., the general concept map of electromagnetism

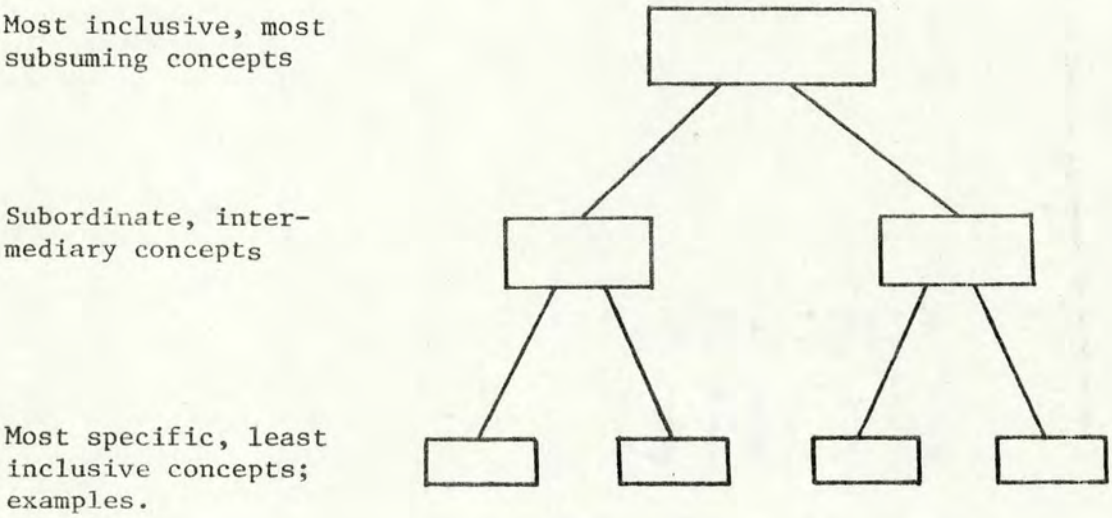


Fig. III-4. A Simplified Schematic Representation of a Concept Map.

used in "Notes III and XVII") we didn't follow the pattern of Figure III-4. The point is, again, that concept maps are simply a diagrammatic representation of a conceptual hierarchy, without rigid rules. Figure III-5 presents a concept map for "induced fields" that we have used at the end of "Notes XIV." Other examples can be found in Notes II, VIII and XVII (Appendices V, VI and VII).

Obviously, in any of these maps, much more connecting lines between concepts could have been drawn and many other related concepts could have been induced. However, one should bear in mind that, in the process of drawing a concept map, there is a trade-off between clarity and completeness.

Concepts maps, as will be shown in Chapter IV, played an important role in this study.

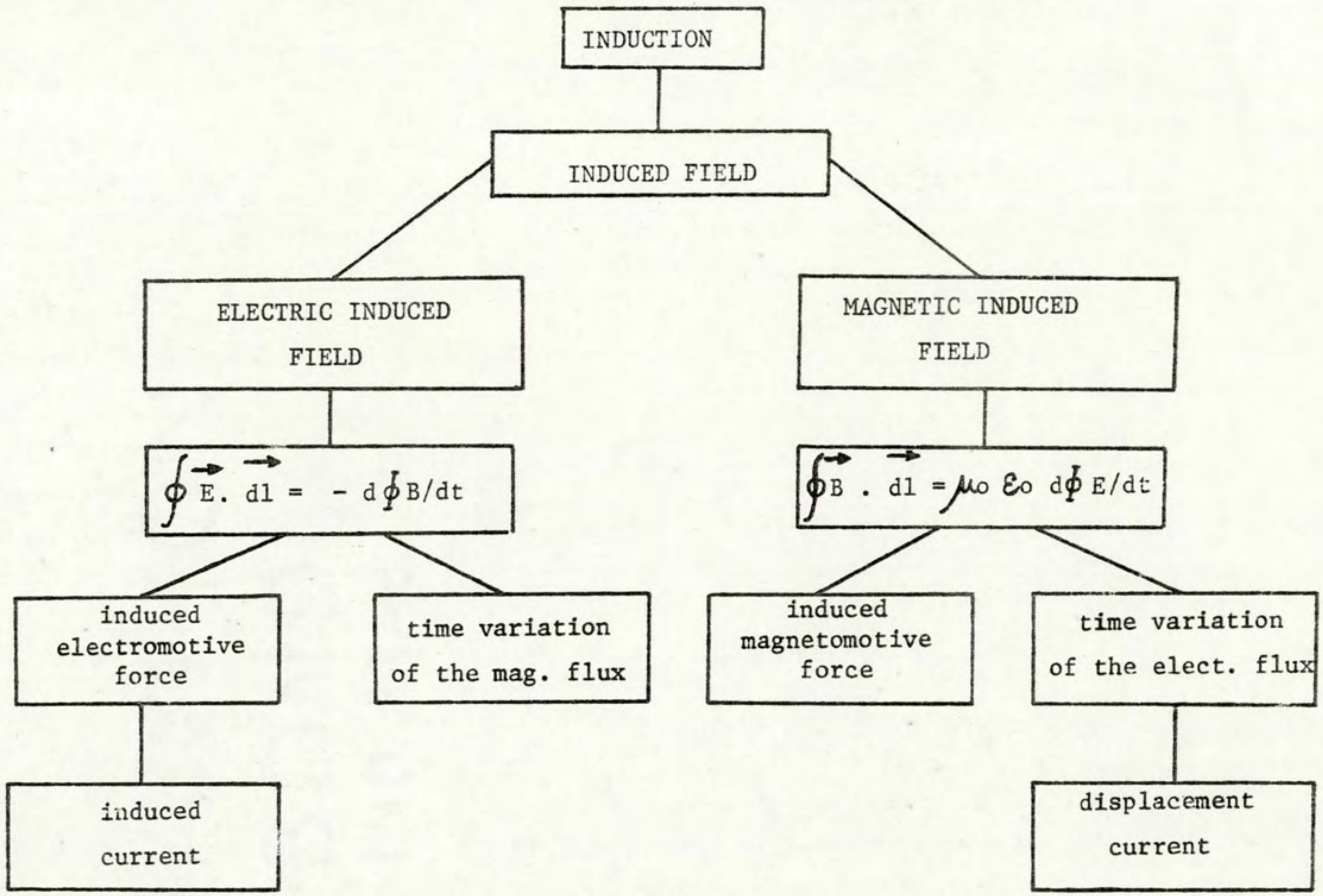


Fig. III-5. A Concept Map for Induced Fields.

III-4.4 The Unit-tests

In addition to the study-guides and "Notes," sets of five equivalent tests were prepared for each PSI unit, except those of laboratory. These tests were based on the unit objectives and consisted of problems and essay-type questions. Due to the different content sequences, the unit-tests were different for the two PSI groups, but, except in the first units, in most cases items corresponding to certain topics were the same. That is, some items of the unit-tests of "unit x" from one PSI group could be found in the unit-tests of "unit y" from the other one. Thus, the type of test and test items were quite similar (and traditional) for both groups.

Later on, we realized that we might have made a logical mistake in using similar evaluation tests: if a new approach was being used with the experimental group, coherent evaluation tests should have been designed for such a group. Instead, we have used evaluation tests primarily designed for a conventional approach. At least to some extent, this could have been unfair for the experimental group. Samples of unit-tests for both groups are in Appendix VIII. The quizzes for the lecture groups were similar in format and content.

III-5 The Concept Association Tests

Three tests of concept association were administered at the beginning (i.e., prior to any instruction), at the middle and at the end of the course. These tests were not used for course evaluation purposes, i.e., they were used only for research purposes and the students of all groups were aware of this.

III-5.1 The word-association test

The first of these tests was a word-association test (Shavelson, 1974): fifteen physical concepts relevant to the study of electromagnetism were selected and the students were asked to associate as many words as they could to each of these concepts in a given period of time. The selected concepts were: Electromotive Force - Electric Resistance - Inductance - Magnetic Field - Potential - Energy - Force - Electric Current - Electric Flux - Electric Field - Electric Charge - Magnetic Flux - Time - Work - Capacitance.

Each of these concepts was at the top of a separate page of a test booklet, and the students were given one minute for each concept to write down, in the corresponding page, as many words as they could in association to the given concept. They were instructed that the associated words should be from the field of physics. The time was controlled by the teacher or by a proctor. A condensed version of this test, including the specific instructions given to the students, is in Appendix IX. This Appendix also includes a brief questionnaire given to the students only at the beginning of the course, and the two other association tests.

III-5.2 The numerical concept association test

The second test, which we labeled "numerical-association test," was based on a "similarity judgments" test that we found in the literature (Johnson, 1967).

Each of the 15 concepts used in the word association test was paired with every other concept to create a list of 105 pairs of concepts. Each pair of concepts in this test was followed by a 7-point rating scale, '1' standing for the highest degree of relationship between the concepts of a certain pair according to the student's point of view, and '7' standing for the smallest degree of relationship. That is, the student should check number 1 when the concepts were highly related and number 7 when he did not see any relationship between them. Numbers 2 to 6 represented intermediate situations. There was no time limit for this test. The test, with the detailed instructions given to the students, is in Appendix IX.

III-5.3 The graphical association test

This test, which could be called a "concept mapping test," was designed specifically for this experiment. We were interested in knowing how the students would draw a concept map for a given set of concepts, i.e., how they would draw a diagram showing a hierarchical order among the given concepts. However, we could not use terms like concept map, superordinate, and subordinate concepts because they would not be potentially meaningful for the students. In addition, we were not willing to introduce a bias toward the shape of the map by suggesting, for example, a vertical hierarchy.

Thus we gave the students a list of 19 concepts (which included most of the previous ones) and very simple instructions to follow. They should display the concepts in a sheet of paper in whatever distribution they prefer, obeying the following rules: the most general concepts should be written inside rectangles; the concepts at an intermediary level of generality should be written inside circles or ellipses; and the least general concepts should be just written without any geometrical figure around them. In addition, related concepts should be linked through lines of arbitrary size and shape. (No example was provided and there was no time limit.) This test is also in Appendix IX.

All students in the lecture groups answered the tests at the same time in the three times they were administered. However, students in the PSI groups answered them individually at the second and third times (after the 10th and 17th units, respectively) because of the individualized nature of the teaching format. In any case, however, students were very cooperative in answering the tests.

III-6 The Laboratory

As explained before, for the lecture groups it was not possible to schedule laboratory classes where the students would perform experiments. For those students, most laboratory classes were just demonstrations.

On the other hand, factors like the use of proctors and opportunity to spread the use of experimental kits over a span of time permitted the insertion of laboratory units in the unit sequence of the PSI groups. Four, among the seventeen units programmed for the PSI groups, were laboratory experiments:

Study of an Electrostatic Field
Linear and Nonlinear Resistors
R.C. Circuit
Electromagnetic Induction.

These experiments usually could be done in a two-hour period and were the same for the experimental and control groups, in spite of differences in the corresponding unit number. However, the lab-guides were different:

Those used by the control group were programmed lab-guides which were used in previous semesters. These guides contained a brief introduction, a set of objectives and a procedure in the form of programmed instruction. The first questions of the procedure were designed to provide a theoretical framework for the experiment, whereas the last ones asked the student to perform experimental tasks. Considering the feedback from the use of these guides in previous semesters we can say that they are well designed and highly efficient in helping the students achieve the objectives of the experiment. That is, they are instructional materials of good quality. (A sample is in Appendix X.)

However, we felt that programmed lab-guides would be inconsistent with the Ausubelian approach. Thus, we prepared special guides and "Notes" for the experimental groups.

The introduction of the "Lab-Notes" consisted of a general discussion about theory and experimentation in physics designed to serve as an advance organizer for the experiment. After this introduction, the theoretical basis of the experiment and some practical difficulties were discussed.

A brief study-guide accompanied the "Lab-Notes" of each experiment. It contained a small introduction, the objectives of the experiment, and a suggested procedure that referred the students to the "Lab-Notes" and encouraged them to try to identify the basic phenomena and concepts involved in the experiment as well as the basic questions to be investigated. This procedure also encouraged the students to do the experiment, to collect data, and so on, but it was not a sequence of steps to be followed.

Thus, the lab-guides were different. However, it must be emphasized that this difference has nothing to do with discovery learning versus reception learning. The experiments were the same, the objectives were almost the same and the lab-guides were different only to the extent that this was necessary to keep them coherent with the corresponding content approaches.

The evaluation of the lab-units was carried out through the presentation and discussion of the results with a proctor (or the teacher). In addition, students answered a short written test which was administered only for research purposes. However, in that case the students were not aware of this and, at least in the first lab-units, they thought that the test was an integrant part of unit evaluation.

This test consisted of five questions which were essentially Gowin's (1970) "five questions." These questions were devised by D. B. Gowin as a method for the critical analysis of documented claims:

1) What is the telling question(s)?
2) What are the key concepts?
3) What are the methods of inquiry?
4) What are the knowledge claims?
5) What are the value claims?

For example, in analyzing or "unpacking" a research paper, the <u>telling question</u> is the question that identifies the phenomenon of interest under study, in such a way that it seems likely that something will be found out by answering this question. The <u>key concepts</u> are the basic concepts, of the field of study to which the paper is concerned, that are involved in the telling question or in the research itself. The <u>methods</u> of inquiry are the sequence of steps, the techniques of investigation, the devices that were used to answer the telling question, i.e., to go from the telling question to the <u>knowledge claims</u>. The <u>value claims</u> are claims about the significance, utility, importance of the knowledge claims.

We thought that this approach would be also useful to evaluate student's performance in the laboratory and we designed a test with the following questions:

> 1) What was(were) the basic question(s) that you tried to answer by performing the experiment, i.e., what were you really investigating? [telling question]
>
> 2) What key concepts of electromagnetism (basic, fundamental concepts) were involved in the formulation of such question(s)? [key concepts]
>
> 3) What basic electromagnetic phenomenon(a) was (were) involved in the experiment that you performed?
>
> 4) What method did you use to answer the basic question(s) that you investigated? [method?]
>
> 5) What results did you get, i.e., what answers did you get for what you were investigating? [knowledge claims?]

Students were asked to answer these questions after each laboratory experiment. The results of this test, as well as the results of the other tests used in this investigation will be presented in the next chapter.

Chapter IV

RESULTS

As remarked in the last paragraph of the Introduction, this study was carried out to investigate the effect of an experimental approach, based on Ausubel's learning theory, to the content of the Physics II course in comparison to the conventional approach based on the textbook (Halliday and Resnick, 1966). However, as can be inferred from the design, and specially from the research instruments, strong emphasis was put in the search of differences in terms of concept association, differentiation and hierarchical organization. In addition, it was assumed that the achievement tests used throughout the course would provide additional measures of concept learning.

Thus, the overall "telling question" of this study could be formulated as follows:

> What differences, in terms of student's ability to apply, to associate, to differentiate, and to hierarchically structure electromagnetic concepts would arise from an Ausubelian approach, in comparison to the traditional one, to the content organization of an introductory college course in electromagnetism?

The laboratory experiment, although an integrant part of the study and coherent with the Ausubelian approach, led to a somewhat different "telling question" that will be discussed in the last section of this chapter.

The key concepts of Ausubel's theory (as well as those of electromagnetism used in the study) embedded in this question were already identified and discussed in the description of the design and in the discussion of Ausubel's theory. The method used to look for answers to the "telling question" was also described in the previous chapters. Thus, the remaining chapters have to do with the results, the answers to the "telling question," the discussion of the results and implications of our findings. That is, in Gowin's language these chapters deal with the "records," the "knowledge claims," and the "value claims."

We will start the discussion of the results with what we could call achievement measures, then, the association tests and, finally, the laboratory test will be discussed.

IV-1 Achievement Measures

Under the label of "achievement measures" we will include all measures other than the association and laboratory tests. Thus, measures like pass and fail indexes, number of tests/unit, and results of the final exam will be included in this category.

Tables IV-1 and IV-2 show the number of students who withdrew and the number of students who completed the course, among those who were originally enrolled in the course. By "students who withdrew" we mean students that abandoned the course for any reason, at any time during the semester (however, almost all withdrawals occurred during the first weeks of the semester) By "students who completed the course" we mean those students that followed course activities up to the end of the semester, passing or not. No significant differences were found at the .05 level of significance between experimental and control groups.

TABLE IV-1

NUMBER OF STUDENTS WHO COMPLETED THE COURSE VS.
NUMBER OF WITHDRAWALS IN THE PSI GROUPS

	Completed the Course	Withdrew from the Course		
Group E1	38	11	49	$\chi^2 = .0014^*$
Group C1	36	9	45	
	74	20	94	

$^*p > .05$, two-tailed.

TABLE IV-2

NUMBER OF STUDENTS WHO COMPLETED THE COURSE VS.
NUMBER OF WITHDRAWALS IN THE LECTURE GROUPS

	Completed the Course	Withdrew from the Course		
Group E2	38	10	48	$\chi^2 = .17^*$
Group C2	27	10	37	
	65	20	85	

$^*p > .05$, two-tailed.

Students who withdrew were not included in the samples discussed in Section III-1 because these students did not really participate in the experiment. However, as they were enrolled in the course and most of them had at least some initial contact with the course, we decided to check if there was any significant differences in this aspect.

The following results concern only the students that constituted the samples. Tables IV-3 and IV-4 show the proportions of passing and failing in each group. No significant differences were found.

TABLE IV-3

NUMBER OF STUDENTS THAT PASSED OR FAILED THE COURSE IN THE PSI GROUPS

	Passed	Failed		
Group E1	37	1	38	$p = .74^*$
Group C1	35	1	36	
	72	2	74	

*Fisher exact, one-tailed.

TABLE IV-4

NUMBER OF STUDENTS THAT PASSED OR FAILED THE COURSE IN THE LECTURE GROUPS

	Passed	Failed		
Group E2	32	6	38	$p = .44^*$
Group C2	24	3	27	
	56	9	65	

*Fisher exact, one-tailed.

The numbers of students who completed all units and those that did not complete them (and took a final exam) in the PSI groups are in Table IV-5. The difference was not statistically significant.

TABLE IV-5

NUMBER OF STUDENTS THAT COMPLETED AND DID NOT COMPLETE ALL THE UNITS IN THE PSI GROUPS

	Completed All Units	Did Not Complete All Units		
Group E1	29	9	38	$\chi^2 = .61^*$
Group C1	31	5	36	
	60	14	74	

$^*p > .05$, two-tailed.

The average number of tests taken per unit in the PSI groups is presented in Table IV-6. The difference was not significant again.

TABLE IV-6

AVERAGE NUMBER OF TESTS TAKEN PER UNIT IN THE PSI GROUPS

Group	N	M	SD	F	t
E1	38	1.46	.23		
C1	36	1.37	.21	1.25^*	1.75^*

$^*p > .05$, two-tailed.

The last of these achievement measures, a very important one under a traditional point of view, is the final exam of the lecture groups. Although the quizzes were different for these groups due to the different content sequences, the final exam was the same. It was a conventional final exam including several essay type questions and problems. The results of this are in Table IV-7. Once more there was no significant difference.

TABLE IV-7

RESULTS OF THE FINAL EXAM FOR THE LECTURE GROUPS

Group	N[1]	M	SD	F	t
E2	33	6.83	1.29		
C2	25	6.56	1.11	1.35*	.84*

*p > .05, two-tailed.

1. Some students who failed the course did not take the final exam.

Summing up, no significant differences were found in terms of achievement. The pass and fail indexes were the same, the results of the final exam were the same, and so on. Should we be surprised?

Probably not, because all these measures were based on conventional achievement tests, and, in such a case, the finding of "no significant difference" is hardly surprising. That is, when conventional instruments are used to evaluate a new approach we should not expect to detect major differences because these instruments probably are not appropriate to detect them. In physics, the situation is quite similar: how could one expect to find different spectral lines in the spectrum of two different light sources using an instrument that does not detect a line spectrum?

In our case, the Ausubelian approach emphasized concepts and conceptual hierarchies, but the unit-tests, quizzes and final exam were conventional achievement tests in physics, emphasizing analytical and numerical problems. Consequently, the fact that no differences were found in terms of achievement should not be surprising. On the other hand, this finding suggests that the Ausubelian approach was at least as efficient as the conventional approach, in terms of conventional achievement measures.

IV-2 The Word-Association Test

As explained in Section III-5.1, in the word-association test students received a list of fifteen concepts and were given one minute per concept to write down words they associated with each concept. The average number of words associated per concept (that is, the average of the averages corresponding to each concept) by each group is in Table IV-8. In this Table, A1 stands for the first time the test was administered (A for association, 1 for first), A2 for the second time and A3 for the third time (beginning, middle and end of course, respectively).

TABLE IV-8

AVERAGE NUMBER OF WORDS ASSOCIATED PER CONCEPT

Group	A1 M	A1 t	A2 M	A2 t	A3 M	A3 t
E1	4.56		5.96		6.45	
C1	4.18	.88*	5.63	.66*	6.00	1.83*
E2	4.01		4.83		5.78	
C2	3.96	.15*	5.30	-1.22*	5.83	-.16*

*$p > .05$, two-tailed. $N_1 = N_2 = 15$

As can be inferred from Table IV-8, there was no significant difference in terms of average number of words associated per concept. However, this is a quite general and condensed analysis of the large amount of data provided by this test. Thus, in order to get further information a more detailed analysis was done.

First of all, we established a distinction between associated words and "significant associations." By associated word we mean any word associated with a given concept; however, by "significant association" we mean words that, according to our judgment, were significantly related to the given concept. Usually, these words were other concepts rather than just common words. For example, "current" was considered a "significant association" for the concept of "electric resistance," whereas "length" and "width" were not. It must be emphasized that the criterion was not "right" or "wrong" associations, nor "words belonging to the terminology of electromagnetism" and "not belonging." Units of measure, for instance, were not considered "significant associations," but they would be "right associations" if this criterion had been used. Our criterion could be better described as distinguishing between "more important" and "less important" (secondary) associations, according to our own judgment of "conceptual relevance."

This criterion was, obviously, subjective but it was applied uniformly to the test of each student in each group. The general result, i.e., the average number of "significant associations" per concept is in Table IV-9. The general pattern of this Table is the same of Table IV-8, with one exception: in case A3 an inversion occurred between groups E2 and C2, that is, group C2 had more associations, but less "significant associations" in case A3. The differences, however, were not statistically significant again.

To search for differences in terms of variability, we looked at the standard deviations. Tables IV-10 and IV-11 show the mean standard deviations for total associations and significant associations, respectively.

TABLE IV-9

AVERAGE NUMBER OF SIGNIFICANT ASSOCIATIONS

Group	A1 M	A1 t	A2 M	A2 t	A3 M	A3 t
E1	2.26		4.28		5.36	
C1	1.93	.75*	4.02	.44*	4.86	1.30*
E2	2.09		3.31		4.85	
C2	2.01	.21*	3.66	-.71*	4.41	1.12*

*p > .05, two-tailed. $N_1 = N_2 = 15$

TABLE IV-10

MEAN STANDARD DEVIATIONS FOR TOTAL ASSOCIATIONS

Group	A1 M_{SD}	A1 t	A2 M_{SD}	A2 t	A3 M_{SD}	A3 t
E1	2.19		2.31		2.05	
C1	1.88	2.92**	2.32	-.10*	1.99	.71*
E2	2.07		1.95		2.23	
C2	2.07	.00*	2.21	2.20**	2.11	.87*

*p > .05; **p < .05, two-tailed. $N_1 = N_2 = 15$

TABLE IV-11

MEAN STANDARD DEVIATIONS FOR SIGNIFICANT ASSOCIATIONS

Group	A1 M_{SD}	A1 t	A2 M_{SD}	A2 t	A3 M_{SD}	A3 t
E1	1.54		1.93		1.99	
C1	1.23	1.88*	1.90	.26*	1.87	1.24*
E2	1.87		1.64		2.05	
C2	1.51	2.05**	1.79	-1.42*	1.87	1.46*

*$p > .05$; **$p < .05$, two-tailed. $N_1 = N_2 = 15$

In the case of the mean standard deviations, a couple of significant differences appeared, but not systematically as can be seen in Tables IV-10 and -11.

Thus, there was no significant differences either in terms of averages or variability for both total and "significant associations," but from Tables IV-8 and -9 we can see that the average number of associations increased systematically from the first to the third test for all groups, and this increase was more accentuated for "significant associations." However, this is not surprising at all because one would certainly expect such increase as the students proceed through the course. On the other hand, one could perhaps expect that the relatively high mean standard deviations at the beginning of the course would decrease with time. But this was not the case because they at least did not decrease. However, before trying to interpret these results let's go into further details. Tables IV-12 and IV-13 show the average number of total associations and "significant associations" for each concept. Asterisks in these tables indicate significant differences at

TABLE IV-12
WORD ASSOCIATION TEST
(Average Number of Words Associated with Each Concept)

CONCEPT	A1		A2		A3		A1		A2		A3	
	E1	C1	E1	C1	E1	C1	E2	C2	E2	C2	E2	C2
Electromotive	3.78	3.58	4.67	5.31	6.34	5.94	3.89	3.52	5.82	4.62*	6.03	5.58
Force ($\mathcal{E}$)	1.97	2.05	2.46	1.94	2.04	1.75	2.42	2.41	2.46	1.39*	2.71	1.38*
Electric	4.36	4.03	5.67	6.72	6.29	6.58	4.00	3.96	[illegible].50	5.88*	6.72	5.63
Resistance (R)	1.84	1.72	2.04	2.51	1.82	2.25	2.09	2.33	2.02	2.23	2.67	1.97
Inductance (L)	2.69	2.42	2.94	2.75	5.66	4.97	2.53	2.56	2.91	3.12	3.97	5.08*
	2.46	1.81	2.10	2.05	2.00	2.05	1.96	1.72	1.85	2.10	1.75	1.56
Magnetic	4.92	4.67	6.69	4.91*	6.17	6.79	4.05	3.78	4.94	4.73	5.41	5.04
Field ($\vec{B}$)	2.23	2.06	2.65	2.25	1.99	2.33	1.92	1.78	2.03	2.18	2.11	1.55
Potential (V)	3.19	3.81	6.56	5.89	6.21	5.58	3.45	3.22	4.71	5.23	5.19	5.33
	1.51	2.01	1.99	2.25	1.43	1.62	2.05	1.65	1.43	1.73	1.97	1.97
Energy (U)	6.81	6.06	6.86	6.67	7.11	6.48	5.32	5.41	5.41	5.38	5.91	6.54
	2.24	1.80	2.64	2.28	2.21	2.06	2.05	2.29	2.27	2.42	2.67	2.72
Force ($\vec{F}$)	5.86	5.61	7.56	6.22*	8.06	6.48*	5.29	5.56	6.94	6.73	6.94	7.13
	1.99	1.81	2.60	2.58	2.01	2.03	2.03	2.34	1.98	2.07	2.46	2.80
Electric	6.17	5.64	7.14	7.14	7.31	6.85	5.26	5.30	5.91	6.81	7.03	7.21
Current (I)	2.37	1.90	2.52	2.83	2.04	2.11	2.31	2.66	2.05	2.14	2.32	2.15
Electric Flux	3.81	3.36	5.14	4.44	5.31	5.09	3.61	3.33	4.03	4.54	4.97	4.38
($\vec{\Phi}_E$)	2.16	1.68	2.03	1.84	2.00	1.57	1.85	1.36	1.87	2.23	2.07	1.84
Electric	4.08	3.97	7.53	6.53	6.74	6.36	3.82	3.52	4.94	5.92*	6.63	6.13
Field ($\vec{E}$)	2.01	1.95	2.29	2.70	1.95	1.95	1.93	1.95	1.59	2.12	2.12	2.31
Electric	4.97	4.42	7.33	7.31	6.74	6.03	4.11	4.22	5.62	6.35	5.84	6.29
Charge (q)	2.29	1.93	2.44	2.75	2.58	1.98	2.18	1.85	2.13	2.46	2.30	2.07
Magnetic	3.81	2.61	4.78	2.97*	5.63	5.21	3.16	2.59	3.71	4.42	4.66	5.46
Flux ($\vec{\Phi}_B$)	1.92	1.90	2.32	2.17	2.06	1.83	1.91	1.82	1.61	2.44*	1.52	2.15
Time (t)	5.92	4.08*	5.28	5.22	6.43	5.06*	3.95	4.30	4.41	4.58	5.56	5.33
	3.26	1.78*	2.29	2.19	2.16	2.28	2.20	2.77	2.13	2.82	2.12	2.14
Work (W)	4.89	5.36	6.44	6.22	6.26	6.45	4.89	4.93	5.56	5.96	5.88	6.67
	2.12	2.07	1.99	2.04	2.19	2.08	1.96	1.90	2.05	2.05	2.23	2.37
Capacitance (C)	3.11	3.08	4.58	6.08*	6.49	6.18	2.89	3.15	3.09	5.23*	6.00	5.63
	2.50	1.76	2.23	2.38	2.21	1.93	2.17	2.25	1.83	2.82*	2.37	2.65

NOTE: A1, A2, and A3 stand for first, second and third times. Means are in the first rows and standard deviations in the second ones. Asterisks indicate significant differences (t test for means, F test for variances, both two-tailed) at .05 level.

TABLE IV-13

WORD ASSOCIATION TEST
(Average Number of Significant Associations with Each Concept)

CONCEPT	A1		A2		A3		A1		A2		A3	
	E1	C1	E1	C1	E1	C1	E2	C2	E2	C2	E2	C2
Electromotive	2.92	2.58	4.36	4.83	6.17	5.82	3.00	2.59	5.12	4.31	5.69	5.38
Force ($\mathcal{E}$)	1.75	1.61	2.33	1.66	1.98	1.57	2.07	2.27	2.07	1.35*	[illegible].47	1.24*
Electric	2.50	2.28	4.69	5.17	5.23	5.48	2.29	2.15	3.21	4.15*	5.72	4.25*
Resistance (R)	1.58	1.39	1.75	1.76	1.66	1.91	1.56	2.05	1.65	1.62	2.25	1.78
Inductance (L)	1.97	1.72	2.67	2.50	5.40	4.61	1.87	1.85	2.59	2.58	3.88	4.42
	1.95	1.28*	1.96	2.05	1.90	1.85	1.66	1.20	1.79	1.63	1.72	1.44
Magnetic	3.72	3.25	5.78	4.17*	5.94	5.50	3.00	3.00	4.29	3.92	5.06	4.79
Field ($\vec{B}$)	1.83	1.46	2.14	1.69	1.97	2.15	1.47	1.49	1.64	1.79	1.72	1.35
Potential (V)	2.03	1.69	4.86	4.67	5.03	4.58	2.03	1.70	3.74	4.28	4.6[illegible]	4.25
	1.78	1.31	1.82	1.85	1.62	1.37	1.64	1.44	1.26	1.34	1.94	1.89
Energy (U)	.83	.78	2.14	1.94	4.94	3.55*	1.05	.96	1.41	1.92	4.00	3.04
	.77	.80	1.69	1.60	2.25	2.18	1.23	1.06	1.16	2.06*	2.45	1.85
Force ($\vec{F}$)	1.17	.36*	3.94	2.83*	5.20	3.76*	1.08	.74	3.26	2.88	4.31	3.83
	1.16	.72*	2.10	1.61	1.95	1.84	1.51	1.16	2.00	2.10	2.18	2.28
Electric	4.25	4.25	6.61	6.14	6.97	6.62	3.97	3.74	4.94	5.69	6.47	6.50
Current (I)	2.13	1.44*	2.19	2.66	2.05	2.03	1.99	2.44	1.74	1.78	2.11	1.56
Electric	2.56	2.03	4.19	3.69	4.89	4.48	2.16	2.48	3.53	3.69	4.72	3.92
Flux (Φ_E)	1.81	1.48	1.45	1.67	1.88	1.42	1.44	1.34	1.71	2.04	1.99	1.72
Electric	3.17	2.50	6.64	5.67	6.40	6.06	2.87	2.70	4.29	5.04	6.44	5.33*
Field ($\vec{E}$)	1.84	1.46	1.99	2.62	1.80	1.82	1.86	1.64	1.66	2.01	1.92	1.95
Electric	3.50	3.08	6.44	6.44	6.31	5.73	3.08	3.30	4.91	5.77	5.75	5.79
Charge (q)	1.75	1.63	2.22	2.31	2.52	1.91	2.14	1.75	1.93	2.32	2.16	2.69
Magnetic	2.47	1.69*	4.06	2.44*	5.11	4.64	2.29	1.85	3.41	3.46	4.44	4.46
Flux (Φ_B)	1.46	1.41	1.91	1.86	1.76	1.67	1.31	1.46	1.50	1.86	1.56	1.69
Time (t)	.36	.08*	1.47	1.69	3.51	3.06	.26	.37	.76	1.12	3.31	2.33
	.72	.37*	1.72	1.41	2.33	2.18	.76	.74	1.10	1.53	2.02	2.08
Work (W)	.28	.17	2.36	2.81	3.20	3.33	.29	.44	1.47	2.08	3.03	2.96
	.70	.45*	1.71	1.79	2.03	2.34	.46	.89*	1.64	1.41	2.25	2.39
Capacitance (C)	2.11	2.42	4.03	5.25*	6.11	5.64	2.11	2.26	2.74	4.01*	5.25	4.88
	1.82	1.63	1.93	2.03	2.14	1.83	1.91	1.77	1.71	1.94	2.06	2.19

NOTE: A1, A2, and A3 stand for first, second and third times. Means are in the first rows and standard deviations in the second rows. Asterisks indicate significant differences (t test for means, F test for variances, both two-tailed) at .05 level.

the .05 level; for unequal variances an approximation of the t test (Darlington, 1975) was used. Tables IV-14 and IV-15 summarize the specific cases where these differences occurred.

TABLE IV-14

SPECIFIC CASES (CONCEPTS*) WHERE SIGNIFICANT DIFFERENCES OCCURRED FOR TOTAL ASSOCIATIONS

	A1		A2		A3	
Group	dif. v	dif. M	dif. v	dif.M	dif. v	dif. M
E1, C1	--	--	--	$\vec{B}$, E1	--	--
	--	--	--	$\vec{F}$, E1	--	$\vec{F}$, E1
	--	--	--	Φ_B,E1	--	--
	t, E1	t, E1	--	--	--	t, E1
	--	--	--	C, C1	--	--
Totals:	1 E1	1 E1	--	3E1; 1C1	--	2 E1
E2, C2	--	--	ε, E2	ε, E2	ε, E2	--
	--	--	--	R, C2	--	--
	--	--	--	--	--	L, C2
	--	--	--	$\vec{E}$, C2	--	--
	--	--	Φ_B, C2	--	--	--
	--	--	C, C2	C, C2	--	--
Totals:	--	--	1E2; 2C2	1E2;3C2	1 E2	1 C2

*Concepts are represented by their symbols; the letter after the symbol indicates which group had higher mean or variance.

TABLE IV-15

SPECIFIC CASES (CONCEPTS*) WHERE SIGNIFICANT DIFFERENCES OCCURRED FOR SIGNIFICANT ASSOCIATIONS

Groups	A1		A2		A3	
	dif. v	dif. M	dif. v	dif. M	dif. v	dif. M
E1, C1	L, E1	--	--	--	--	--
	--	--	--	$\vec{B}$, E1	--	--
	--	--	--	--	--	U, E1
	$\vec{F}$, E1	$\vec{F}$, E1	--	$\vec{F}$, E1	--	$\vec{F}$, E1
	I, E1	--	--	--	--	--
	--	Φ_B, E1	--	Φ_B, E1	--	--
	t, E1	t, E1	--	--	--	--
	W, E1	--	--	--	--	--
	--	--	--	C, C1	--	--
Totals:	5 E1	3 E1	--	3E1;1C1	--	2 E1
E2, C2	--	--	ε, E2	--	ε, E2	--
	--	--	--	R, C2	--	R, E2
	--	--	U, C2	--	--	--
	--	--	--	--	--	$\vec{E}$, E2
	W, C2	--	--	--	--	--
	--	--	--	C, C2	--	--
Totals:	1 C2	--	1E2;1C2	2 C2	1 E2	2 E2

*Concepts are represented by their symbols; the letter after the symbol indicates which group had higher mean or variance.

From Tables IV-14 and IV-15 we can see that the relatively small number of significant differences found for specific concepts supports the overall conclusion of no significant differences. In addition, most of the differences found in case A2 (middle of course) can be easily explained as a consequence of the different content sequences. For example, the fact that the average number of associations (both total and significant) for concepts $\vec{B}$ and Φ_B

was higher for group E1 in case A2 is due to the fact that at the time of A2 the control group had not yet studied these concepts. On the other hand, the higher mean of group E2, at the same time, for concept C is due to the fact that this group had just studied a unit on "Capacitance" prior to the test.

Since mid-course differences can be explained by differences in content sequence, let's focus our attention on end-of-course differences. In terms of variances, the only significant difference occurred for the concept ε in experimental group E2. One possible explanation is that, in this group, some students understood the concept of electromotive force as a superordinate concept subsuming the induced electromotive force, whereas others (as it is likely to be the case in the control group) saw this concept as a subordinate concept associated with current and circuits only. In such a case, a higher variability among experimental subjects would be explained.

Concerning the means, however, several differences can be found in Tables IV-14 and -15 and the remarkable fact is that in only one case the mean of the control groups was higher. It happened at the time of A3 for total associations with concept L (inductance) in group E2 showing more associations, which could be explained by the fact that the next to the last unit of the control group was specifically a unit on "Inductance." It is true that the last unit of the experimental group was a unit on "Energy" and this fact could account for the higher mean of "significant associations" with this concept in group E1; however, this case was not the only one in which the experimental groups had higher means. Considering both total and significant associations, the experimental groups had higher means in six cases and the control groups in only one case. Furthermore, among those six cases, four were concerned with "significant associations."

It is true that, in addition to the possible explanation for the difference in concept U, the difference in $\vec{F}$ could be explained simply by the initial difference; however, these four differences in "significant associations" are coherent with the general average number of such associations reported in Table IV-9: in this Table, although not significantly different, the averages of the experimental groups were higher than the averages of the control groups.

Thus, one could accept these differences as evidence that the Ausubelian approach produced more "significant (meaningful?) associations" per concept. However, we must be careful in doing this because the overall result was "no significant difference" and the criterion to identify the "significant associations" was quite subjective. We certainly must gather more evidences before drawing conclusions about any possible effect of the Ausubelian approach.

Besides counting the number of total and "significant associations," another possible way of analyzing the results of the word-association test is to measure the overlapping of words associated with a certain pair of concepts. That is, given the lists of words associated with two concepts, we can compute how many of these words are common. For example, supposing that the following words were associated with force ($\vec{F}$) and electric field ($\vec{E}$):

($\vec{F}$) Force	($\vec{E}$) Electric Field
Work	Electric Charge
Electric Field	Magnetic Field
Magnetic Field	Force
Distance	Dipole
Electric Charge	Energy
Energy	Potential
Gravitational Field	Capacitance
Electromotive Force	Current
Current	Circuit
Nuclear Force	

we would find six common words (force, electric field, magnetic field, electric charge, energy, and current) in the two lists (which include the two given concepts). The degree of overlapping would give us an indirect measure of the degree of interrelation as well as of differentiation between the concepts according to the student's point of view. A more sophisticated measure would be to take into account the rank order of words, under one concept, which are shared in common with the other one, and calculate a "relatedness coefficient" (Shavelson, 1972). However, we preferred the first type of measure because, in spite of being extremely time consuming, we thought it would be less time consuming than the use of the relatedness coefficient and perhaps more suitable for quantitative comparisons.

Thus, the overlapping was computed for every one of the 105 possible pairs of concepts in each one of the 139 individual tests of the students involved in the study. Then, the average overlapping for each concept in each group was calculated, and, finally, the general averages and mean standard deviations were computed. The results are in the following Tables. Tables IV-16, IV-17, and IV-18 show the means and standard deviations for each pair of concepts. Asterisks in these Tables indicate significant differences at the .05 level between means or variances; for unequal variances an approximation of the t test (Darlington, 1975) was used. Tables IV-19 and IV-20 summarize the cases where these differences were found. Only total associations were considered in these Tables.

Even considering that Tables IV-19 and -20 are supposed to be a summary of the differences shown in Tables IV-16, -17 and -18, they are still difficult to interpret due to the large amount of information they contain. However, let's take a general look at these Tables. In the PSI groups, we can see that the experimental group had initially greater variabilities and means in many cases, but these differences gradually disappeared with time, in such a way that, at the end of the course, there were no differences at all. In the lecture groups, the experimental group also had initially greater variabilities in many cases, but only two differences in means. However, at the middle of the course, an inversion occurred: the control group had larger variabilities and means in many cases. On the other hand, at the end of the course, almost all differences disappeared, as occurred with the PSI groups.

TABLE IV-16 — WORD ASSOCIATION TEST A1: OVERLAPPING
(Average Number of Overlapping Associated Words for Each Pair of Concepts)

	ℰ	R	L	$\vec{B}$	V	U	$\vec{F}$	I	Φ_E	$\vec{E}$	q	Φ_B	t	W	C
ℰ		1.45 .89* 1.87 1.19	.50 .37 .80 .74	.61 .44 .82 .85	1.08 .85 1.28 1.06	1.13 .81 1.34 1.39	.89 .67 1.13 1.00	1.58 .89* 1.72 1.05	.66 .44 1.05 .75	.74 .56 1.16 1.05	.71 .67 1.31 1.04	.24 .41* .54 .84	.29 .30 .61 .61	.58 .56 .79 1.05	.79 .46* 1.44 .94
R	1.32 .67* 1.16 .96		.39 .22* .89 .51	.29 .11* .73 .32	.55 .33 1.03 .73	1.16 .52* 1.39 .75	.39 .11* 1.00 .42	2.00 1.37* 2.09 1.24	.79 .48* 1.30 .70	.65 .26* 1.10 .53	.76 .41* 1.46 .75	.26 .15* .60 .36	.24 .11* .68 .32	.63 .26* .88 .59	.61 .44* 1.08 .70
L	.59 .19* .93 .47	.38 .25* .64 .44		.55 .44 .95 .75	.26 .22 .60 .51	.42 .19 .64 .62	.28 .15 .72 .60	.71 .52 1.11 1.01	.42 .30* .89 .61	.42 .37 .68 .69	.45 .41 .86 .69	.32 .22 .66 .51	.05 .04 .23 .19	.11 .07* .39 .27	.61 .30* .97 .61
$\vec{B}$	.76 .42* 1.06 .60	.41 .31 .64 .52	.70 .50* 1.27 .85		.42 .52 .86 1.16	.47 .48 .80 .98	.61 .41 .95 .69	.95 .59 1.23 .93	.55 .56 .92 .97	1.11 .67 1.23 1.04	.63 .41* 1.02 .69	1.55 .89* 1.29 1.09	.21 .04* .62 .19	.38 .41 .75 .64	.26 .26 .64 .71
V	1.16 .47* 1.44 .56	.73 .44 1.07 .94	.30 .11* .85 .40	.46 .42 .87 .84		1.05 1.30 1.11 1.30	.63 .78 .94 .97	1.08 .78 1.19 1.37	.58 .56 1.11 .93	.68 .70 1.16 1.07	.79 .67 1.44 1.21	.18 .37* .56 .88	.29 .07* .69 .27	.89 1.04 1.06 1.16	.47 .48 .98 1.05
U	.46 .53 .73 .70	.43 .78* .69 1.05	.19 .17 .52 .45	.22 .28 .48 .66	.81 .89 .84 .98		1.34 1.22 1.36 1.31	1.03 .89 1.33 1.34	.61 .41 .97 .80	.39 .56 .75 .89	.76 .52 1.30 1.05	.21 .22 .47 .64	.34 .33 .78 .55	2.00 1.67 1.41 1.54	.58 .33 1.15 .83
$\vec{F}$	.76 .39* .80 .60	.19 .17 .52 .38	.14 .03* .35 .17	.59 .50 .72 .77	.54 .58 .93 .94	.92 1.25 1.09 1.27		.71 .44 1.25 .89	.34 .26 .85 .66	.84 .67 1.20 .96	.58 .44 1.08 .93	.32 .30 .66 .72	.92 .81* 1.50 1.00	2.00 2.19 1.45 1.47	.26 .15 .72 .60
I	1.81 .94* 1.41 1.04	1.62 1.89 1.26 1.39	.78 .61 1.08 .80	1.00 .83* 1.25 .74	1.14 .94 1.34 1.07	.46 .69 .87 1.01	.51 .39* .87 .55		1.39 1.33 1.57 1.49	1.53 .89 1.43 1.12	1.39 1.07 1.70 1.49	.63 .59 .85 .97	.26 .37 .60 .69	.55 .74* .76 1.40	.95 .74 1.31 1.40
Φ_E	.89 .33* 1.05 .63	.86 .47* 1.29 .61	.43 .44 .77 .88	.65 .78 .86 .80	.81 .33* 1.13 .59	.16 .22 .44 .42	.43 .14* .96 .54	1.46 1.17 1.32 1.30		1.29 1.15 1.63 1.41	1.11 .74* 1.48 .94	.84 .74 1.00 1.20	.71 .19 .58 .40	.29 .19 .57 .48	.50 .70 .92 1.14
$\vec{E}$	.97 .36* 1.32 .59	.65 .39* .92 .60	.54 .44* .96 .65	1.19 1.72 1.17 1.26	.76 .56 1.04 .81	.24 .19* .72 .40	.65 .50 .98 .81	1.38 1.08* 1.64 1.00	1.32 .83* 1.38 .91		1.26 1.56 1.69 1.67	.95 .63 .90 1.11	.29 .11* .84 .32	.53 .63 1.03 .74	.61 .63 .89 .93
q	.92 .33* 1.16 .59	1.03 .42* 1.19 .69	.47 .47 .80 .84	.76 .94 .98 1.01	.92 .39* 1.26 .77	.22 .22 .58 .48	.57 .42 .93 .69	1.70 1.00* 1.45 1.12	1.11 .67 1.20 .89	1.59 1.22 1.50 1.24		.53 .52 .86 .98	.29 .26 .61 .59	.47 .41 .89 .89	.89 .70 1.50 1.17
Φ_B	.41 .19* .69 .40	.43 .25* .73 .50	.46 .36 .87 .68	1.32 .92 1.13 1.00	.24 .22 .55 .48	.05 .17 .33 .45	.38 .19* .86 .58	.65 .28* 1.01 .51	.81 .56 1.10 .77	1.08 .50* 1.12 .65	.78 .36* 1.11 .64		.13 .19* .41 .62	.21 .26* .41 .59	.21 .37* .58 .93
t	.24 .19 .72 .52	.30 .14* .57 .35	.16 .00* .55 .00	.35 .25* .89 .60	.22 .19 .63 .58	.54 .53 .96 .94	.89 1.31 1.41 1.26	.32 .11* .58 .32	.57 .14* .93 .35	.38 .19 .64 .52	.46 .31* .93 .58	.43 .14* .83 .35		.71 .56 1.39 1.01	.13 .15 .34 .36
W	.51 .42 .84 .69	.35 .53 .68 .81	.14 .03* .54 .17	.41 .42 .76 .81	.65 .89 1.01 1.09	1.46 1.78 1.37 1.35	1.86 2.42 1.46 1.34	.41 .56 .90 .69	.27 .14* .84 .42	.43 .42 .80 .69	.41 .28* .98 .51	.16 .25 .50 .60	1.03 1.17 1.52 1.08		.26 .33 .55 .73
C	.51 .44 .69 .61	.51 .50 .80 .91	.73 .33* 1.17 .68	.35 .50 .68 .61	.35 .50 .72 .88	.27 .25 .56 .69	.22 .11* .48 .32	.65 .81 .82 1.06	.49 .36 .84 .68	.46 .61 .73 .77	.70 .69 .85 .86	.19 .19 .46 .40	.14 .06* .54 .23	.16 .11 .44 .40	

NOTE: Statistics concerning the PSI groups are below the diagonal - lecture groups are above. Numbers in the first row of each matrix element are the means of experimental and control groups, in that order. The corresponding standard deviations are in the second row. Asterisks indicate significant differences at the .05 level between the means (t test, two-tailed) or between the variances (F test, two-tailed).

TABLE IV-17 — WORD ASSOCIATION TEST A2: OVERLAPPING

(Average Number of Overlapping Associated Words for Each Pair of Concepts)

	$\mathcal{E}$	R	L	$\vec{B}$	V	U	$\vec{F}$	I	Φ_E	$\vec{E}$	q	Φ_B	t	W	C
$\mathcal{E}$		1.79 2.00 1.72 1.67	.56 .85 .89 1.05	1.29 1.23 1.27 1.14	1.47 2.42* 1.31 1.25	1.18 1.42 1.25 1.30	1.41 1.46 1.33 1.27	1.71 2.54* 1.59 1.36	.82 .96* 1.38 .96*	1.29 1.58 1.31 1.36	1.53 2.19 1.44 1.36	.71 .65 1.29 .94	.29 .63* .52 .98*	1.12 1.27 1.15 1.19	.91 1.31 1.29 1.23
R	1.94 2.50 1.74 1.46		.76 .69 1.05 1.19	.74 .96 .93 1.08	1.09 1.35 1.06 1.13	.85 1.23* .86 1.37*	.47 .58* .66 .99*	1.65 2.73* 1.18 1.25	.56 .77* .70 1.24*	.85 .73 .99 .96	1.03 1.38 1.17 1.10	.67 .65 .79 1.06	.26 .81* .57 1.23*	.79 .85 .95 1.16	1.00 .96 1.21 1.48
L	1.06 .50* 1.47 .94*	.89 .25* 1.28 .55*		1.00 1.04 1.28 1.22	.41 .65 .82 .85	.26 .54* .51 .86*	.65 .77 .95 1.18	1.09 1.08 1.24 1.35	.50 .81 .75 1.06	.76 .92 1.13 1.02	.85 1.08 1.28 1.35	.88 .54* 1.20 .71*	.24 .42 .50 .70	.21 .38* .48 .70*	.62 .73 1.04 .87
$\vec{B}$	.78 .78 .80 .87	.78 .56 .96 .81	1.00 1.11 1.07 1.28		.85 1.08 1.05 1.09	.44 1.00* .66 1.57*	1.44 1.42 1.48 1.39	2.03 1.69 1.27 1.52	1.06 1.00 .95 1.36	1.71 1.73 1.36 1.48	1.85 1.38 1.31 1.47	2.09 2.12 1.38 1.37	.53 .81* .83 1.52*	.74 .85 1.02 1.22	.59 .96* .86 1.43*
V	1.89 2.53 1.33 1.48	1.38 1.81 1.15 1.58	.94 .61 1.12 .99	1.22 1.22 1.31 1.02		1.29 2.27* 1.14 1.12	1.27 2.23* 1.11 1.66*	1.82 2.19 1.27 1.67	1.24 1.27 1.16 1.15	2.21 2.65 1.34 1.44	1.94 2.27 1.37 1.28	.76 .81 1.10 1.50	.35 .81* .54 1.17*	1.56 2.54* 1.26 1.56	1.06 2.00* 1.28 1.50
U	.94 1.50 1.26 1.30	.67 1.81* 1.04 .92	.31 .44 .86 .91	.53 .83* .84 1.25	1.56 1.78 1.48 1.71		1.35 2.35* 1.43 1.62	.79 1.92* 1.12 1.94*	.41 .96* .92 1.46*	.79 1.85* 1.07 1.52	.74 2.00* 1.21 1.79*	.32 .73* .53 1.69*	.56 .81 1.08 1.52	2.15 2.73 1.28 1.22	.35 1.58* .77 1.88*
$\vec{F}$	1.19 1.17 1.19 1.03	.58 .39* 1.23 .69*	.47 .61 .88 .87	1.36 1.56 1.57 1.46	1.39 1.64 1.68 1.40	1.64 2.06 1.64 1.51		1.74 1.59 1.48 1.63	1.18 1.54 1.19 1.65	1.74 2.54 1.52 1.79	1.44 2.31* 1.31 1.67	.85 1.04 1.08 1.46	1.47 1.27* 1.26 1.87*	2.32 2.96 1.41 1.64	.53 1.73* .83 1.80*
I	2.56 2.86 1.71 1.42	3.33 3.08 1.80 1.63	1.19 .89 1.33 1.30	1.58 1.50 1.36 1.21	2.31 2.39 1.69 1.42	.83 1.19 .97 1.33	1.03 1.42 1.25 1.32		1.94 1.54 1.61 1.61	2.21 1.88 1.51 1.51	2.44 2.85 1.44 2.03	1.65 1.31* 1.30 1.91*	.82 1.04 1.24 1.37	1.12 1.73 1.20 1.34	1.15 1.85 1.35 1.59
Φ_E	.86 .67 1.13 1.04	.94 .64* 1.53 .87*	.47 .67 .81 1.12	1.33 1.03 1.12 .97	1.42 1.50 1.27 1.11	.67 .64 1.12 1.10	1.19 1.39* 1.65 1.10*	1.67 1.94 1.69 1.52		2.21 2.31 1.25 1.16	1.88 2.42* 1.07 1.55*	1.44 1.15 1.19 1.59	.44 .62* .70 1.42*	.59 1.38* .86 1.65*	.91 1.65* 1.11 1.52
$\vec{E}$	1.56 1.25 1.52 1.18	1.44 1.28 1.21 1.37	.97 .94 1.13 1.31	2.19 1.94 1.67 1.60	2.47 2.61 1.93 1.57	1.06 1.28 1.19 1.26	1.97 2.58 1.93 1.81	2.53 2.44 1.93 1.89	2.89 2.83 1.49 1.28		2.68 3.69* 1.39 1.26	1.44 1.23 1.44 1.66	.56 .65 .83 1.09	1.26 2.46* 1.46 1.50	1.18 2.46* 1.17 1.45
q	1.64 1.67 1.51 1.45	1.61 1.58 1.52 1.50	1.11 1.06 1.21 1.47	2.03 1.83 1.75 1.23	2.39 2.81 1.69 1.79	1.17 1.28 1.61 1.43	2.11 2.42 2.12 1.70	2.81 3.11 1.94 1.79	2.14 2.91* 1.27 1.21	3.61 3.97 2.21 1.92		1.41 1.31 1.37 1.46	.62 1.04* .92 1.75*	1.03 2.08* 1.36 1.44	1.12 2.65* 1.30 1.79
Φ_B	.64 .19* 1.02 .40*	.56 .17* .69 .38*	.53 .83* .65 1.16*	2.44 1.92 1.68 1.46	.81 .25* 1.09 .55*	.53 .25* 1.16 .50*	1.17 .64* 1.58 .80*	1.17 .75 1.03 1.13	1.44 .78* 1.36 .99	1.58 .94* 1.70 1.17*	1.50 .89 1.50 1.06		.50 .81* .83 1.20*	.44 .65* .79 1.26*	.59 .73 .99 1.34
t	.69 .75 .98 .84	.58 .83 .84 .88	.28 .31 .61 .79	.75 .64 .97 .80	.53 .92 .88 1.13	.67 1.03 1.10 1.13	1.25 1.36 1.23 1.42	.81 1.39* .95 1.44*	.61 .47 .96 .81	.72 1.06 1.09 1.24	1.06 1.19 1.24 1.35	.61 .25* .93 .60*		1.00 1.19* 1.18 1.81*	.41 .59* .70 1.42*
W	1.33 1.69 1.57 1.41	.92 .97 1.48 1.13	.42 .36 .87 .72	.75 .92 1.05 1.02	2.36 2.47 1.84 1.75	2.36 2.89 1.48 1.41	2.53 2.92 1.92 1.65	1.28 1.39* 1.67 1.08*	.78 1.00 .99 .89	1.67 2.03 1.74 1.61	1.81 2.06 1.69 1.51	.50 .39* 1.11 .64*	1.33 1.25 1.45 1.25		.47 1.69* .83 1.44*
C	1.36 1.44 1.44 1.27	1.44 1.08* 1.76 1.08*	.86 .72 1.17 1.28	.64 .86 1.10 .96	1.61 2.17 1.40 1.25	.94 1.19 1.33 1.47	.72 1.28 1.16 1.21	1.64 2.14 1.42 1.31	1.06 1.58 1.22 1.00	1.61 2.11 1.40 1.21	1.81 2.31 1.37 1.41	.42 .31 .69 .86	.53 1.08* 1.00 1.46*	1.08 1.78* 1.00 1.62*	

NOTE: Statistics concerning the PSI Groups are below the diagonal-lecture groups are above. Numbers in the first row of each matrix element are the means of experimental and control groups, in that order. The corresponding standard deviations are in the second row. Asterisks indicate significant differences at the .05 level between the means (t test, two-tailed) or between the variances (F test, two-tailed).

TABLE IV-18 — WORD ASSOCIATION TEST A3: OVERLAPPING
(Average Number of Overlapping Associated Words for Each Pair of Concepts)

	$\mathcal{E}$	R	L	$\vec{B}$	V	U	$\vec{F}$	I	Φ_E	$\vec{E}$	q	Φ_B	t	W	C
$\mathcal{E}$		2.66 2.38 2.13 1.31*	1.69 2.13 1.69 1.39	1.66 1.63 1.43 1.17	2.38 2.33 1.72 1.61	1.50 1.50 1.72 1.35	1.56 1.54 1.34 1.18	3.19 2.83 2.16 1.31*	1.53 1.25 1.59 1.29	2.00 1.46 2.02 1.22*	2.00 1.71 1.81 .91*	1.47 1.38 1.41 1.10	1.03 1.42 .93 1.32	1.38 1.75 1.41 1.45	2.06 1.54 1.74 1.06*
R	2.46 2.82 1.60 1.47		1.31 1.21 1.20 1.25	1.25 1.47 1.05 .80	1.13 1.71 1.84 1.16*	1.56 1.46 1.76 1.74	1.16 1.08 1.39 1.83	3.38 2.79 2.04 1.53	1.34 .75 1.49 1.03*	1.53 .83 1.68 1.20	1.75 1.33 1.74 1.55	.88 .83 .94 .82	.81 1.08 .74 1.50*	1.34 1.54 1.52 1.56	1.72 1.25 1.67 1.59
L	2.23 1.70 1.37 1.38	1.37 1.00 1.26 1.06		2.09 1.96 1.42 1.33	1.13 1.25 1.41 1.19	.97 1.00 1.28 1.10	1.19 1.00 1.28 1.25	2.22 2.17 1.52 1.66	1.38 1.17 1.36 1.43	1.59 1.42 1.29 1.61	1.56 1.08 1.27 1.28	1.72 1.92 1.35 1.14	1.16 1.38 1.22 1.13	.63 1.08 1.07 1.18	1.06 1.33 1.11 1.49
$\vec{B}$	2.11 2.21 1.37 1.67	1.06 1.09 1.03 .95	2.37 2.18 1.33 1.38		1.38 1.25 1.16 1.33	1.09 1.25 1.33 1.48	1.97 1.71 1.56 1.56	2.44 2.13 1.50 1.45	1.78 1.58 1.21 1.38	2.28 2.00 1.51 1.41	2.09 1.58 1.40 1.25	2.69 2.29 1.26 1.16	1.53 1.42 1.27 1.38	1.06 1.21 1.19 1.44	1.34 1.33 1.15 1.27
V	2.49 2.42 1.84 1.66	1.77 2.12 1.54 1.52	1.26 1.00 1.34 1.09	1.12 1.58 .98 1.23		1.44 2.29 1.79 1.71	1.56 1.96 1.22 1.63	2.25 2.71 1.88 1.68	1.59 1.33 1.50 1.31	2.22 2.21 1.68 1.35	2.38 2.04 1.84 1.57	1.06 .83 1.11 .92	1.09 1.08 1.40 1.14	1.72 2.54 1.42 2.06*	2.44 1.79 1.76 1.38
U	1.57 1.48 1.31 1.39	.89 1.39 .93 1.22	1.46 1.09 1.48 1.21	1.35 1.27 1.25 1.28	1.86 1.85 1.48 1.33		1.66 2.38 1.64 2.08	1.84 1.96 1.94 1.76	1.09 .96 1.35 1.04	1.72 1.46 1.63 1.67	1.75 1.67 1.83 1.81	.97 .96 1.26 1.12	.63 1.21 .94 1.28	2.06 3.25* 1.63 2.03	1.69 1.67 1.60 1.58
$\vec{F}$	2.06 1.97 1.51 1.40	.86 1.06 1.24 1.16	1.20 1.15 1.41 1.35	1.82 1.94 1.34 1.48	1.57 2.15 1.33 1.60	1.66 2.06 1.71 1.39		1.78 2.17 1.54 2.04	1.47 1.33 1.39 1.20	2.25 2.04 1.57 1.81	2.22 2.21 1.93 2.04	1.47 1.17 1.22 1.20	1.56 1.88 1.22 1.70	2.34 2.71 1.86 1.68	1.09 1.67 1.17 1.74*
I	3.34 3.45 1.53 1.58	3.09 3.52 1.52 1.42	2.23 1.88 1.55 1.58	2.24 2.48 1.54 1.42	2.43 2.88 1.70 1.52	1.54 1.55 1.77 1.77	2.00 2.12 1.66 2.00		2.13 1.71 1.77 1.43	2.69 2.63 1.80 1.58	3.00 2.88 1.80 1.78	1.94 1.83 1.41 1.55	1.59 1.83 1.41 1.81	1.38 2.33* 1.64 1.88	2.03 2.63 1.45 1.69
Φ_E	2.20 1.76 1.49 1.66	.83 1.12 .95 1.41*	1.51 1.48 1.20 1.50	2.18 2.45 1.42 1.46	1.51 1.76 1.12 1.23	1.26 1.06 1.17 1.14	1.80 2.03 1.68 1.57	2.11 2.55 1.32 2.06*		2.91 2.96 1.59 1.49	2.50 2.50 1.50 1.47	1.81 1.63 1.33 1.31	1.34 .96 1.26 1.55	1.06 .92 1.22 1.25	1.88 1.79 1.45 1.47
$\vec{E}$	2.49 2.24 1.60 1.70	1.69 1.48 1.35 1.35	1.57 1.82 1.24 1.55	2.36 2.76 1.43 1.52	2.29 2.82 1.36 1.47	1.77 1.79 1.48 1.60	2.23 3.00 1.63 1.75	3.00 3.03 2.04 1.94	3.34 3.55 1.45 1.54		3.50 3.67 1.39 1.71	1.94 1.71 1.29 1.76	1.50 1.54 1.44 2.06	1.78 [illegible] 1.58 1.71	2.59 2.50 1.41 1.44
q	2.23 2.36 1.68 1.45	1.40 1.79 1.24 1.36	1.63 1.52 1.57 1.60	2.03 2.24 1.53 1.28	1.97 2.45 1.52 1.66	1.57 1.55 1.56 1.23	2.37 2.70 1.80 1.72	2.71 3.03 2.08 1.88	2.57 3.09 1.58 1.26	3.54 4.15 1.50 1.39		1.88 1.50 1.36 1.64	1.34 1.54 1.29 2.06*	1.84 2.00 2.50 2.06	2.53 2.54 1.59 1.41
Φ_B	2.03 1.85 1.42 1.64	.77 1.00 .94 1.25	2.00 2.38 1.39 1.37	2.76 2.91 1.23 1.44	.97 1.21 .98 1.22	1.03 1.09 1.07 1.10	1.43 1.64 1.36 1.41	2.09 2.21 1.42 1.62	2.51 2.37 1.62 1.66	2.34 2.33 1.49 1.76	1.51 1.73 1.34 1.57		1.31 1.46 1.35 1.53	.94 1.33 .88 1.61*	1.19 .96 1.12 1.27
t	1.80 1.61 1.65 1.41	.97 1.48 .98 1.12	1.43 1.15 1.31 1.37	1.65 1.45 1.67 1.48	1.00 1.45 1.26 1.25	1.20 .97 1.75 1.31	2.31 1.94 2.15 1.73	2.31 1.94 1.78 1.64	1.80 1.58 1.45 1.60	1.91 1.70 1.96 1.79	1.71 1.76 1.58 1.54	1.89 1.45 1.60 1.72		.97 1.88* 1.33 1.54	1.06 1.42 .91 1.72*
W	1.54 1.64 1.38 1.29	1.00 1.58 1.11 1.62*	.74 1.06 1.12 1.27	.94 1.27 1.37 1.40	2.14 2.55 1.52 1.62	2.29 2.73 1.69 1.57	2.46 3.21 1.63 1.78	1.54 2.12 1.77 1.88	1.03 1.45 1.15 1.44	2.00 2.52 1.75 1.92	1.80 2.33 1.73 1.78	1.03 1.15 1.36 1.28	1.46 1.70 1.60 1.74		1.22 1.79 1.39 1.84
C	2.46 2.06 1.48 1.27	1.91 1.70 1.27 1.45	1.80 1.52 1.43 1.46	1.32 1.64 1.12 1.11	2.34 2.36 1.66 1.58	1.89 1.64 1.47 1.39	1.54 1.88 1.42 1.56	2.83 2.79 2.09 1.73	1.94 2.42 1.26 1.32	2.80 3.18 1.57 1.47	2.71 2.85 1.58 1.52	1.23 1.52 1.17 1.23	1.49 1.48 1.31 1.66	2.20 1.94 1.49 1.64	

NOTE: Statistics concerning the PSI groups are below the diagonal-lecture groups are above. Numbers in the first row of each matrix element are the means of experimental and control groups, in that order. The corresponding standard deviations are in the second row. Asterisks indicate significant differences at the .05 level between the means (t test, two-tailed) or between the variances (F test, two-tailed).

TABLE IV-19

WORD ASSOCIATION TEST: OVERLAPPING-PSI GROUPS
(Pairs of Concepts Where Significant Differences Occurred)

A1		A2		A3	
v	M	v	M	v	M
	ε/R, E1				
ε/L, E1					
ε/$\vec{B}$, E1					
ε/V, E1	ε/V, E1				
	ε/$\vec{F}$, E1				
	ε/I, E1				
ε/Φ_E, E1	ε/ϕ_E, E1				
ε/$\vec{E}$, E1	ε/$\vec{E}$, E1				
ε/q, E1	ε/q, E1				
ε/Φ_B, E1		ε/ϕ_B, E1	ε/ϕ_B, E1		
R/L, E1		R/L, E1	R/L, E1		
R/U, C1			R/U, C1		
		R/$\vec{F}$, E1			
R/Φ_E, E1		R/ϕ_E, E1		R/ϕ_E, C1	
R/$\vec{E}$, E1					
R/q, E1	R/q, E1				
R/Φ_B, E1		R/ϕ_B, E1	R/ϕ_B, E1		
R/t, E1					
				R/W, C1	
		R/C, E1			
L/$\vec{B}$, E1					
L/V, E1					
L/$\vec{F}$, E1					
L/$\vec{E}$, E1					
		L/ϕ_B, C1			
L/t, E1					
L/W, E1					
L/C, E1					
		B/U, C1			
$\vec{B}$/I, E1					
$\vec{B}$/t, E1					
V/ϕ_E, E1	V/ϕ_E, E1				
V/q, E1	V/q, E1				
		V/ϕ_B, E1	V/ϕ_B, E1		
U/$\vec{E}$, E1					
		U/ϕ_B, E1			

TABLE IV-19 — Continued

A1		A2		A3	
v	M	v	M	v	M
$\vec{F}/I$, E1					
$\vec{F}/\phi_E$, E1		$\vec{F}/\phi_E$, E1			
$\vec{F}/\phi_B$, E1		$\vec{F}/\phi_B$, E1			
$\vec{F}/C$, E1					
				I/ϕ_E, C1	
$I/\vec{E}$, E1					
	I/q, E1				
I/ϕ_B, E1					
I/t, E1		I/t, C1			
		I/W, E1			
$\phi_E/\vec{E}$, E1					
			ϕ_E/q, C1		
			ϕ_E/ϕ_B, E1		
ϕ_E/t, E1	ϕ_E/t, E1				
ϕ_E/W, E1					
$\vec{E}/\phi_B$, E1	$\vec{E}/\phi_B$, E1	$\vec{E}/\phi_B$, E1			
q/ϕ_B, E1					
q/t, E1					
q/W, E1					
ϕ_B/t, E1		ϕ_B/t, E1			
		ϕ_B/W, E1			
t/C, E1		t/C, C1			
		W/C, C1	W/C, C1		

TOTALS:

	A1		A2		A3	
	v	M	v	M	v	M
E1 >	41	13	15	5	--	--
C1 >	1	--	5	3	3	--
	42	13	20	8	3	--

NOTE: Concepts are represented by its symbols; letters indicate which group had higher mean or variance.

TABLE IV-20

WORD ASSOCIATION TEST: OVERLAPPING-LECTURE GROUPS
(Pairs of Concepts Where Significant Differences Occurred)

A1		A2		A3	
v	M	v	M	v	M
			$\mathcal{E}/V$, C2		
$\mathcal{E}/R$, E2				$\mathcal{E}/R$, E2	
$\mathcal{E}/I$, E2			$\mathcal{E}/I$, C2	$\mathcal{E}/I$, E2	
		$\mathcal{E}/\phi_E$, E2			
				$\mathcal{E}/\vec{E}$, E2	
				$\mathcal{E}/q$, E2	
$\mathcal{E}/\phi_B$, C2					
		$\mathcal{E}/t$, C2			
$\mathcal{E}/C$, E2				$\mathcal{E}/C$, E2	
R/L, E2					
$R/\vec{B}$, E2					
				R/V, E2	
R/U, E2	R/U, E2	R/U, C2			
$R/\vec{F}$, E2		R/F, C2			
R/I, E2			R/I, C2		
R/ϕ_E, E2		R/ϕ_E, C2		R/ϕ_E, E2	
$R/\vec{E}$, E2					
R/q, E2					
R/ϕ_B, E2					
R/t, E2		R/t, C2	R/t, C2	R/t, C2	
R/W, E2					
R/C, E2					
		L/U, C2			
L/ϕ_E, E2					
		L/ϕ_B, E2			
L/W, E2		L/W, C2			
L/C, E2					
		$\vec{B}/U$, C2			
$\vec{B}/q$, E2					
	$\vec{B}/\phi_B$, E2				
$\vec{B}/t$, E2		$\vec{B}/t$, C2			
		$\vec{B}/C$, C2			
		$V/\vec{F}$, C2			
V/ϕ_B, C2					
V/t, E2		V/t, C2			
			V/U, C2		
			$V/\vec{F}$, C2		
			V/W, C2	V/W, C2	
			V/C, C2		
			$U/\vec{F}$, C2		

TABLE IV-20 -- Continued

A1		A2		A3	
v	M	v	M	v	M
		U/I, C2	U/I, C2		
			U/$\vec{E}$, C2		
		U/ϕ_E, C2			
		U/q, C2	U/q, C2		
		U/ϕ_B, C2			
					U/W,C2
		U/C, C2	U/C, C2		
			$\vec{F}$/q, C2		
$\vec{F}$/t, E2		F/t, C2			
		F/C, C2	$\vec{F}$/C, C2	$\vec{F}$/C, C2	
		I/ϕ_B, C2			
I/W, C2					I/W,C2
ϕ_E/q, E2		ϕ_E/q, C2			
		ϕ_E/t, C2			
		ϕ_E/W, C2	ϕ_E/W, C2		
$\vec{E}$/t, E2					
			ϕ_E/C, C2		
		q/t, C2		q/t, C2	
			$\vec{E}$/q, C2		
ϕ_B/t, C2		ϕ_B/t, C2			
ϕ_B/W, C2		ϕ_B/W, C2		ϕ_B/W, C2	
ϕ_B/C, C2					
		t/W, C2			t/W,C2
		t/C, C2		t/C, C2	
		W/C, C2			
			$\vec{E}$/W, C2		
			$\vec{E}$/C, C2		
			q/W, C2		
			q/C, C2		
			W/C, C2		

TOTALS:

	A1		A2		A3	
	v	M	v	M	v	M
E2>	24	2	2	-	7	-
C2>	6	-	29	23	6	3
	30	2	31	23	13	3

NOTE: Concepts are represented by its symbols; letters indicate which group had higher mean or variance.

In order to try to interpret these results, let's see if the general averages and mean standard deviations show the same pattern. Table IV-21 presents the average overlapping for all 105 pairs of concepts in each group, and Table IV-22 shows the mean standard deviation for all pairs.

TABLE IV-21

AVERAGE NUMBER OF OVERLAPPING ASSOCIATED WORDS CONSIDERING ALL PAIRS OF CONCEPTS

Group	A1 M	A1 t	A2 M	A2 t	A3 M	A3 t
E1	.64		1.30		1.86	
C1	.51	2.27**	1.39	-.87*	1.99	-1.46*
E2	.67		1.08		1.71	
C2	.54	2.35**	1.44	-4.02**	1.72	- .12*

*$p > .05$; **$p < .05$, two-tailed. $N_1 = N_2 = 105$

TABLE IV-22

MEAN STANDARD DEVIATIONS FOR ALL PAIRS OF CONCEPTS

Group	A1 M_{SD}	A1 t	A2 M_{SD}	A2 t	A3 M_{SD}	A3 t
E1	.91		1.32		1.46	
C1	.71	4.99**	1.21	2.31**	1.48	- .62*
E2	1.00		1.11		1.47	
C2	.86	2.94**	1.37	-6.73**	1.47	.00*

*$p > .05$; **$p < .05$, two-tailed. $N_1 = N_2 = 105$

These Tables, indeed, show the same trend of Tables IV-19 and -20. The experimental groups had averages and variabilities significantly higher than the control groups at the beginning of the course, but, at the end, there were no significant differences between the groups of each pair. The inversion in case A2 for the lecture groups, is also clear in Tables IV-21 and -22.

A possible interpretation of these results can be found, according to the following line of reasoning: supposing that the degree of overlapping is an indication of the degree of relatedness between the concepts of a given pair, to the extent that concept learning occurs during the course, we can expect an increase in the overlapping of associated words for any pair of concepts because all of them are relevant to the study of electromagnetism and, consequently, are related. Obviously, the degree of relationship between the concepts is variable, but the general tendency would be an increase in overlapping. However, if strongly accentuated this tendency would work against concept differentiation. That is, considering that the overlapping is an indirect measure, a high degree of overlapping would indicate a low degree of differentiation.

Thus, on one hand we would expect an increasing degree of relationship among concepts from the beginning to the end of the course, but, on the other hand, if concept differentiation is desirable, we would not expect this effect to be very strong when measured indirectly through overlapping of words associated with these concepts.

Coming back to Table IV-21, we can see that, as expected, the average overlapping increased in all groups. However, this increase was more accentuated in the control groups and even caused a significant inversion in the lecture groups at time A2. The increase in variability in all groups would be also expected since some students would learn more meaningfully than others, but, again, the increase was more accentuated in the control groups and a significant inversion occurred in A2 (Table IV-22).

Thus, we could say that according to our previous reasoning, the degree of concept differentiation was greater and more uniform in the experimental groups. Consequently, we could interpret results as an evidence that the Ausubelian approach was more efficient in fostering concept differentiation than the traditional approach or, at least, that it provided for concept differentiation earlier in the course, since the differences were not statistically significant at the end of the course.

However, we must be careful again because the same argument could be used to say that the evidence is that the Ausubelian approach was less efficient in helping the students to see a closer relationship among the concepts.

The numerical association test, which is a more direct measure of the degree of relationship between concepts, will provide an alternative evaluation to decide between these rather contradictory interpretations.

IV-3 The Numerical Concept Association Test

As already explained, in the numerical association test students were given a list of 105 pairs (the same used to compute the overlapping), in random order, and they should check a number from 1 to 7 indicating the degree of relationship for each pair, 1 standing for the highest relationship and 7 for the lowest. The means and standard deviations for each pair are in Tables IV-23, IV-24, and IV-25. In these Tables, asterisks indicate significant differences at .05 level. An approximation of the t test was used (Darlington, 1975) for the case of unequal variances. The cases where these significant differences occurred are summarized in Tables IV-26 and IV-27. These Tables are still difficult to interpret, but in the case of the PSI groups (Table IV-26) we can easily see that, particularly at the end of the course, the means and variances in the control group were significantly higher than the experimental one in many cases. To be precise, at opportunity A3, the mean of the control group was higher in 23 cases (i.e., in more than 20% of the cases), whereas in none of the cases the inverse occurred. In terms of variability the trend was similar: the variances of the control group were higher in 13 cases (more than 10%), whereas in only 2 cases (2%) the experimental group had greater variance. Thus, in the PSI groups the experimental one tended to present smaller means (higher relationship between concepts) and variances. However, this trend was not observed in the lecture groups where few significant differences were found.

The general means, i.e., the means of the means corresponding to each pair, in each group are presented in Table IV-28 and the mean standard deviations are shown in Table IV-29. These tables offer a general view of the results of the numerical association test which confirm both the trend observed for the PSI groups in Table IV-26 and the lack of significant differences in the lecture groups.

TABLE IV-23 — NUMERICAL ASSOCIATION TEST A1

	$\mathcal{E}$	R	L	$\vec{B}$	V	U	$\vec{F}$	I	Φ_E	$\vec{E}$	q	Φ_B	t	W	C
$\mathcal{E}$		3.34 3.52 1.72 2.13	3.97 3.95 1.54 1.99	4.14 4.05 1.84 2.08	2.67 2.60 1.47 1.87	2.57 2.48 1.63 1.63	3.27 1.65* 2.12 1.16*	2.51 1.80 1.82 1.32	3.71 3.54 1.78 1.74	3.89 2.31* 1.70 1.35	3.50 2.80 1.83 1.55	4.71 3.36* 1.74 2.30	4.55 5.54* 1.90 1.84	2.78 2.38 1.93 1.70	4.36 4.16 2.03 2.06
R	2.94 3.31 1.58 1.86		3.68 4.33 1.65 1.90	5.14 5.88 1.97 1.48	3.51 4.07 1.92 2.40	3.44 2.92 1.86 1.89	3.76 4.63 1.82 2.18	2.16 1.46* 1.88 .83*	3.32 2.92 1.83 1.92	4.08 4.27 2.15 1.95	3.47 3.04 2.08 2.17	4.83 6.13* 1.76 1.14*	5.43 5.44 2.05 2.34	3.24 3.88 2.02 2.15	3.92 3.73 1.89 2.05
L	3.73 3.64 1.61 1.93	3.85 3.86 2.08 2.09		3.09 3.95 1.87 2.27	3.86 3.77 1.78 2.20	3.88 4.35 1.89 1.94	3.89 3.85 1.90 1.83	3.08 2.50 1.50 1.54	3.68 3.45 1.70 1.84	3.68 3.55 1.61 1.97	3.42 3.43 1.91 2.09	3.67 3.96 1.71 2.03	4.85 4.71 1.94 2.05	4.89 5.56 1.99 1.89	4.08 4.14 2.01 2.12
$\vec{B}$	3.37 3.94 1.78 1.84	4.78 5.34 1.69 1.75	3.56 3.53 2.08 2.05		3.11 2.79 1.68 1.89	3.00 2.75 1.60 1.70	2.95 3.00 2.01 2.30	3.34 2.21* 2.53 1.72*	3.17 3.76 1.90 2.24	3.11 3.88 2.04 2.35	3.37 3.33 2.19 2.29	2.76 1.92* 2.07 1.32*	5.53 5.82 2.09 1.87	3.74 3.79 1.78 2.17	4.72 6.11* 1.75 1.15*
V	2.51 2.75 1.63 2.05	3.06 4.08* 1.76 2.29	3.91 3.81 1.62 2.03	2.76 2.94 1.62 1.53		2.31 1.93 1.72 1.30	2.97 3.15 1.66 1.83	2.39 1.92 1.64 1.47	3.74 3.30 1.75 1.97	2.94 2.54 1.78 1.69	2.54 2.35 1.54 1.90	4.36 3.02* 1.84 2.04	5.03 5.64 2.15 2.06	2.97 2.30 1.80 1.44	3.51 3.24 1.72 2.02
U	2.43 2.11 1.71 1.62	2.47 3.03 1.63 1.68	4.29 4.11 1.68 1.60	3.06 3.20 1.83 1.68	1.89 2.17* 1.09 1.58*		2.43 2.08* 1.65 1.02*	2.61 2.40 1.85 1.58	3.65 3.68 1.87 1.86	2.92 3.46 1.75 2.11	2.76 2.38 1.84 1.56	3.37 4.00 1.73 2.09	4.54 4.07 2.13 2.48	1.84 1.22* 1.85 .42*	3.88 3.52 1.51 1.95
$\vec{F}$	2.53 2.17 1.95 1.63	4.17 4.06 1.92 2.14	4.36 4.11 2.23 2.14	2.29 3.43* 1.49 1.99	2.75 3.33 1.79 1.66	2.19 2.56* 1.21 2.06*		3.21 3.50 1.91 2.15	4.34 4.33 1.58 2.04	3.08 2.38 2.18 1.58	3.09 2.50 1.95 1.68	3.36 3.16 2.36 2.06	4.19 4.48 2.07 2.46	2.03 1.07* 2.07 .27*	4.29 5.08 1.95 1.72
I	2.03 1.78 1.32 .87*	1.37 1.72 .73 1.34*	2.09 2.89* .90 1.74*	1.80 2.03 1.21 1.69	2.06 2.33* 1.21 1.85*	1.69 2.56* .76 1.58*	3.29 3.94 1.71 1.84		2.26 1.96* 1.93 1.27*	2.26 2.24 1.90 1.88	2.11 1.44* 2.10 1.22*	3.27 3.92 2.08 2.22	4.68 3.89 2.29 2.39	3.54 3.33 1.98 1.97	4.05 3.38 1.75 1.88
Φ_E	3.23 2.81 1.90 1.80	2.74 2.44 1.71 1.73	3.53 3.60 1.97 1.87	3.26 2.92 1.99 1.78	4.06 3.47 1.79 1.75	3.71 3.28 1.84 1.49	4.00 4.47 2.10 1.70	1.80 1.72 1.23 1.06		2.70 2.44 1.79 1.83	2.71 2.48 1.90 1.53	3.79 4.12 2.08 2.39	3.68 3.00 2.21 2.30	4.06 4.00 1.88 2.02	4.22 4.36 1.51 1.62
$\vec{E}$	3.20 3.50 1.61 1.56	3.11 3.97 2.00 1.80	3.50 3.33 1.91 1.94	2.64 2.61 1.78 1.84	1.97 2.22 1.22 1.33	2.86 2.89 1.91 1.55	2.41 2.47 1.84 1.89	1.78 1.86 1.35 1.38	2.39 1.97* 2.14 1.44*		2.30 1.56 1.90 1.09*	3.89 4.81 2.11 1.90	5.62 6.12 1.95 1.72	3.47 3.54 1.81 2.25	3.76 3.00 1.72 1.97
q	3.23 2.97 1.82 1.61	2.89 3.08 1.79 2.06	3.73 3.31 1.99 1.98	2.97 3.28 1.83 2.16	2.86 2.64 1.63 1.57	2.49 2.14 1.69 1.31	3.11 2.86 1.86 2.11	1.20 1.78* .41 1.64*	2.50 2.50 1.70 1.75	1.47 1.83 1.11 1.46		3.95 3.74 1.89 2.01	4.79 5.38 2.46 2.08	4.08 3.71 1.96 2.22	2.83 3.48 1.65 2.23
Φ_B	3.76 3.97 1.78 1.76	4.59 5.14 1.46 1.69	3.82 3.86 2.16 2.16	1.92 1.92 1.46 1.44	4.55 4.28 1.62 1.70	3.74 3.92 1.66 1.73	3.31 3.72 1.95 2.16	3.12 2.86 1.89 1.57	3.66 3.50 2.09 1.76	3.77 3.14 1.91 1.66	3.74 3.53 1.90 1.96		3.89 3.65 2.35 2.44	4.20 4.52 1.88 2.08	4.54 5.52 2.12 1.69
t	4.79 4.75 2.09 2.13	5.09 4.89 2.02 2.39	4.85 4.71 1.94 2.05	5.46 5.89 1.75 1.74	4.74 5.83* 2.13 1.70	4.54 4.36 2.25 2.42	3.69 4.22 2.29 2.45	3.19 3.03 2.39 2.32	3.03 3.92 2.18 2.42	5.14 5.89 2.07 1.83	4.94 4.89 2.16 2.39	3.29 4.11 2.37 2.36		4.76 5.11 2.31 2.44	4.42 4.83 2.14 1.88
W	2.44 1.94 1.76 1.28	3.40 2.69 2.00 1.67	5.40 5.67 1.77 1.49	3.58 3.42 1.99 2.05	2.72 2.75 1.77 1.87	1.08 1.39* .37 1.08*	1.31 1.38 1.04 1.40	3.56 2.81 1.61 1.75	3.97 3.81 1.93 2.10	3.57 3.44 1.69 1.92	3.91 3.19 1.98 1.98	4.57 4.28 2.00 1.75	3.89 4.33 2.55 2.62		4.08 4.38 1.93 2.00
C	4.24 4.46 2.23 2.06	3.71 3.89 1.98 2.27	4.03 4.23 2.15 2.37	4.68 4.74 1.84 1.77	3.51 3.24 1.72 2.02	3.88 3.69 1.82 1.71	4.21 4.86 1.75 1.90	3.51 2.89 1.76 1.78	3.91 3.80* 2.17 1.49*	3.66 3.60 2.18 2.02	3.24 2.83 1.95 1.54	4.76 4.56 2.00 1.86	4.38 3.86 2.28 2.13	4.39 4.46 2.03 1.93	

NOTE: Statistics concerning the self-paced groups are below the diagonal-lecture groups are above. Numbers in the first row of each matrix element are the means of experimental and control groups, in that order. The corresponding standard deviations are in the second row. Asterisks indicate significant differences at the .05 level between the means (t test, two-tailed) or between the variances (F test, two-tailed).

TABLE IV-24 —NUMERICAL ASSOCIATION TEST A2

	ℰ	R	L	$\vec{B}$	V	U	$\vec{F}$	I	Φ_E	$\vec{E}$	q	Φ_B	t	W	C
ℰ		3.21 3.12 1.87 1.88	3.77 3.67 1.82 2.01	3.97 4.32 1.79 2.01	3.35 2.69 1.95 2.04	2.68 2.69 1.63 1.59	2.53 2.92 1.88 2.31	2.24 2.15 1.54 1.76	3.59 3.42 1.96 1.72	3.30 3.12 1.81 1.99	3.09 2.56 1.58 1.42	4.53 4.29 1.80 2.26	5.36 4.96 1.65 2.01	2.65 2.46 1.57 1.75	4.00 4.92 1.69 1.79
R	2.91 2.56 1.71 1.89		4.63 4.81 1.79 1.99	4.84 5.50 2.00 1.77	4.36 3.38 2.03 2.48	3.48 2.96 2.10 1.97	4.27 4.76 1.79 1.85	1.52 1.65 1.20 1.29	3.88 4.15 2.06 2.15	4.32 4.38 1.80 2.08	2.91 2.44 1.64 1.69	5.19 5.42 1.99 1.77	4.81 5.62 2.36 1.98	3.73 2.81 2.13 1.60	3.84 4.56 1.91 2.08
L	3.97 3.89 1.96 1.60	4.59 5.34 1.66 1.73		3.76 3.73 1.95 2.03	4.19 3.71 1.84 1.93	4.26 4.00 1.81 1.45	3.94 4.24 1.97 2.02	2.73 2.88 1.44 1.75	4.73 3.86 1.82 1.42	4.48 3.86 1.88 1.86	4.56 3.04* 1.95 1.99	3.94 3.77 1.87 1.88	5.13 5.05 1.93 1.87	5.26 4.50 1.50 2.21*	4.45 4.20 1.89 2.12
$\vec{B}$	4.18 4.57 2.02 1.77	4.89 5.72* 1.70 1.49	3.03 3.17 1.99 1.87		3.42 3.88 2.02 2.07	3.13 2.39 1.76 1.53	2.00 2.60 1.33 1.98*	1.79 2.12 1.53 1.81	3.42 3.39 2.22 1.99	2.68 2.88 1.70 1.56	2.65 2.92 1.72 1.55	1.61 1.96 1.46 1.71	4.94 5.96 2.17 1.61	2.76 3.16 1.15 1.49	4.67 5.00 1.95 2.10
V	2.71 1.53* 1.88 1.06*	3.16 2.54 1.82 2.01	4.32 4.62 1.45 1.76	3.03 3.62 1.83 1.91		2.00 2.12 1.56 1.40	3.21 3.36 1.63 1.75	2.68 2.50 1.79 1.96	2.56 2.96 1.71 1.59	2.79 1.60 1.52 1.38	2.03 1.81 1.24 1.23	4.00 4.64 1.94 1.94	6.06 5.54 1.35 2.10*	2.03 1.96 1.87 1.73	3.53 2.04* 1.56 1.54
U	2.47 1.72* 1.33 .91*	3.06 3.14 1.69 1.94	3.97 4.31 1.78 1.64	3.22 2.86 1.91 1.40	1.64 2.00 1.17 1.04		2.56 2.50 1.80 1.63	2.67 2.35 1.59 1.55	3.58 3.27 1.58 1.73	2.94 2.35 1.50 1.44	2.62 2.08 1.46 1.49	4.15 3.74 1.82 1.79	4.41 4.42 1.92 2.39	1.32 1.31 1.15 1.19	4.16 2.88* 1.55 1.42
$\vec{F}$	2.62 2.58 2.03 1.65	3.94 4.58 1.76 1.68	4.18 4.06 1.89 1.63	1.89 2.34 1.09 1.49	2.74 2.61 1.29 1.46	2.06 2.08 1.35 1.23		3.03 3.64 1.88 2.16	3.79 3.54 1.65 1.58	1.67 1.69 1.27 1.35	2.24 2.00 1.33 1.78	3.36 3.12 1.83 1.54	4.18 3.96 1.96 2.47	1.70 1.42 1.70 1.24	4.64 4.29 1.92 2.03
I	1.88 1.64 1.30 1.02	1.42 1.44 .69 .73	2.73 3.60* 1.59 1.82	1.61 2.06* .99 1.47*	1.89 1.80 1.17 1.23	2.17 2.22 .92 1.20	2.86 3.39 1.78 1.48		2.50 2.50 1.96 1.61	2.47 1.92 1.88 1.20*	1.88 1.35 1.76 1.29	2.59 3.19 1.46 2.17*	3.59 2.73 2.40 2.41	2.82 2.84 1.80 1.52	3.56 3.91 1.94 1.68
Φ_E	3.09 3.86 1.67 1.66	3.44 4.39* 1.58 1.71	4.03 4.18 1.91 1.93	2.67 3.74* 1.53 1.85	2.62 3.72* 1.21 1.65	3.32 3.97 1.74 1.42	3.22 3.94* 1.53 1.45	1.78 3.06* .93 1.79*		1.88 1.96 1.45 1.75	2.21 1.86 1.67 1.42	3.59 4.08 1.94 2.08	4.30 3.85 2.39 2.60	3.77 3.58 1.56 1.98	3.66 4.14 1.77 1.61
$\vec{E}$	2.82 2.22 1.47 1.27	2.92 4.03* 1.50 1.98	3.53 3.94 1.88 1.95	2.06 2.44* 1.07 1.59*	1.28 1.89* .51 1.28*	2.33 2.31 1.41 1.04	1.58 1.49 1.00 .70	1.47 1.94* .84 1.45*	1.34 1.25 .84 .55*		1.70 1.32 1.51 1.25	3.56 4.48 2.05 1.88	5.27 6.08 1.99 1.85	2.79 2.31 1.71 1.49	3.94 2.75* 2.16 2.09
q	2.97 2.42 1.73 1.44	2.44 3.00* 1.16 1.80*	3.73 3.63 1.81 2.10	2.49 3.31 1.77 1.88	1.75 2.22* .84 1.44*	2.14 2.56 1.15 1.21	1.83 1.61 1.18 1.15	1.11 1.22* .40 .72*	1.75 2.11 1.46 1.39	1.19 1.17 .52 .45		3.56 3.87 1.89 1.79	5.97 4.92 1.64 2.42*	3.67 2.36* 1.49 1.52	3.39 2.25 1.82 1.82
Φ_B	4.41 4.83 2.00 1.81	5.50 5.56 1.65 1.65	3.21 3.69 1.82 1.89	1.42 1.60 1.08 1.01	4.03 4.89* 1.84 1.70	3.64 3.92 1.85 1.52	2.81 3.25 1.17 1.56	2.64 3.28 1.64 1.43	2.75 4.00* 1.38 1.84	3.31 4.22* 1.64 1.61	3.81 4.19 1.97 1.98		4.91 4.28 2.21 2.54	4.30 3.90 1.49 2.10	4.61 5.55* 2.04 1.82
t	4.19 4.89 1.94 1.98	3.94 4.53 2.27 2.02	4.39 4.91 1.84 1.98	4.75 6.14* 1.95 1.36	4.97 5.31 1.92 1.89	3.63 4.17 2.13 2.31	3.31 4.25 2.11 2.35	2.50 1.86 1.80 1.61	3.36 5.28* 2.04 2.15	4.61 5.67* 2.00 1.88	4.39 4.11 2.33 2.40	3.94 5.58* 2.18 1.70		5.06 5.08 1.98 2.42	5.09 5.38 1.65 2.24
W	2.09 1.92 1.24 1.16	2.97 3.00 1.61 2.01	4.50 4.69 1.65 1.86	3.47 3.08 1.84 1.50	1.61 1.83 1.20 1.13	1.11 1.06 .32 .23	1.14 1.14* .35 .54*	2.89 2.71 1.33 1.38	3.78 4.03 1.99 1.70	2.57 2.31 1.31 1.28	2.78 2.47 1.38 1.08	4.46 4.47 1.79 1.65	4.19 4.36 2.32 2.37		4.88 3.36* 1.90 1.93
C	4.09 3.44 1.93 1.71	3.91 4.17 1.80 1.90	4.21 4.53 1.92 2.08	5.15 5.03 1.86 1.68	2.69 2.22 1.95 1.81	3.62 2.58* 1.94 1.34*	4.20 4.53 1.81 1.42	3.21 2.94 1.71 1.43	3.03 3.50 1.65 1.96	2.51 2.81 1.65 1.69	2.03 1.58* 1.64 .81*	4.70 5.39 1.94 1.44	4.43 4.53 2.05 2.16	4.29 3.34* 1.79 1.61	

NOTE: Statistics concerning the self-paced groups are below the diagonal line-lecture groups are above. Numbers in the first row of each matrix element are the means of experimental and control groups, in that order. The corresponding standard deviations are in the second row. Asterisks indicate significant differences at the .05 level between the means (t test, two-tailed) or between the variances (F test two-tailed).

TABLE IV-25 — NUMERICAL ASSOCIATION TEST A3

	ℰ	R	L	$\vec{B}$	V	U	$\vec{F}$	I	Φ_E	$\vec{E}$	q	Φ_B	t	W	C
ℰ		2.63 2.46 1.66 1.84	2.72 2.21 1.61 1.64	3.97 2.48* 1.86 1.65	1.84 1.54 1.25 1.06	2.47 1.96* 1.68 1.16*	3.74 3.25 2.32 2.45	1.69 1.39* 1.20 .72*	3.44 3.54 1.76 1.61	3.31 2.96 2.05 1.80	2.94 2.67 1.88 1.95	4.06 2.23* 1.93 1.88	4.28 4.00 2.08 2.23	2.31 1.92 1.69 1.47	3.44 3.96 1.88 2.18
R	2.76 3.52 1.46 2.05		4.97 4.83 1.52 1.83	5.34 5.46 1.60 1.93	3.50 2.88 1.97 2.13	3.35 2.42 2.06 1.53	4.47 4.88 1.83 1.96	1.47 1.38 .98 .77	3.66 4.70 1.84 2.16	4.41 4.92 1.48 1.82	3.00 2.79 1.98 1.86	5.50 5.78 1.63 1.59	4.94 5.00 1.98 2.02	3.48 2.67 1.84 1.95	4.03 4.87 2.11 2.01
L	3.15 3.28 2.00 2.10	4.17 4.75 1.89 1.90		2.52 1.65 1.79 1.37	4.23 3.73 1.84 1.91	3.97 3.05* 2.04 1.33*	4.48 4.43 1.75 1.95	2.56 1.46* 1.50 .78*	3.59 4.21 2.14 2.02	3.56 3.92 1.74 2.22	3.47 3.79 1.88 1.86	2.41 1.91 1.56 1.28	3.91 3.17 2.15 2.12	4.75 4.22 1.61 1.62	4.34 4.04 1.70 1.83
$\vec{B}$	3.20 2.82 1.86 1.86	5.09 5.15 1.91 1.56	1.71 2.06 1.15 1.41		3.78 3.88 1.84 2.36	2.91 2.38 1.75 1.41	2.19 2.04 1.51 1.46	1.31 1.48* .54 1.04*	2.22 2.92 1.58 2.10	2.03 1.42* 1.20 .65*	3.50 3.33 1.78 1.95	1.53 1.88* .88 1.73*	3.38 3.35 1.93 2.08	3.19 2.88 1.57 1.96	5.39 4.91 1.86 2.17
V	1.86 2.27 1.35 1.44	3.00 3.15 2.06 2.08	4.12 4.41 1.70 1.64	4.17 3.45 2.19 1.66		2.09 1.83 1.47 1.24	3.72 3.13 1.69 1.51	2.44 2.25 1.74 1.75	3.25 3.30 1.68 1.92	1.66 2.00* .90 1.74*	2.23 1.71* 1.45 .95*	4.78 4.63 1.96 1.97	5.25 5.46 1.92 1.98	2.53 1.83* 1.85 1.24*	2.22 2.13 1.29 1.82
U	2.44 2.03 1.52 1.05*	3.24 3.18 1.81 1.96	2.88 3.30 1.74 1.61	1.89 2.61* 1.39 1.09	1.80 2.39* .90 1.39*		2.34 1.54* 1.54 1.02*	2.31 2.17 1.47 1.34	3.38 3.77 1.98 2.02	1.97 1.87 1.00 1.22	2.52 1.75 1.61 1.29	3.66 3.04 1.73 1.90	3.88 3.63 2.06 2.26	1.22 1.33* .66 1.09*	2.59 2.68 1.64 1.64
$\vec{F}$	2.20 3.27* 1.78 2.10	3.68 4.36 1.75 1.65	3.73 4.90* 1.91 1.96	1.97 2.81* 1.22 1.86*	2.97 2.94 1.36 1.46	2.06 2.55 1.30 1.12		3.00 2.25 1.74 1.67	3.75 3.83 1.70 1.79	2.03 1.92 1.20 1.67	1.88 1.96* 1.13 1.66*	3.91 3.46 1.91 2.06	4.75 4.54 2.20 2.45	1.34 1.00* .97 .00*	4.13 4.22 1.72 2.07
I	1.54 1.73 .74 .94	1.44 1.88* .70 1.39*	1.89 2.91* 1.30 1.80	1.57 1.70 .85 1.19	1.97 2.67* 1.17 1.61	2.15 2.73 1.06 1.31	2.86 3.59 1.75 1.97		2.72 2.88 1.57 2.01	1.75 2.17 .92 1.24	1.39 1.71* .76 1.73*	2.50 2.22 1.46 1.44	2.28 1.83 1.67 1.63	3.19 2.58 1.60 1.59	3.16 3.83 1.67 1.71
Φ_E	3.00 3.52 1.91 1.66	3.40 4.52* 1.74 1.80	3.60 4.24 1.96 1.75	1.89 2.21* .96 1.39*	2.94 3.70 1.50 1.65	2.97 3.56 1.45 1.24	3.06 4.15* 1.35 1.35	2.09 2.91* 1.06 1.59*		2.09 1.42* 1.63 .83*	2.50 1.79* 1.76 1.18*	2.75 3.33 1.68 1.81	2.78 4.08* 2.12 2.47	4.16 4.08 1.55 1.77	3.28 3.09 1.61 2.09
$\vec{E}$	2.11 2.73* 1.21 1.31	3.49 4.21 1.77 1.78	3.34 3.85 2.04 1.77	1.51 1.64 .85 .86	1.40 1.97* .77 1.49*	1.49 2.52* .78 1.35*	1.40 2.27* .81 1.40*	1.43 1.79* .61 1.05*	1.29 1.30 .68 .53		1.41 1.04* 1.01 .20*	2.25 2.54 1.41 2.00	4.16 4.63 2.00 2.32	2.69 2.54 1.45 1.56	2.06 2.38 1.32 1.84
q	2.85 2.91 1.88 1.40	2.71 3.45 1.38 2.06	4.03 4.03 1.88 1.64	3.00 3.36 1.86 1.71	1.89 2.39 1.32 1.32	2.18 2.76 1.55 1.41	1.71 2.39 1.25 1.68	1.54 1.24 1.09 .66*	2.11 2.48 1.35 1.58	1.29 1.55 .94 1.20		4.50 3.25* 2.02 2.09	5.25 4.08 2.30 2.52	3.00 2.88 1.55 1.70	2.19 1.67 1.60 1.34
Φ_B	2.77 2.33 1.70 1.55	4.82 5.36 1.71 1.65	2.15 2.38 1.48 1.60	1.44 1.45 .79 .90	4.34 4.30 1.70 1.76	2.80 3.48 1.45 1.42	3.20 3.58 1.66 1.71	2.29 2.27 1.23 1.51	2.43 2.91 1.58 1.28	2.09 2.33 1.31 1.59	4.03 3.79 2.22 2.00		2.75 3.17 1.90 2.22	4.50 4.30 1.97 1.55	5.03 5.30 1.87 1.69
t	3.50 3.63 2.02 2.24	4.09 4.53 2.19 1.88	3.15 4.39* 1.71 2.25	3.86 4.76 2.10 2.03	4.76 4.50 1.80 2.05	3.49 4.18 1.99 2.07	3.43 4.91* 1.74 1.89	1.97 2.06 1.42 1.87	2.77 3.48 1.96 2.37	4.26 4.67 2.13 2.03	4.49 4.39 2.29 2.45	2.91 3.70 1.93 2.05		4.09 4.29 2.29 2.60	6.03 5.09* 1.45 2.31*
W	2.09 2.00 1.46 1.22	2.74 3.55* 1.34 1.95*	4.21 4.79 1.97 1.67	3.40 3.55 1.67 1.58	1.80 2.00 .93 1.27	1.14 1.67* .60 1.11*	1.37 1.33 .94 .78	2.80 3.12 1.23 1.58	3.63 4.56* 1.78 1.61	2.40 3.09* 1.01 1.65*	2.57 3.45 1.70 2.06	4.37 4.45 1.61 1.68	4.14 4.71 2.17 2.08		4.09 3.83 2.07 2.12
C	2.80 3.50 1.69 1.85	3.54 4.15 1.70 1.89	3.37 3.72 1.99 1.76	4.51 4.76 1.76 1.73	1.69 2.52* .93 1.77*	2.54 2.79 1.24 1.62	3.91 4.79* 1.70 1.93	2.59 3.50* 1.62 1.78	2.80 3.73* 1.41 1.61	2.00 2.36 1.46 1.48	1.83 1.73 1.07 1.28	4.69 4.33 1.84 1.81	3.77 4.48 2.17 2.08	3.03 3.81* 1.48 1.67	

NOTE: Statistics concerning the self-paced groups are below the diagonal-lecture groups are above. Numbers in the first row of each matrix element are the means of experimental and control groups, in that order. The corresponding standard deviations are in the second row. Asterisks indicate significant differences at the .05 level between the means (t test, two-tailed) or between the variances (F test, two-tailed).

TABLE IV-26

NUMERICAL ASSOCIATION TEST--PSI GROUPS
(Pairs of Concepts Where Significant Differences Occurred)

A1		A2		A3	
V	M	V	M	V	M
q/I, C1	q/I, C1	q/I, C1		q/I, E1	
					$\vec{F}$/L, C1
I/V, C1					I/V, C1
			W/C, E1		W/C, C1
		I/$\vec{B}$, C1			
		I/$\vec{E}$, C1		I/$\vec{E}$, C1	
	t/V, C1				
					F/C, C1
I/L, C1	I/L, C1		I/L, C1		I/L, C1
				C/V, C1	C/V, C1
Φ_E/$\vec{E}$, E1		Φ_E/$\vec{E}$, E1			
			t/$\vec{E}$, C1		
					W/ϕ_E, C1
		Φ_E/I, C1	Φ_E/I, C1	Φ_E/I, C1	ϕ_E/I, C1
				$\vec{F}$/$\vec{E}$, C1	$\vec{F}$/$\vec{E}$, C1
			Φ_E/R, C1		ϕ_E/R, C1
		$\vec{E}$/V, C1	$\vec{E}$/V, C1	$\vec{E}$/V, C1	
			ϕ_B/$\vec{E}$, C1		
	R/V, C1				
			$\vec{B}$/R, C1		
		$\vec{E}$/$\vec{B}$, C1			
			$\vec{E}$/R, C1		
			t/ϕ_E, C1		
		U/ε, E1	U/ε, E1	U/ε, E1	
			ϕ_B/V, C1		
					I/C, C1
			ϕ_E/V, C1		
			ϕ_E/ϕ_B, C1		
U/I, C1	U/I, C1				
		$\vec{F}$/W, C1			
	$\vec{F}$/$\vec{B}$, C1			$\vec{F}$/$\vec{B}$, C1	$\vec{F}$/$\vec{B}$, C1
				W/$\vec{E}$, C1	W/$\vec{E}$, C1
			t/$\vec{B}$, C1		
			t/ϕ_B, C1		
					$\vec{F}$/ε, C1
			$\vec{F}$/ϕ_E, C1		$\vec{F}$/ϕ_E, C1

TABLE IV-26 -- Continued

A1 v	A1 M	A2 v	A2 M	A3 v	A3 M
			$\phi_E/\vec{B}$, C1	$\phi_E/\vec{B}$, C1	
		q/R, C1			
					$U/\vec{B}$, C1
		ε/V, E1	ε/V, E1		
Φ_E/C, E1					ϕ_E/C, C1
				$U/\vec{E}$, C1	$U/\vec{E}$, C1
				W/R, C1	
		q/C, E1			
		q/V, C1			
					$t/\vec{F}$, C1
					$\vec{E}$/ε, C1
					t/L, C1
U/V, C1				U/V, C1	U/V, C1
I/R, C1				I/R, C1	
U/W, C1				U/W, C1	U/W, C1
I/ε, E1					
		U/C, E1	U/C, E1		
$U/\vec{F}$, C1					

TOTALS:

	A1 v	A1 M	A2 v	A2 M	A3 v	A3 M
E1 >	3	-	5	4	2	-
C1 >	8	6	9	16	13	23
	11	6	14	20	15	23

NOTE: Concepts are represented by its symbols; letters indicate which group had higher mean or variance.

TABLE IV-27

NUMERICAL ASSOCIATION TEST-LECTURE GROUPS
(Pairs of Concepts Where Significant Differences Occurred)

A1		A2		A3	
V	M	V	M	V	M
		W/L, C2			
				$\vec{F}$/q, C2	
q/I, E2				q/I, C2	
				t/C, C2	
			W/C, E2		
I/$\vec{B}$, E2	I/$\vec{B}$, E2			I/$\vec{B}$, C2	
		t/q, C2			
		I/$\vec{E}$, E2			
		t/V, C2			
				I/L, E2	I/L,E2
			C/V, E2		
	t/ε, C2				
				Φ_E/$\vec{E}$, E2	
Φ_B/$\vec{B}$, E2				ϕ_B/$\vec{B}$, C2	
				$\vec{E}$/V, C2	
				$\vec{E}$/$\vec{B}$, E2	$\vec{E}$/$\vec{B}$,E2
					t/Φ_E,C[illegible]
				U/ε, E2	
	Φ_B/V, E2				
			$\vec{E}$/C, E2		
$\vec{F}$/W, E2	$\vec{F}$/W, E2			F/W, E2	
q/$\vec{E}$, E2				q/$\vec{E}$, E2	
					$\vec{B}$/ε,E2
		$\vec{F}$/$\vec{B}$, C2			
$\vec{B}$/C, E2	$\vec{B}$/C, C2				
			W/q, E2		
			q/L, E2		
$\vec{F}$/ε, E2	$\vec{F}$/ε, E2				
		Φ_B/I, C2			
					q/Φ_B,E[illegible]
Φ_B/R, E2	Φ_B/R, C2				
			q/C, E2		
				q/V, E2	
				U/L, E2	

TABLE IV-27 -- Continued

A1 v	A1 M	A2 v	A2 M	A3 v	A3 M
				q/Φ_E, E2	
				W/V, E2	
I/R, E2					
U/W, E2				U/W, C2	
				$I/\mathcal{E}$, E2	
			U/C, E2		
	$\Phi_B/\mathcal{E}$, E2				$\Phi_B/\mathcal{E}$,E2
$U/\vec{F}$, E2				$U/\vec{F}$, E2	$U/\vec{F}$,E2
Φ_E/I, E2					
	$\vec{E}/\mathcal{E}$, E2				

TOTALS:

	A1 v	A1 M	A2 v	A2 M	A3 v	A3 M
E2 >	12	6	1	7	12	6
C2 >	--	3	5	-	7	1
	12	9	6	7	19	7

NOTE: Concepts are represented by its symbols; letters indicate which group had higher mean or variance.

Assuming that most of the fifteen concepts selected for this test were relatively unknown to the students and considering that all of them are relevant to the study of electromagnetism and, thus, related at least to some extent, we could expect a decrease in means and variability from A1 to A3. That is, we would expect that as the course developed, students would see more relationship between the concepts and would give more uniform ratings. This expectation is supported by the results shown in Tables IV-28 and -29: they seem to indicate a gradual decrease in means and variability in all groups.

However, these Tables also seem to indicate that, in the case of the PSI groups, such a decrease was more accentuated for the experimental group: at the end of the course the mean of the experimental group was significantly lower than the mean of the control group; the mean standard deviation was also lower but not statistically significant. In the case of the lecture groups, no significant differences were found. At the end of the course, the variability of the experimental group was slightly lower, but the mean was slightly higher. However, in both cases the difference was not significant.

Thus, these results could be interpreted as evidence that, particularly in the case of the PSI groups, the Ausubelian approach was more efficient in helping the students to relate the concepts involved in the course. But, in accepting this evidence, we are ruling out the second possible interpretation formulated for the results of the overlapping in the word association test. (At this opportunity we said that the Ausubelian approach could have fostered more concept differentiation than the traditional approach, but could have been less efficient in helping the students to relate the concepts.)

TABLE IV-28

GENERAL MEAN SCORES IN THE NUMERICAL ASSOCIATION TEST

Group	A1 M	A1 t	A2 M	A2 t	A3 M	A3 t
E1	3.33		3.08		2.80	
C1	3.41	-.57*	3.33	-1.54*	3.24	-3.17**
E2	3.59		3.51		3.21	
C2	3.52	.50*	3.39	.77*	3.03	1.13*

*$p > .05$; **$p < .05$, two-tailed. $N_1 = N_2 = 105$

TABLE IV-29

MEAN STANDARD DEVIATIONS IN THE NUMERICAL ASSOCIATION TEST

Group	A1 M	A1 t	A2 M	A2 t	A3 M	A3 t
E1	1.77		1.57		1.51	
C1	1.82	-1.02*	1.56	.17*	1.62	-1.99*
E2	1.89		1.77		1.66	
C2	1.86	.65*	1.82	-1.24*	1.71	- .85*

*$p > .05$, two-tailed. $N_1 = N_2 = 105$

Taking a rather general position and putting together the bits of evidence gathered from the word association and the numerical association tests, we could interpret them as evidence that the Ausubelian approach fostered significant associations, concept differentiation, and concept relatedness to a larger extent than the traditional approach, and that these effects were more accentuated when individualized instruction was used. In addition, we could say that, in terms of traditional achievement measures the Ausubelian approach was, at least, as efficient as the traditional one.

However, we must agree that, so far, this evidence is not too strong and that the analysis of the results was kept at a rather macroscopic level. That is, the statistical analysis of the association tests was quite simple and more refined techniques like multidimensional scaling or relatedness coefficients could lead to evidences less (or more) supportive of Ausubel's theory. However, we think that a first attempt to make sense out of the large (and variable) amount of data provided by the association tests should not be highly sophisticated. ("Sophisticated" statistical approaches tend to obscure the meaning of the raw data.) In addition, we also had the graphical association test and the laboratory test to gather further evidence about any possible effect of the Ausubelian approach in comparison to the traditional one.

IV-4 The Graphical Concept Association Test

In this test, students were given a list of nineteen concepts, including the key concepts of electromagnetism as well as subordinate concepts, and asked to draw a "concept map" (although this terminology was not used) according to the following rules: The more general concepts should be written inside rectangles, concepts at an intermediate level of generality should be written inside circles or ellipses, and the least general concepts should be just written without any geometrical figure surrounding them. In addition, lines of arbitrary size and shape should be used to connect related concepts.

This test provided remarkable differences between experimental and control groups. To support this assertion and the interpretation of the results of this test we randomly selected five sets of tests (A1, A2, A3), corresponding to five students from each group. Thus, a total of 20 sets was randomly selected: eight of them, two for each group, are reproduced in Figures 1 to 8 and the remaining 12 are in Appendix XII. All maps in Figures 1 to 8 and in Appendix XII are exact reproductions of the actual maps drawn by the students.

Analyzing these maps qualitatively, it is hard to see any major differences in the maps drawn at the beginning of the course (A1). However, at opportunity A2 (middle of course) we can see in the experimental groups a tendency toward a vertical conceptual hierarchy where the most general concepts are at the top. In addition, there is also a tendency of identifying "electromagnetic field" and "electromagnetic force" as the most inclusive concepts.

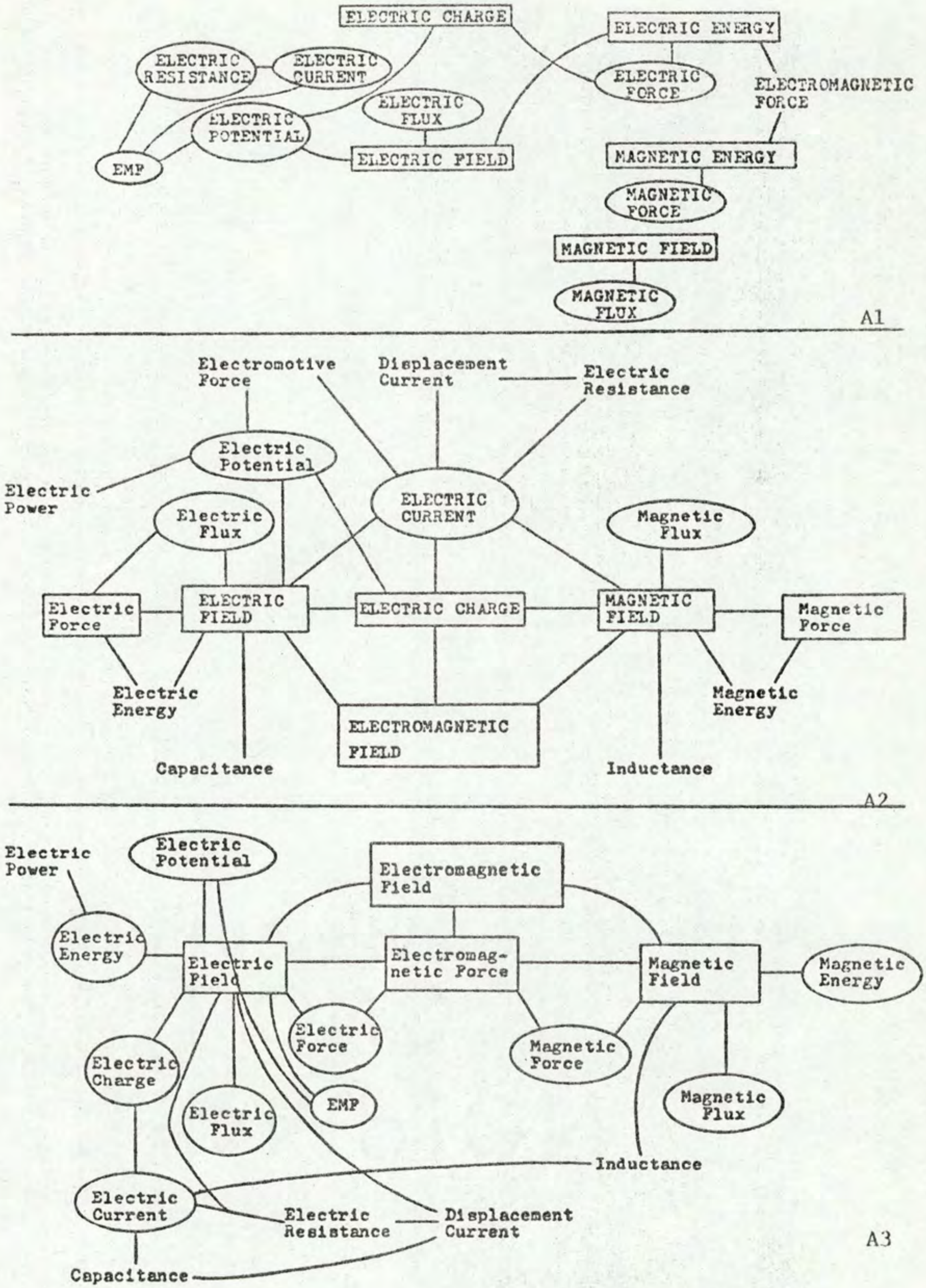


Fig. IV-1. Concept Maps, Student # 19, Group El.

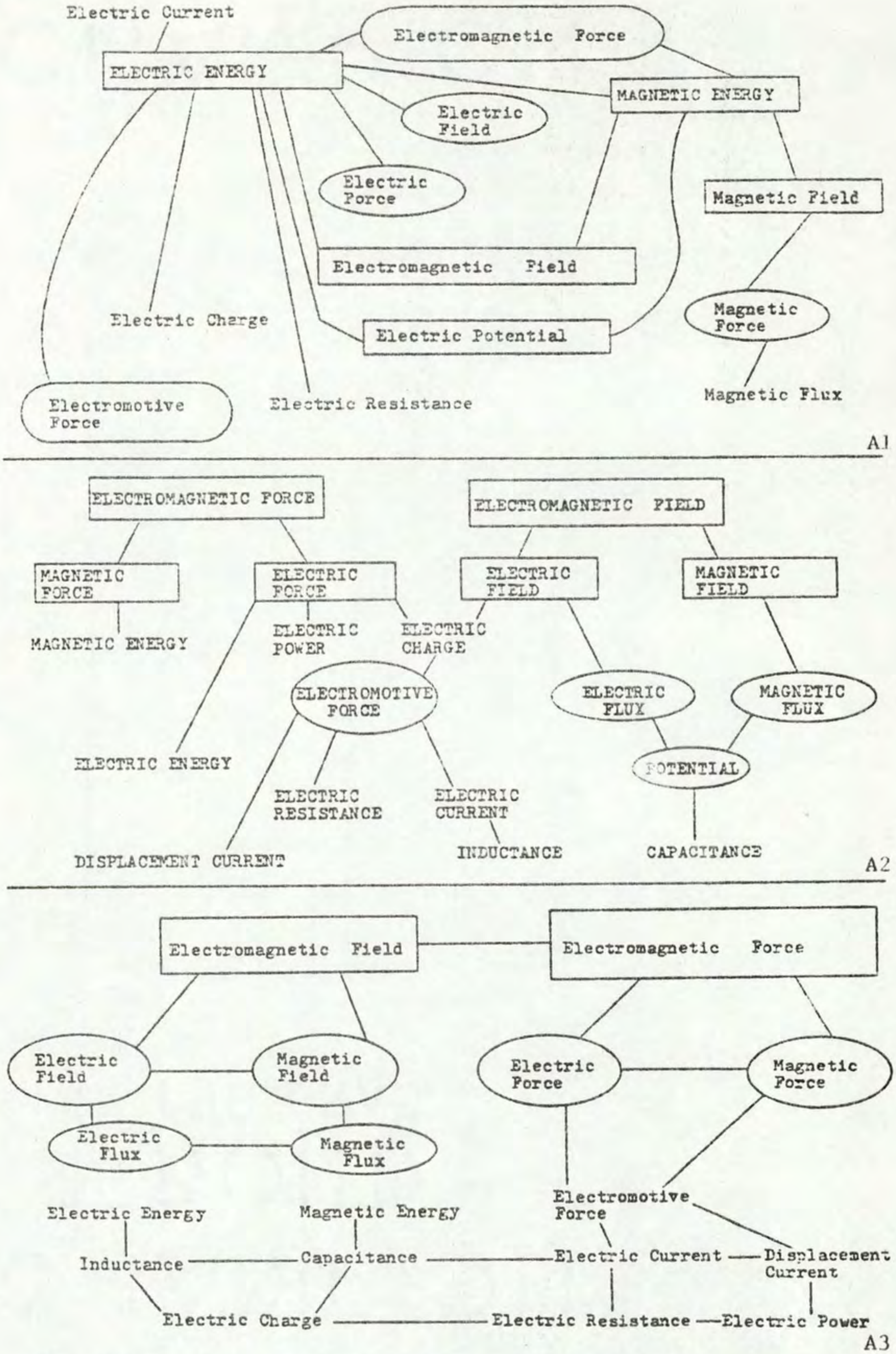
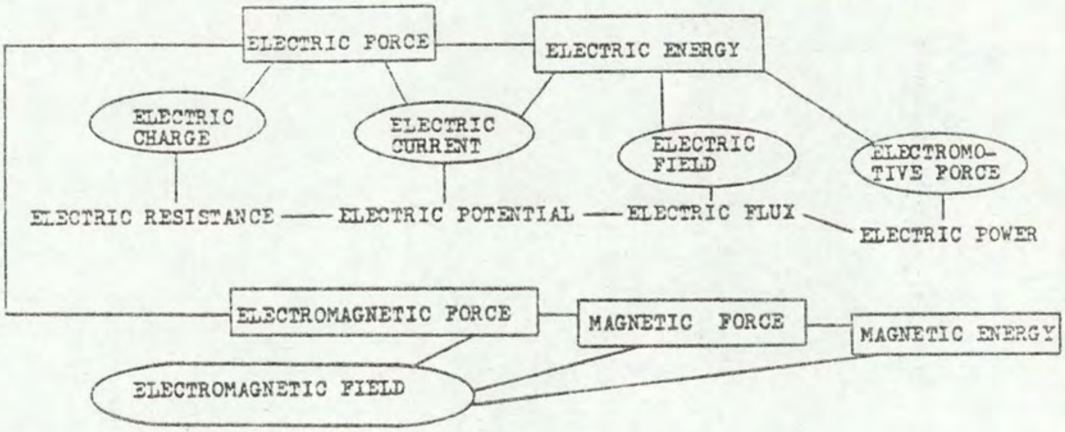
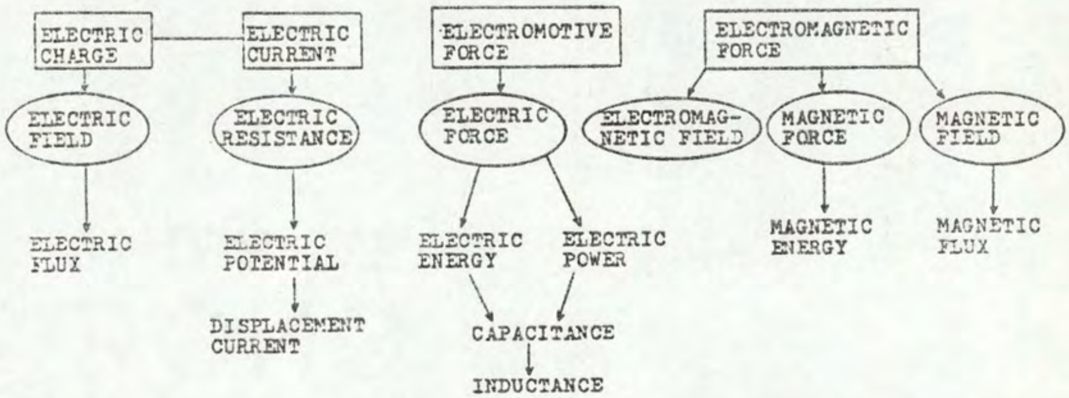


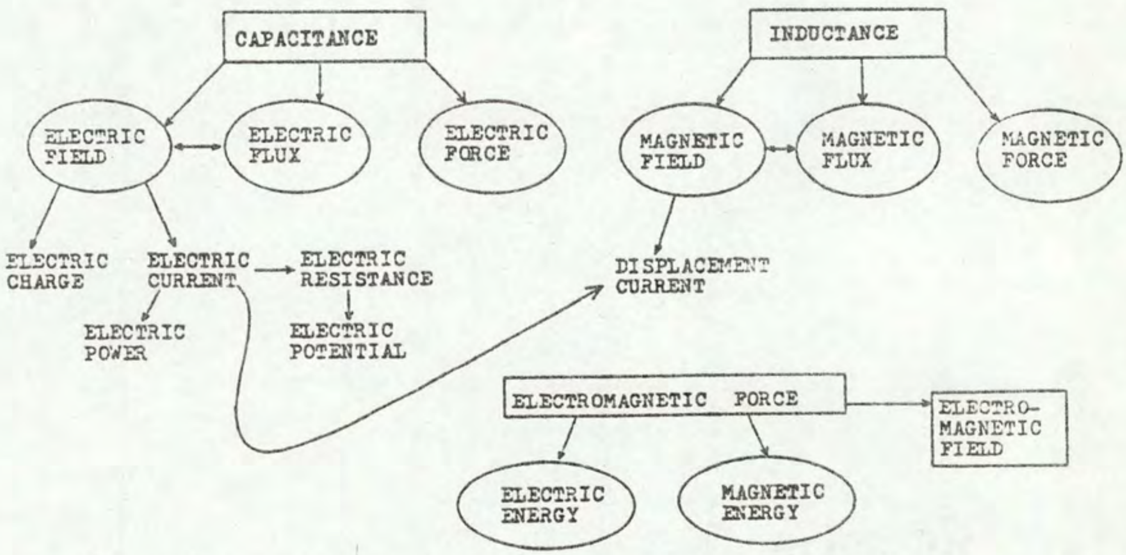
Fig. IV-2. Concept Maps, Student # 26, Group E1.



A1



A2



A3

Fig. IV-3. Concept maps, Student # 19, Group C1.

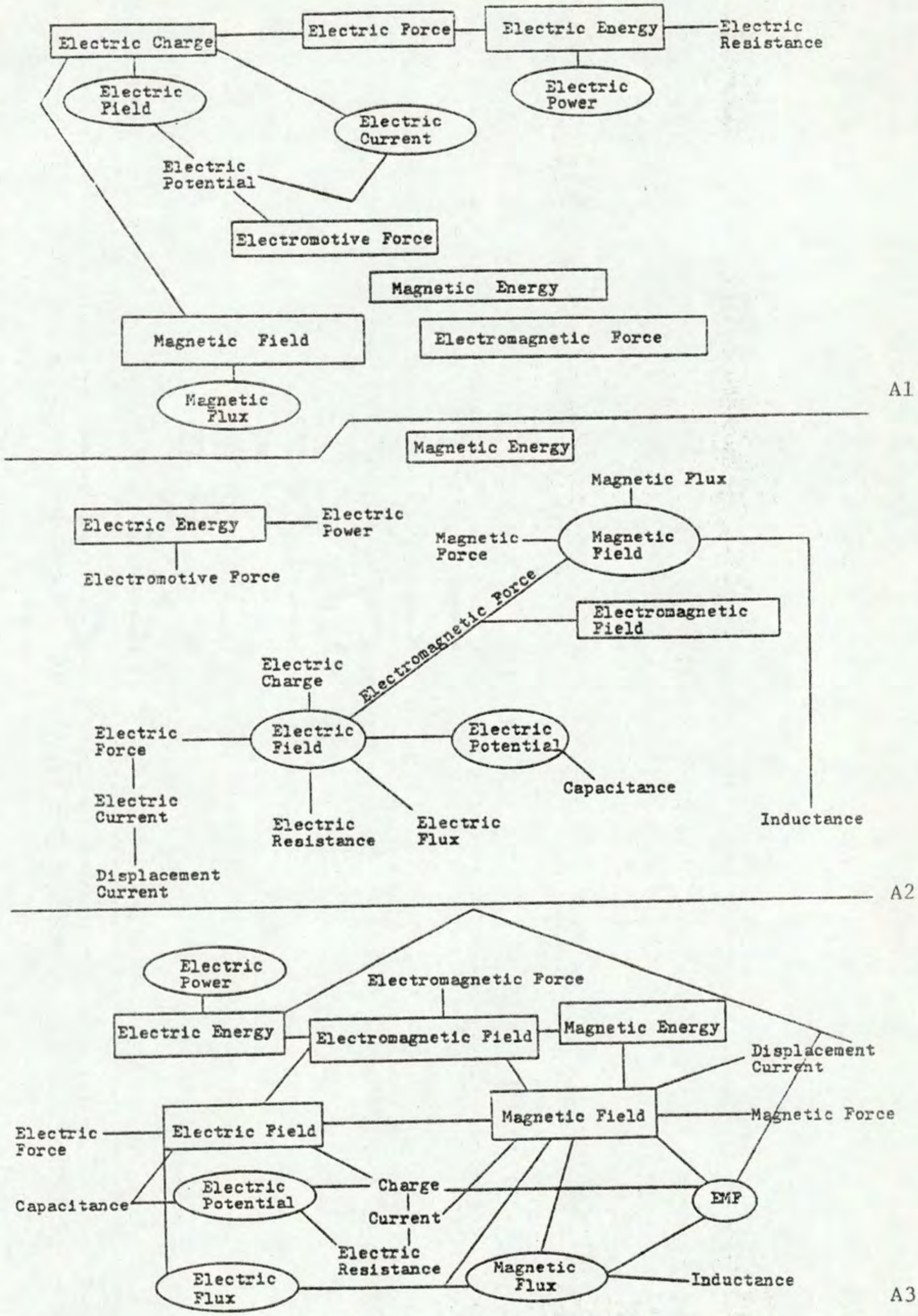
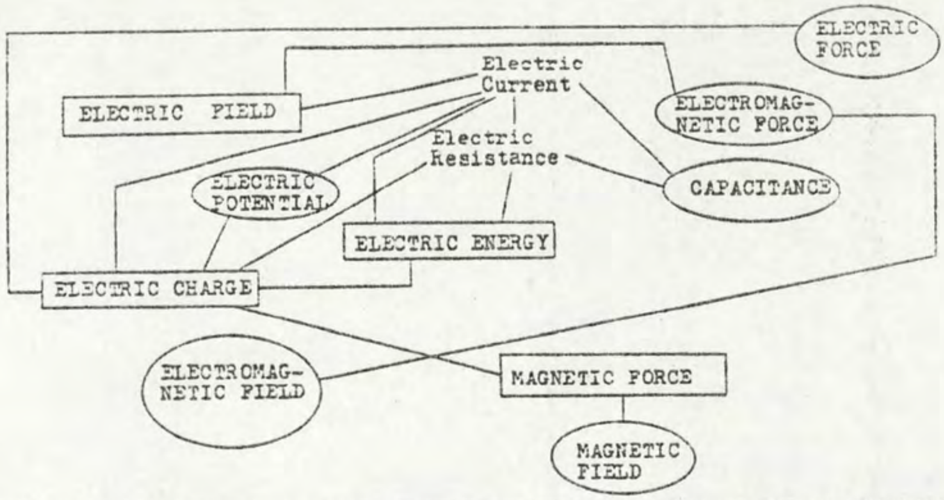
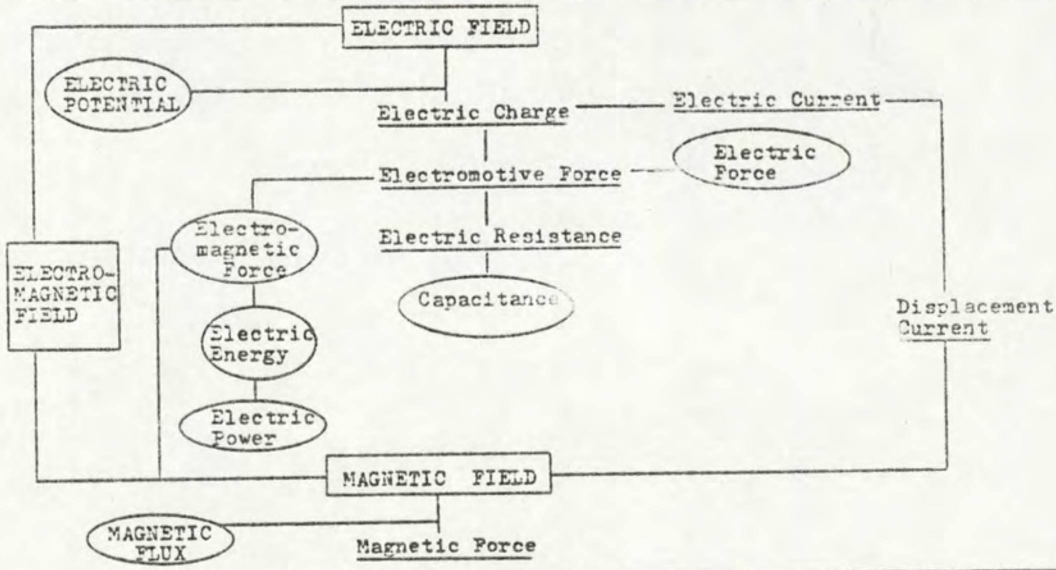


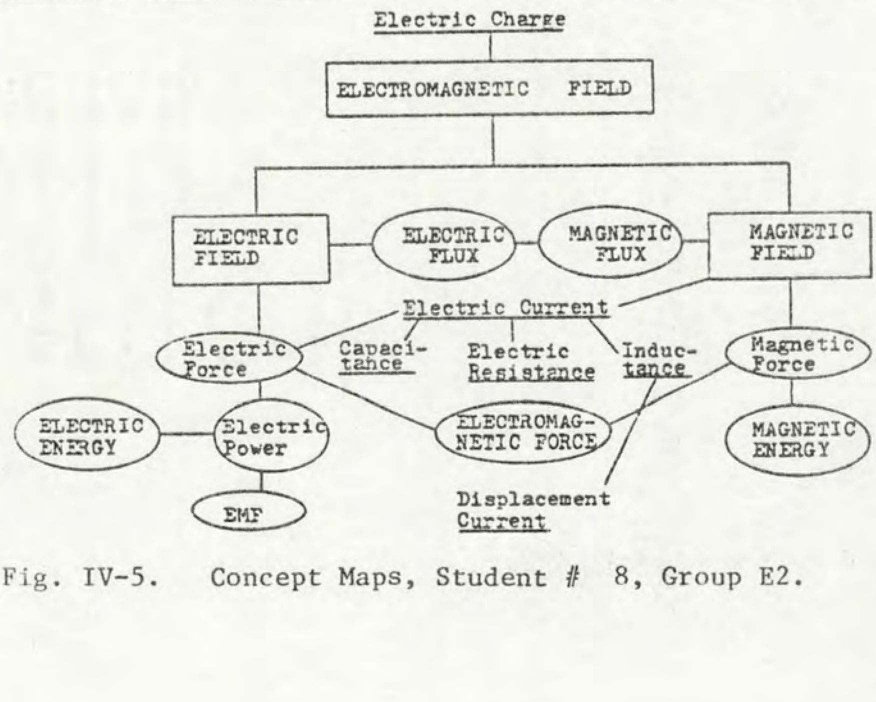
Fig. IV-4. Concept Maps, Student # 8, Group C1.



A1

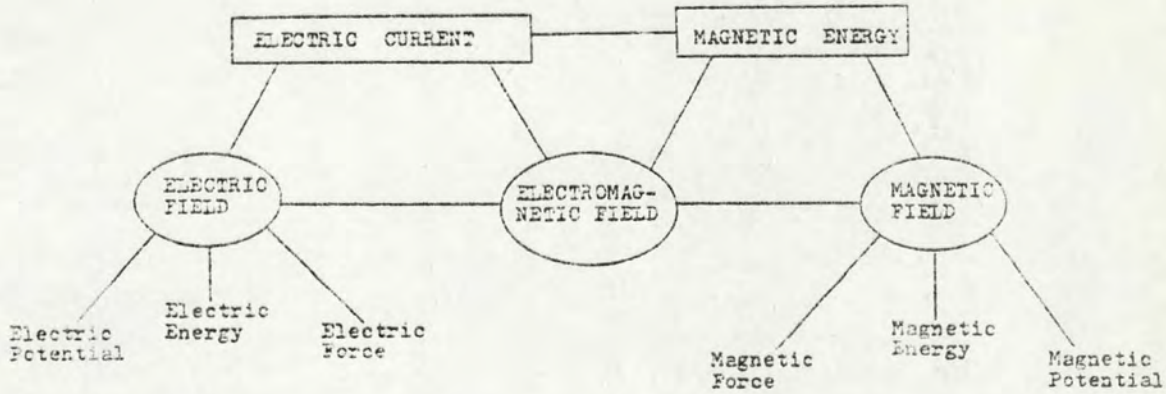


A2

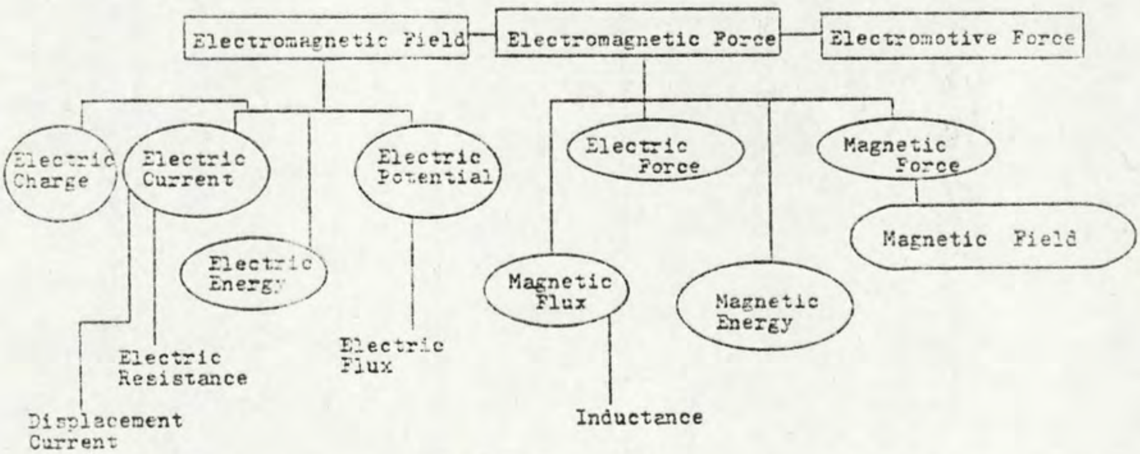


A3

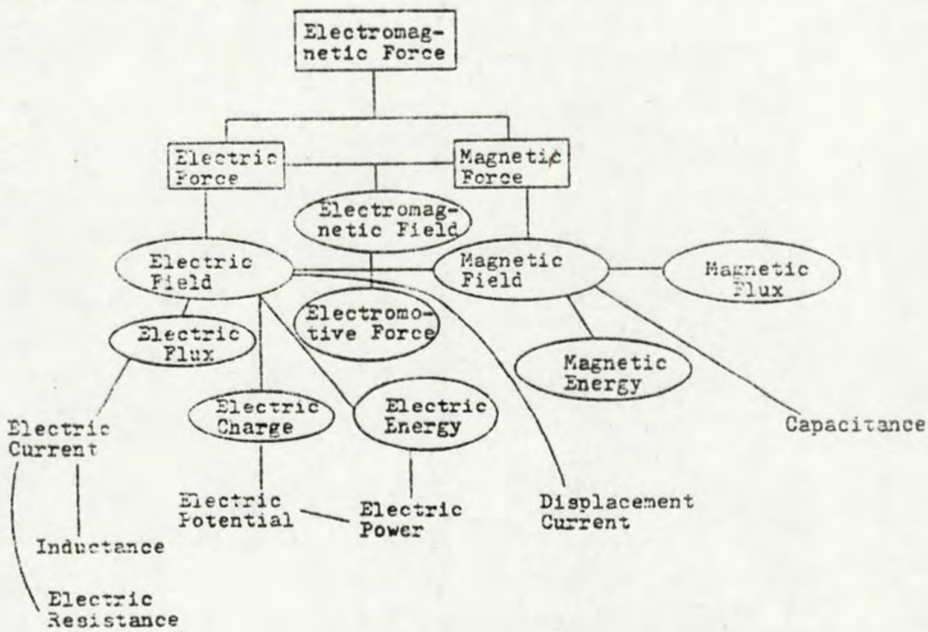
Fig. IV-5. Concept Maps, Student # 8, Group E2.



A1



A2



A3

Fig. IV-6. Concept Maps, Student # 25, Group E2.

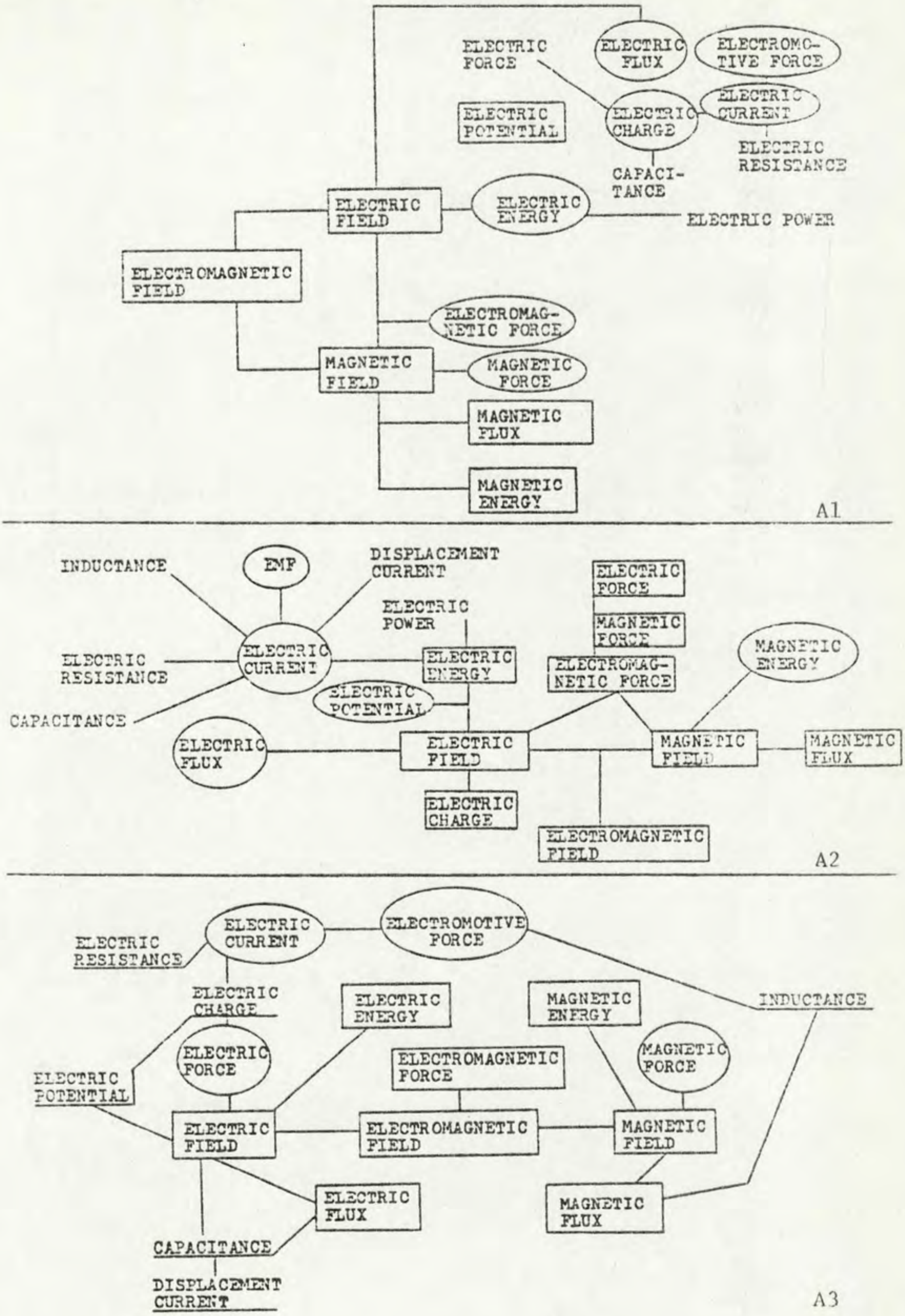


Fig. IV-7. Concept Maps, Student # 3, Group C2.

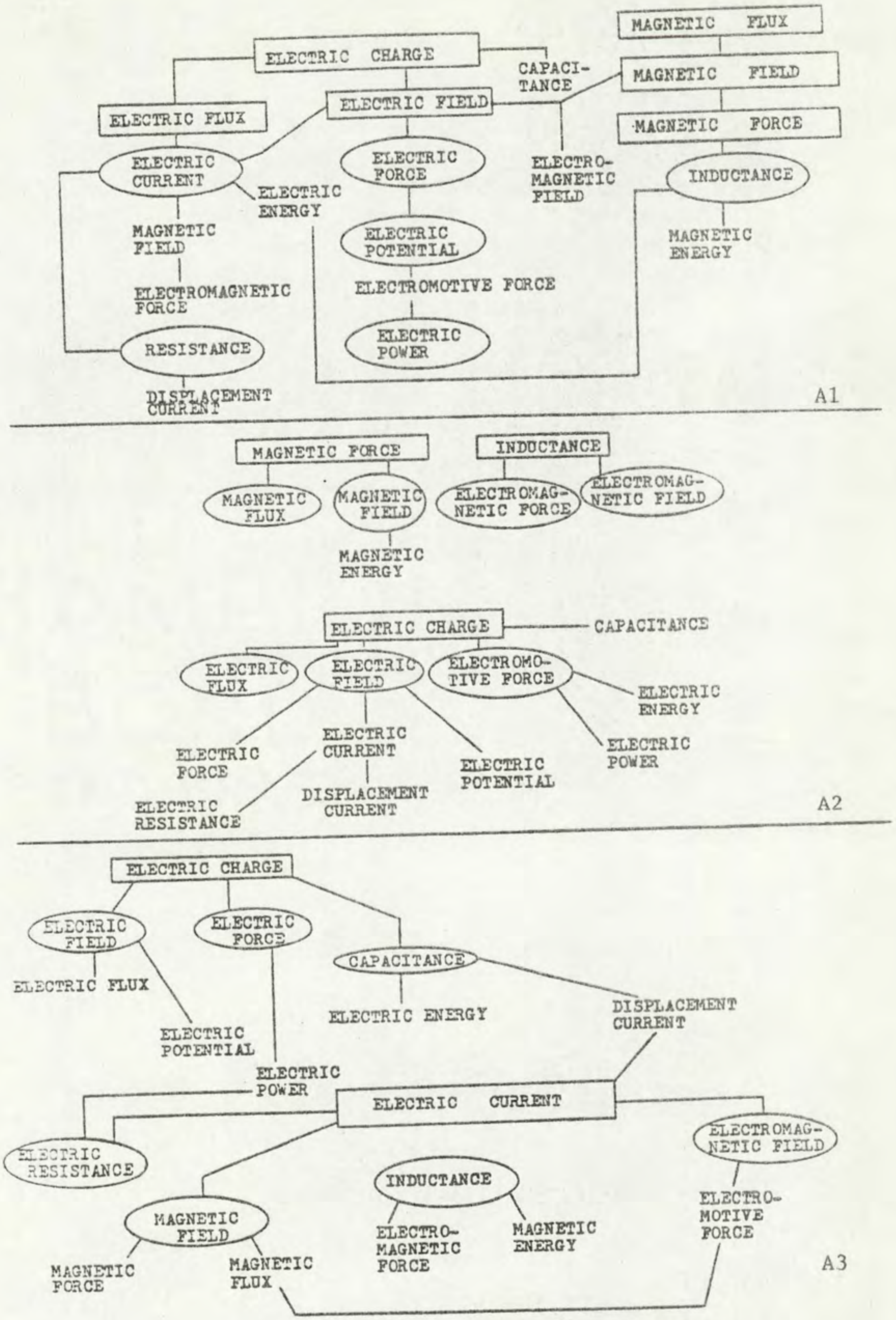


Fig. IV-8. Concept Maps, Student # 10, Group C2.

This trend is confirmed in tests A3 (end of course): in this case most students drew a map showing a clear vertical hierarchy where the most general and inclusive concepts are at the apex and the intermediate and the least inclusive concepts occupy the corresponding hierarchical positions. However, the fact that this effect was highly accentuated in the experimental groups might be not surprising because concept maps were frequently used in the instructional materials and, in most cases, the same type of vertical hierarchy was used (e.g., concept maps used in "Notes II", Appendix V). But we must realize that the concept maps with vertical hierarchy used in the "Notes" were usually concerned with one key concept, e.g., a "map for forces," a "map for fields," a "map for potential," and so on, whereas the map that students should draw included several key concepts. As a matter of fact, the list of concepts given to the students included almost all relevant concepts of electromagnetism and they were drawing nothing else than a general concept map of electromagnetism. But, as general maps, their maps were very different from the general map of electromagnetism used in "Notes III' and "Notes XVII" (Appendix VII) which did not have a vertical hierarchy and included equations and laws as well.

Let's accept, however, the argument that the differences in the final maps are due to the fact that concept maps were used in the written materials of the experimental groups (in the lecture groups the teacher did not discuss the maps in the lectures, i.e., they were used only in the written materials) and not used in the corresponding materials of the control groups. Isn't this an evidence that the instructional materials were effective?

Well, we think that this question deserves some discussion, and we will open a parenthesis to provide an additional theoretical framework that might help us to answer such a question:

We see the act of teaching as a deliberate intervention by someone (the teacher) in the life of someone else (the learner) with the purpose of providing a learning opportunity. That is, the teacher deliberately intervenes in the student's life because he wants him to learn something, but he can only organize his teaching in such a way that the probability of occurrence of learning is maximized. There is not a necessary causal relationship between teaching and learning. Learning is an idiosyncratic act, a responsibility of the learner, or, as Gowin (1977) says, "learning is a responsibility that cannot be shared." Thus, in a formal course, for example, the teacher must concentrate his effort in finding the means of organizing and transmitting the subject matter that will maximize the probability of learning occurrence.

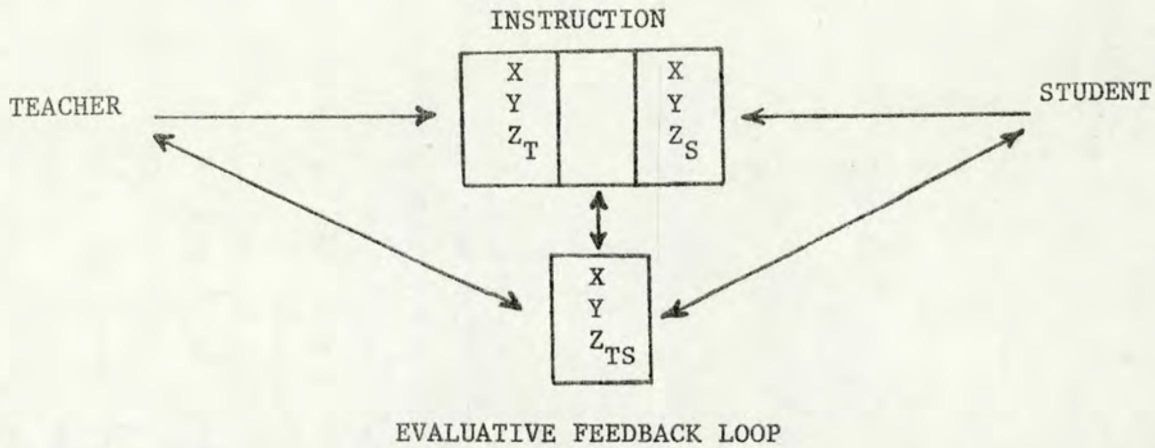
In such a case, we think that the most basic point of concern is to consider how students learn. Once this aspect has been taken into account one can concentrate on instructional strategies. To focus on instructional methods without a theoretical perspective on how students learn is, at best, a process of trial and error.

In addition, we think that a research project that supposedly has to do with learning cannot lack a theoretical basis in terms of a learning theory. But since a research project dealing with learning almost inevitably involves teaching too, wouldn't it make sense to have also a theoretical framework concerned with teaching?

The present study was conducted under the theoretical framework of Ausubel's learning theory, but in terms of teaching we have only described the two teaching methods which are not exactly what one could call a teaching theoretical framework. However, a useful framework that we will use for further interpretation of our findings, is provided by Gowin's theory of education (1977):

In his theory he sees teaching as a triadic, episodic exchange of meanings between teacher and student. That is, the teacher selects XYZ materials from the subject matter and presents them to the student in a form coherent with the meanings that these materials have to him (the teacher). The student, in turn, presents to the teacher the meanings that he acquired from the materials. If there is no agreement on meanings, the teacher presents the materials again to the student, perhaps in a different way, i.e., taking into account the student's meanings, who once more gives his meanings to the teacher. Successful teaching occurs when there is agreement on meanings, that is, when the achievement of shared meanings has occurred between teacher and student. Each exchange of meanings between teacher and student brings them closer to the achievement of shared meanings. (We like to establish an analogy between this model and an exchange force, i.e., the exchange of meanings is like an exchange force that approximates student and teacher until the achievement of shared meanings.) This exchange of meanings, as schematized by Gowin, is in Figure IV-9.

The achievement of shared meanings, however, does not mean that learning occurred. It might be seen as a necessary but not sufficient condition. In the same way that to grasp the meaning of a game does not



KEY: XYZ_T = Teacher's meanings of materials XYZ
XYZ_S = Student's meanings of materials XYZ
XYZ_{TS} = Teacher's and student's shared meanings of materials XYZ

Fig. IV-9. The Triadic Relationship Involved in the Act of Teaching.

necessarily implies in learning how to play the game, to grasp the meaning of a concept does not mean to learn the concept. Learning is an idiosyncratic activity which depends on practice, willingness to learn, and relatability to cognitive structure (to use Ausubel's language).

This theoretical perspective concerned with teaching is coherent with Ausubel's theory which is concerned with learning because the achievement of shared meanings is more likely to occur if the XYZ materials selected by the teacher are potentially meaningful to the student.

Closing the parenthesis and coming back to the concept map tests, we could say that in the process of giving the instructional materials to the students, with our meanings, and receiving their meanings through the unit-tests, quizzes, and association tests, an exchange of meanings has occurred during the course. But the achievement of shared meanings has occurred to a greater extent in the experimental groups, and, in such a sense, the instructional materials used in these groups were more effective. For example, the hierarchical disposition of concepts found in most of the maps drawn by students in the experimental groups, at the end of the course is highly supportive of Ausubel's theory. But one could argue that we used the same disposition in most of our maps in the "Notes." That's true, but we were presenting to the students our meanings in such a disposition only in "submaps" of electromagnetism because our general map was different. The students, on the other hand, in the case of the maps, were presenting their meanings in a general map which at the end of the course ended up as more coherent with Ausubel's theory than ours, and if we were to draw a general map including only the concepts given to the students it would be very similar to most of their final maps. That is, the achievement of shared meanings occurred in this case.

However, when we first mentioned concept maps, in Section III-4.3, we said that different map makers could draw different maps, equally meaningful, for the same set of concepts. The point is, can we say that the maps drawn by students in the experimental groups are better just because we agree with the conceptual hierarchy they display? Isn't this just a matter of preference? Isn't it possible that these students just learned our technique of map making?

To answer these questions let's consider not the shape of the map, but the degree of concept differentiation according to the rules to be followed when drawing them:

In the experimental groups, concepts like "electromagnetic field", electromagnetic force", "magnetic field", "magnetic force", "electric force", and "electric field" were usually identified as the most general (inside rectangles), whereas concepts like "inductance", "capacitance", "electric power", and "displacement current" were identified as least inclusive (just written) and the remaining ones as intermediate (ellipses). In the control groups, there was not such a rather uniform trend. For example, in Figure IV-3 "capacitance" and "inductance" in map A3 were considered as general as "electromagnetic force" and "electromagnetic field". In Figure IV-8, map A3, "electric current" was considered the most general concept and occupies a central position in the map.

We think that the maps speak for themselves and indicate a greater degree of concept differentiation in the experimental groups, but, in order to provide a quantitative measure concerning these maps, we established some criteria and scored each map. However, the following analysis must be seen just as an attempt to quantify qualitative results. The maps are qualitative measures and, as such, must be primarily analyzed qualitatively as we have tried to do so far.

The following criteria were used to analyze the maps quantitatively:

I. Identification of concepts like electromagnetic force and field, magnetic force and field, and electric force and field as more general concepts, according to the given rules.

II. Identification of concepts like electric current, electric and magnetic flux, electric charge, electric potential, electric and magnetic energy, and electromotive force (fem), as second level (intermediate) of generality concepts.

III. Identification of concepts like inductance, capacitance, resistance, electric power, and displacement current as least general (3rd level) concepts.

IV. Overall quality of the map taking into account factors like (a) distinction between levels of generality;
(b) meaningful links between concepts; and
(c) neat and meaningful disposition of concepts in a diagram.

For each of these criteria, all maps were scored on a zero to 3 scale where '1' would stand for "poor", '2' for "regular" and '3' for "excellent". The results are in Tables IV-30, IV-31, IV-32 and IV-33.

TABLE IV-30

CONCEPT MAPPING TEST

Criterion I: Identification of More General Concepts

	A 1			A2			A 3		
Group	N	M	t	N	M	t	N	M	t
E1	37	1.11		35	2.11		35	2.46	
C1	35	1.23	-1.23*	34	1.53	4.41**	33	1.48	7.15**
E2	36	1.17		30	1.77		32	2.16	
C2	26	1.12	.54*	26	1.23	3.95**	23	1.43	4.62**

*p > .05; **p < .05, two-tailed.

TABLE IV-31

CONCEPT MAPPING TEST
Criterion II: Identification of Concepts of Intermediate Level of Generality

	A 1			A 2			A 3		
Group	N	M	t	N	M	t	N	M	t
E1	37	1.03		35	1.60		35	1.89	
C1	35	1.06	-.63*	34	1.18	3.67**	33	1.30	4.18**
E2	36	1.00		30	1.40		32	1.53	
C2	26	1.04	-1.02*	26	1.08	2.78**	23	1.26	1.68*

*$p > .05$; **$p < .05$, two-tailed.

TABLE IV-32

CONCEPT MAPPING TEST
Criterion III: Identification of Least General Concepts

	A 1			A 2			A 3		
Group	N	M	t	N	M	t	N	M	t
E1	37	1.11		35	1.83		35	1.97	
C1	35	1.20	-.95*	34	1.44	2.53**	33	1.52	2.81**
E2	36	1.08		30	1.57		32	1.56	
C2	26	1.12	-.51*	26	1.31	1.42*	23	1.43	.71*

*$p > .05$; **$p < .05$, two-tailed.

TABLE IV-33

CONCEPT MAPPING TEST
Criterion IV: Overall Quality of the Map

	A 1			A 2			A 3		
Group	N	M	t	N	M	t	N	M	t
E1	37	1.54		35	2.09		35	2.43	
C1	35	1.51	.25*	34	1.71	3.25**	33	1.61	5.76**
E2	36	1.36		30	1.90		32	2.19	
C2	26	1.38	-.16*	26	1.46	3.09**	23	1.57	4.04**

*$p > .05$; **$p < .05$, two-tailed.

Considering that the results presented in Tables IV-30 to -33 are consistent with our previous qualitative analysis, we think that the criteria we have used for the quantitative analysis have at least some validity, and these results can be used as additional evidence that there were significant differences in the maps. From these Tables we can see that, at the beginning of the course, the degree of concept differentiation was poor in all groups and no significant difference was found in any case. At opportunity A2 (middle of course) all groups showed an improvement in comparison to A1, but this effect was more accentuated in the experimental groups and in all but one case the difference was statistically significant. At the end of the course (A3) a further improvement was observed in almost all cases, and again the effect was more accentuated in the experimental groups with all but two differences being statistically significant.

Distinguishing between PSI and lecture groups, we can see that in the PSI case the means of the experimental group, in tests A2 and A3 were always higher and the difference statistically significant. In the lecture case, the pattern was the same except that the difference was not significant for criterion II in test A3, and for criterion III in tests A2 and A3.

Thus, the results of the concept map test provide strong evidence that the Ausubelian approach fostered concept differentiation to a greater extent than the traditional approach, and support the evidences found in the results of the word association and numerical association tests concerning this aspect, as well as those concerning significant associations and concept relatedness.

The last section of this chapter is concerned with the results of the laboratory test, which are the only remaining results to be reported.

IV-5 The Laboratory Test

As we mentioned at the beginning of this chapter, in the laboratory part of this study, we tried to answer a "telling question" somewhat different (but highly related) from the general "telling question" of the study. This question could be formulated as follows:

> In laboratory experiments about electromagnetic phenomena, what differences would arise from the use of laboratory-guides coherent with Ausubel's theory, as compared with linearly programmed laboratory-guides, in terms of students ability to:
>
> 1. Identify the basic question(s) under study;
> 2. Identify the key electromagnetic concepts involved in the experiment;
> 3. Identify the basic electromagnetic phenomena involved in the experiment;
> 4. Describe the method used to answer the basic question(s);
> 5. Report the results.

To search for answers to this question a five-item test was designed based on Gowin's "five questions" and administered after each experiment in the PSI groups. This test as well as the lab-guides used in both groups are described in Section III-6.

A zero to 5 five points scale was used to score the laboratory test corresponding to a maximum of '1' point per item. The results corresponding to each item in each test, as well as the totals, are reported in Tables IV-34 to IV-37.

Except in question 2 of the first three experiments, the differences between the variances reported in Tables IV-34 to -37 were not statistically significant. However, in the case of the means a higher number of significant differences occurred, mainly in the first two experiments. Table IV-38 contains only the means to provide a general view of the results of the laboratory test. In this table, asterisks indicate significant differences at .05 level.

From Table IV-38 we can see that, in general, the experimental group had better means, but the significant differences disappeared (with a single exception) in the last two experiments. In addition we can see that the difference favoring the experimental group was particularly accentuated in question 2, i.e., in the identification of the key concepts involved in the experiment. This finding provides further support for the evidences gathered from the association tests favoring the Ausubelian approach.

In the identification of the basic questions, the experimental group did better in the first two experiments, but, following the overall trend,

TABLE IV-34

LABORATORY TEST
First Experiment: Simulated Electrostatic Field

Group	N	M	SD	F	t
		1st Question: Basic Question(s)			
E1	35	.77	.25		
C1	36	.56	.33	1.83*	3.02**
		2nd Question: Key Concepts			
E1	35	.96	.19		
C1	36	.68	.43	6.33**	3.57**
		3rd Question: Basic Phenomena			
E1	35	.26	.37		
C1	36	.21	.37	1.00*	.57*
		4th Question: Method			
E1	35	.54	.22		
C1	36	.50	.21	1.25*	.78*
		5th Question: Results			
E1	35	.51	.28		
C1	36	.47	.36	1.63*	.52*
		TOTAL			
E1	35	3.04	.80		
C1	36	2.42	.89	1.23*	3.08**

*$p > .05$; **$p < .05$, two-tailed.

TABLE IV-35

LABORATORY TEST
Second Experiment: Linear and Nonlinear Resistors

Group	N	M	SD	F	t
		1st Question: Basic Question(s)			
E1	36	.90	.23		
C1	34	.68	.32	2.00*	3.32**
		2nd Question: Key Concepts			
E1	36	.94	.16		
C1	34	.79	.35	4.00**	2.28**
		3rd Question: Basic Phenomena			
E1	36	.56	.26		
C1	34	.25	.28	1.14*	4.80**
		4th Question: Method			
E1	36	.56	.26		
C1	34	.47	.21	1.40*	1.59*
		5th Question: Results			
E1	36	.64	.28		
C1	34	.47	.30	1.13*	2.45**
		TOTAL			
E1	36	3.60	.71		
C1	34	2.66	.74	1.08*	5.42**

*$p > .05$; **$p < .05$, two-tailed.

TABLE IV-36

LABORATORY TEST
Third Experiment: RC Circuit

Group	N	M	SD	F	t
		1st Question: Basic Question(s)			
E1	31	.73	.34		
C1	35	.73	.33	1.09*	.00*
		2nd Question: Key Concepts			
E1	31	.92	.26		
C1	35	.80	.39	2.14**	1.49*
		3rd Question: Basic Phenomena			
E1	31	.32	.28		
C1	35	.41	.39	1.88*	-1.06*
		4th Question: Method			
E1	31	.45	.20		
C1	35	.46	.19	1.33*	- .21*
		5th Question: Results			
E1	31	.71	.38		
C1	35	.60	.38	1.00*	1.17*
		TOTAL			
E1	31	3.13	.92		
C1	35	3.00	.89	1.08*	.58*

*$p > .05$; **$p < .05$, two-tailed.

TABLE IV-37

LABORATORY TEST
Fourth Experiment: Electromagnetic Induction

Group	N	M	SD	F	t
		1st Question: Basic Question(s)			
E1	38	.64	.23		
C1	34	.63	.26	1.40*	.17*
		2nd Question: Key Concepts			
E1	38	.79	.36		
C1	34	.72	.37	1.08*	.81*
		3rd Question: Basic Phenomena			
E1	38	.75	.40		
C1	34	.56	.44	1.19*	1.92*
		4th Question: Method			
E1	38	.58	.25		
C1	34	.53	.24	1.00*	.86*
		5th Question: Results			
E1	38	.63	.22		
C1	34	.57	.24	1.20*	1.11*
		TOTAL			
E1	38	3.39	.89		
C1	34	3.01	.83	1.16*	1.87*

*$p > .05$, two-tailed.

TABLE IV-38

MEAN SCORES IN THE LABORATORY TEST

Question	Group	1st Exp.	2nd Exp.	3rd Exp.	4th Exp.
1	E1	.77**	.90**	.73	.64
	C1	.56	.68	.73	.63
2	E1	.96**	.94**	.92	.79
	C1	.68	.79	.80	.72
3	E1	.26	.56**	.32	.75
	C1	.21	.25	.41	.56
4	E1	.54	.56	.45	.58
	C1	.50	.47	.46	.53
5	E1	.51	.64**	.71	.63
	C1	.47	.47	.60	.57
TOTAL	E1	3.04**	3.60**	3.13	3.39
	C1	2.42	2.66	3.00	3.01

** $p < .05$, two-tailed.

there was no difference in the remaining two experiments. This initial difference is probably due to the emphasis given, in the experimental lab-guides, on the importance of identifying the basic question of an investigation. (The lack of difference in the final experiments remains to be explained.)

However, particularly surprising to us were the rather poor mean scores of both groups in questions 3, 4, and 5, namely, the identification of basic phenomena, the method and the results. It is true that in the first case both groups, specially the experimental one, showed an improvement in the last experiment, but the mean scores of the first three experiments were quite low. On the other hand, in the description of the method the scores were systematically around 50% of the maximum. Students in both groups rarely described the method as a systematic procedure, as a sequence of steps, or, in other words, they rarely described what they had done experimentally, what techniques they used, etc. Most students simply gave answers like "experimental method" or "scientific method".

The report of the results was also poorly done. It seems that most students see the results of an experiment just like the results of a simple exercise and not as pieces of knowledge with theoretical implications. (By the way, if the lab-units were evaluated on the basis of this test, most students would have been required to repeat those units. But, instead of a written test, the same evaluation procedure of previous semesters was used: an oral discussion of the results, usually focusing on the interpretation of the graphs drawn by the students.)

The fact that the initially significant differences in favor of the experimental group disappeared in the last two experiments might be due to the combined effect of several factors, such as:

1. In contrast to the association tests, students were not told that the laboratory test would be used only for research purposes. Consequently, at the beginning they thought that the tests were an integrant part of unit evaluation, but, later on, they realized that the tests were not being used for such a purpose. Consequently, the last tests were answered without the same motivation.

2. The third experiment ended up as not very appropriate to the course because circuits were not emphasized in both groups and the RC circuit is a quite particular type of circuit.

3. The last experiment was one of the last units and most students performed it at the end of the semester when the external pressure from other courses is at a maximum.

Summing up, in spite of possible design weaknesses, the "laboratory experiment," on one hand, provided additional evidence supporting the Ausubelian approach in terms of identification of key concepts. However, on the other hand, it apparently indicated that many students, in both groups, perform an experiment without knowing what basic phenomena underlie the experiment, and don't see experimentation as a process of making knowledge. In addition, it also provided evidence that most students use the terms "scientific method" and "experimental method" rather loosely equating them with the mere use of lab equipment.

Chapter V

SUMMARY, DISCUSSION, AND POSSIBLE IMPLICATIONS

Following previous experience and research with different teaching methods which indicated the need of a theoretical framework in order to achieve more meaningful results, an experiment based on Ausubel's theory of learning was conducted in an introductory college physics course.

V-1 The Experiment

A one semester introductory course in electromagnetism was taught to four groups of students of science and engineering. Two pairs of experimental-control groups were formed, one taught under a self-paced format (PSI) and the other under the traditional lecture approach. The content of the course was the same, but for the experimental groups, it was organized according to an Ausubelian framework, whereas the control groups followed the same traditional organization used in previous semesters which is that found in most textbooks on the subject.

The purpose of the experiment was to look for differences that could arise from the use of these different approaches in terms of student's ability to apply, to associate, to differentiate and to hierarchically organize concepts of electromagnetism. To achieve such a purpose, besides traditional achievement tests, association tests were administered at the beginning, at the middle and at the end of the course. In addition, a laboratory test was administered after each laboratory experiment.

V-2 The Evidence

The statistical analysis of the results followed a rather simple pattern trying to make sense out of the large amount of data provided by the various tests used in the experiment. More sophisticated statistical procedures were judged unnecessary for the purpose of the experiment and left to future follow-up studies. The evidence gathered from the various tests can be summarized as follows:

1. Achievement measures: in terms of traditional achievement measures, no significant differences were found. Thus, to the extent that traditional achievement tests like unit-tests, quizzes and final exams are measuring concept learning, ability in problem solving, etc., the evidence was that both approaches were equally effective.

2. Word association test: the average number of words associated per concept increased in all groups, but particularly in the case of "significant associations" there was a slight evidence that this effect was more accentuated in the experimental groups. In terms of "overlapping of associated words," the averages also increased for all groups, but there was evidence which could be interpreted as indicating that the degree of concept differentiation was greater and more uniform in the experimental groups.

3. Numerical association test: in this case, the numeral averages decreased in all groups indicating a closer association between concepts. However, in the case of the self-paced groups such a decrease was significantly more accentuated in the experimental group. That is, the evidence was that, at the end of the course, the experimental group saw a greater degree of association between related concepts.

4. Graphical association test ("concept mapping" test): the results of this test provided strong support for the previous evidence, concerning concept association and differentiation. The maps drawn by students in the experimental groups were qualitatively different from those of the control group students, and indicated better concept differentiation, more meaningful association and a hierarchical disposition coherent with Ausubel's theory. In addition, a tentative quantitative analysis of these maps, in terms of differentiation between more general, intermediate and least general concepts, favored the experimental groups significantly in almost all cases, specially in the self-paced groups.

5. Laboratory test: there was evidence that students in the experimental group were more able to identify the key concepts involved in the experiments, but, on the other hand, there was evidence that students, in both groups, were not able to identify the basic phenomena underlying the experiments, to describe the method used, and to report the results as pieces of knowledge obtained through experimentation.

V-3 The Knowledge Claims

Based not on any particular evidence, but rather on the whole set of consistent evidence gathered throughout the study, the answers we found for our basic "telling question" were:

- There were no differential effects, due to the traditional or the Ausubelian approaches, on students' performance when measured by traditional achievement evaluation instruments.

- There was a differential effect due to the Ausubelian approach in terms of concept association, concept differentiation and hierarchical organization of concepts. Students under the Ausubelian approach showed a higher degree of differentiation and relatedness among electromagnetic concepts. In addition, their conceptual hierarchies of electromagnetism, as inferred from their concept maps, were more meaningful both under the point of view of Ausubel's theory and from the standpoint of physics as well.

  However, there were indications that this effect was more accentuated in the self-paced experimental group than in the lecture one.

As far as the "laboratory telling question" is concerned, the answers, although supported only by the results of a very simple test, were:

- In general, students in the experimental groups were more able to identify the key electromagnetic concepts involved in the laboratory experiments.

- In the first two experiments, students in the experimental groups were more able to identify the basic question under investigation, but there was no significant difference in the last two experiments.

- In general, there were no significant differences in terms of identification of basic phenomena involved in the experiments, method description, and results report. In addition, except in the last experiment, both groups performed rather poorly in these aspects.

V-4 An Interpretation in the Light of Ausubel's Theory

A higher degree of concept differentiation, on one hand, and a higher degree of concept relatedness, on the other hand, are indications that both progressive differentiation and integrative reconciliation were achieved to a greater extent in the experimental groups. In addition, the conceptual hierarchies displayed in the concept maps drawn by students in the experimental groups were more coherent with Ausubel's assumption that "an individual's organization of the content of a particular subject-matter discipline in his own mind consists of a hierarchical structure, in which the most inclusive ideas occupy a position at the apex of the structure, and subsume progressively less inclusive and more highly differentiated propositions, concepts, and factual data," (1968) than those of the control group students.

Considering that these results can be interpreted by and are supportive of Ausubel's theory, we are inclined to accept them as evidence that meaningful learning occurred to a greater extent in the experimental groups. But, the lack of significant differences in terms of achievement at least do not support this assumption. However, under an Ausubelian point of view, the type of achievement tests used for course evaluation would not provide evidence of meaningful learning, and the lack of significant difference would be a plausible expectation.

According to Ausubel (1968, p. 111):

> In seeking evidence of meaningful learning, whether through verbal questioning or problem-solving tasks, the possibility of rote memorization should always be borne in mind. Long experience in taking examinations makes students adept at memorizing not only key propositions and formulas, but also causes, examples, reasons, explanations, and ways of memorizing and solving "type problems." The danger of rote simulation of meaningful comprehension may be best avoided by asking questions and posing problems that are both novel and unfamiliar in form and require maximal transformation of existing knowledge.

Thus, given that the achievement tests used for course evaluation usually did not ask the type of questions and problems proposed by Ausubel in seeking evidence of meaningful learning, such an evidence could hardly be expected from these tests.

In spite of the fact that no attempt was made to compare the self-paced and lecture approaches, since the corresponding pairs of samples were regarded as different, it was observed that the effect of the Ausubelian approach was usually more accentuated for the self-paced experimental group than in the lecture experimental group in comparison with the corresponding control groups. This observation can also be tentatively interpreted under an Ausubelian framework (1968, p. 262):

> On theoretical grounds it seems rather self-evident that individualized instruction should be incomparably more efficient than instruction in groups for most aspects of subject matter learning. When instruction is geared to the individual pupil's general level of sophistication in a particular discipline, to his mastery of relevant antecedent concepts and principles, to his particular preconceptions and misconceptions, to his general and specific intellectual aptitudes, to the level of abstraction at which he operates, to his idiosyncratic cognitive style and relevant personality attributes, to salient aspects of his progress in mastering a current learning task (for example, consolidation, precision and clarity of new meanings), and to a pace of presentation that is comfortable for him, it necessarily follows that

> learning outcomes should be superior to those that eventuate when instruction is geared to a hypothetical set of characteristics and requirements reflective of the mean pupil in a group.

Among the factors, pointed by Ausubel, favoring individualized over group instruction, the emphasis on mastery of antecedent content and a pace of presentation comfortable to the student were certainly present in the individualized groups and not in the lecture ones. Thus, under an Ausubelian point of view these factors would contribute for superior learning outcomes in the individualized groups and could also provide an explanation for the fact that the Ausubelian approach might have been more effective in this case.

V-5 An Interpretation in the Light of Gowin's Model of Teaching

Gowin sees teaching as a triadic, episodic exchange of meanings of pieces of knowledge between teacher and student, and the occurrence of successful teaching as the achievement of shared meanings between them.

Thus, the coherence between the final conceptual grasp of the students and teacher's expectation in the experimental groups might be interpreted in terms of a greater extent of achievement of shared meanings in these groups. That is, taking into account the fact that the traditional approach was considered a good one and that the same conceptual understanding would be expected at the end of the course, the exchange of meanings toward the achievement of shared meanings was more effective in the Ausubelian approach. This interpretation is coherent with the previous one because the Ausubelian approach intentionally tried to present the subject matter in a way that would be potentially meaningful to the student.

Similarly, the fact that the Ausubelian approach might have been more effective in the case of individualized instruction can be interpreted as a result of a much more intense exchange of meanings in this case due to the greater amount of tests taken by students and specially to the large personal interaction between students and teacher (and proctors).

V-6 Possible Implications for Physics Instruction and Research in Physics Education

It is a well known fact that, both at secondary and university levels, physics is considered a difficult subject and is usually avoided by the students, when not required. At the secondary level most students prefer biology or chemistry and leave physics for those who are going to be future physicists or engineers. At the college level, introductory physics courses designed to provide a physics background for students of technical and scientific (other than physics) careers are usually a teaching problem avoided by many physics professors. In Brazil, for example, even students of mathematics and some branches of engineering have an aversion to physics.

On the other hand, physics is a discipline with an undeniable scientific tradition, a well defined conceptual structure, a solid theoretical basis and other attributes that probably could be used to make it potentially meaningful to most students.

Perhaps physics is indeed difficult to learn, but it may be also the case that something is wrong with the teaching of physics. Fortunately, however, many physicists and physics teachers are deeply concerned with this problem and have already started to search for solutions and to do research in physics education. But, paradoxically, strong efforts were made attempting to find new methods, to make physics attractive or to make physics simple, and almost no attempt has been made to teach physics according to a learning model or a learning theory. It seems that physics educators are willing to do research in physics education in order to improve the learning and teaching of physics, but they are reluctant to use learning theories as research guides as they do with physical theories in their research laboratories. Perhaps they do that because, comparing the outcomes of research in physics with the outcomes of the educational research, the latter seem to be trivial, confusing or contradictory. However, perhaps what they do not realize is that the lack of a theoretical basis is probably what is wrong with the research in education and that by doing research in physics education without a theoretical framework they are doing the same type of educational research which they usually ignore.

The situation, however, seems to be changing. For example, physics educators had recently "discovered" Piaget and are concerned with the possibility that mistakes are being made in physics instruction if physics is being taught at levels inconsistent with Piaget's developmental stages. But, without denying the importance of Piaget's theory for physics instruction, we would like to emphasize that it is not the only theory potentially relevant for such a purpose. We believe that Ausubel's theory, for example, which served as the theoretical basis of this study, is also potentially relevant for physics instruction and for research in physics education.

We have argued that the results of this study can be interpreted in terms of Ausubel's theory and are supportive of this theory. However, this study should not be seen as an attempt to check the validity of Ausubel's theory when applied to the teaching and learning of physics. Our study is just a first attempt to use this theory as a theoretical framework for research in physics education. As such, it has limitations and may have low external validity. Thus, instead of claiming that our findings do have implications for physics instruction and for research in physics education, a much more defensible position would be to say that we have shown that it is possible to organize the teaching of physics in accordance with Ausubel's learning theory, keeping at least the same level of achievement in comparison to traditional approaches, with the addition of possible advantages in terms of concept learning.

Thus, we recognize that the empirical evidence that we gathered in this study, although supportive of Ausubel's theory, is not strong enough to infer definite implications for the teaching of physics and for research in such a field. For example, this study did not evaluate meaningful learning in an Ausubelian sense which involves long-range retention and transferability of learning studies.

In addition, most studies on Ausubel's theory are concerned with the effect of advance organizers, and we think that, for physics instruction, progressive differentiation and integrative reconciliation,[1] for example, are more relevant aspects of this theory because they deal more directly with concept learning. The point is that, at least in physics instruction there is a shortage of studies bearing on Ausubel's theory. However, if his theory is valid most difficulties found in the teaching of physics might

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1. These learning principles will be more explicitly described in the second edition of Ausubel's book to be published in 1978 (personal communication).

be due to the lack of emphasis on progressive differentiation, on integrative reconciliation, on relatability to cognitive structure and so on. Thus, instead of looking for methods and trying to make physics attractive or simple, we should, primarily, make physics potentially meaningful to the students in an Ausubelian sense.

Certainly, more research is needed in this field, but we think that the theory-based experiment is the missing link in physics education research, and Ausubel's theory can provide a theoretical framework for such a research. For example, many other aspects of Ausubel's theory, such as the emphasis on relatability to cognitive structure, meaningful versus rote learning, assimilation and obliterative assimilation, forgetting, and retention might be objects of research projects in the field of physics education with potentially relevant results for physics instruction.

V-7 The Non-traditional Tests

In addition to its attention to Ausubel's theory, another aspect of this study that might also have implications for physics instruction and for research on physics education is the use of non-traditional tests to evaluate qualitative aspects of concept learning and laboratory experiments.

Our findings seem to indicate, on one hand, that traditional achievement tests such as unit-tests, quizzes and final exams are not adequate to detect evidence of more qualitative differences among students in terms of concept learning, and, on the other hand, that non-traditional tests such as concept association tests might be appropriate for such a purpose.

Concept learning is certainly a main objective of physics instruction, but relevant qualitative aspects of this learning probably are not being evaluated when conventional questions and type-problems are posed to the students. The concept association tests, on the other hand, as far as we can infer from this study, provide at least some evidence of qualitative differences in concept learning. Particularly the numerical concept association and the concept mapping tests ended up as valuable instruments to detect qualitative differences in the learning outcomes. (As a matter of fact, the concept maps were also useful for instructional purposes.)

It is true that these instruments might not be the best ones and questions of reliability and validity might be raised. However, these tests are easily designed, are easily administered, and the students have no difficulties in understanding them. If a large number of concepts is used, the statistical analysis may be time consuming when a computer is not available. This may be a problem in some cases, but we think that, in most cases, the use of non-traditional tests of this type, in addition to the traditional ones, can result in instructional advantages without overloading instructors and students. The point is that, by using non-traditional evaluation instruments and paying attention also to qualitative aspects of students' learning, significant differences might be detected and provide the feedback needed to change or improve the instructional approaches.

As far as the laboratory is concerned, our findings, although fragile and preliminary, might indicate that laboratory evaluation should not only focus on the tables, graphs and numerical values obtained by the students in their experiments. These quantitative results will have little value, in our opinion, if the students are not able to identify the basic questions under investigation, the key concepts and basic phenomena involved in the experiments; if they are not able to describe the methods they have used and if they do not see a laboratory experiment as a process of making knowledge. If this happens, then, the strong emphasis placed on laboratory experiments in many physics courses is at best unjustified.

V-8 A General Map of the Study

As we did in most of the special "Notes" prepared for the experimental groups, we will conclude the present description with a general map. However, this map, which is shown in Figure V-1, is not exactly a concept map such as those used in the "Notes," but rather a "concept-event-fact map" designed to schematize the pattern of inquiry followed in this study. The following quote (Gowin, 1977) will help us to begin to "unpack" this map:

> The pattern of inquiry can be looked at as a meaning structure. The elements of this structure are events, facts, concepts. What inquiry makes by its actions are the specific connections between a given event, the records made of that event, the factual judgments derived from studying the records, the concepts that focus the regularities in the events and the concepts and conceptual systems that are used to interpret the factual judgments in order to arrive at an explanation of the event. To have created this meaning structure in a particular inquiry is to have done a coherent piece of research.

Gowin (1970) defines concept as "a sign which points to a commonality in events and which permits the concept user to make relatively stable responses to those varied events." For example, the concept of liquid points to common qualities of milk, oil, water, while ignoring the many differences

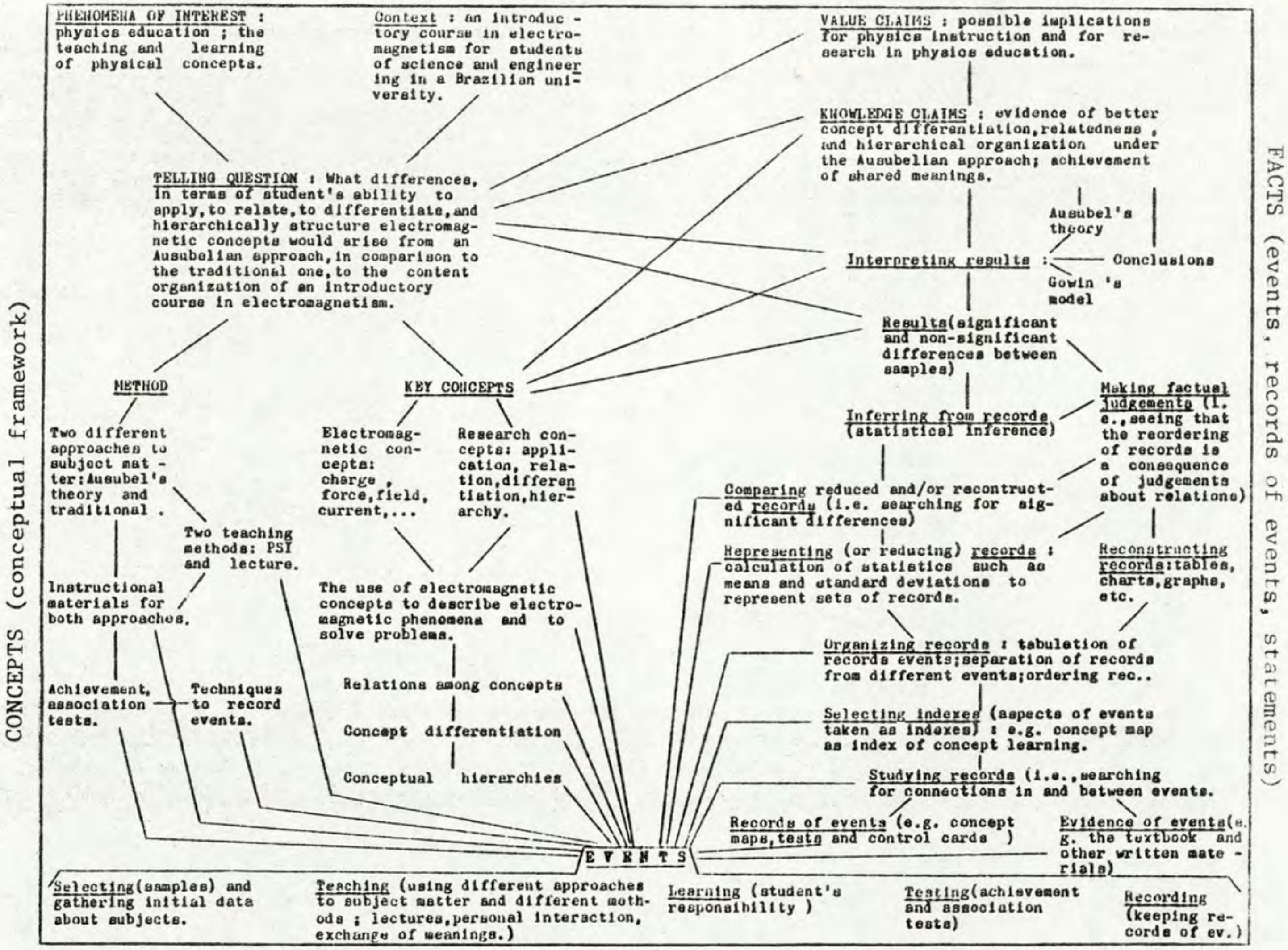

Fig. V-1. A General Map of the Study.

between them. In addition, he defines (1969) conceptual system as "a set of logically related concepts, usually permitting a pattern of reasoning in relating one concept to another." Facts may be given three distinct but related meanings (Gowin, 1969 and 1970): fact in the first sense means an event which just naturally occurs or is made to occur by the researcher; fact in the second sense refers to the record of the event (an event cannot be studied if no record is left); facts in the third sense consist of statements, typically in verbal or mathematical form, which are based on records of events occurring in the phenomena of interest.

Thus, the process of inquiry has to do with the connection between events, facts and concepts. It is in that sense that the map of Figure V-1 is a "concept-event-fact map." The left side of this map is concerned with concepts and conceptual systems: there we find the telling question of the study, i.e., the basic question about the phenomena of interest that was experimentally investigated; we also find the key concepts embedded in the telling question and the method used in the search for answers to such a question. The method, in a broad sense, involves the conceptual framework of Ausubel's theory and the conceptual systems implicit in teaching and evaluation. Thus, this side of the map deals, explicitly or not, with concepts and, in a sense, it represents the whole conceptual framework that led to the events that we made happen in this study.

The events are at the basis of the map and also of the research as a whole: had events such as teaching and testing not happened, it would be impossible to gather empirical evidence that would support the answers to the telling question. But had the events happened without leaving records they could not have been studied. So, records of events are an essential feature of inquiry. As a matter of fact, "much time of actual inquiries is spent in inventing techniques and devices for making a record which will serve as an index to the phenomena of interest," (Gowin, 1970).

The right side of the map is concerned with facts in all three senses (events, records of events, and the statements): once records of events are made they can be studied, organized, reduced, reconstructed, rearranged, etc. This action of "processing the records" eventually leads to what is labeled as "results" which, in turn, lead to the knowledge claims which are answers to the telling question. (Sometimes the records are so extensively "processed" and the results are so far from the events that the connection between factual statements and events is lost.) The results and their interpretations, the conclusions, the knowledge claims and the value claims are products of inquiry and, as such, they should be connected to the conceptual framework, the telling question and the phenomena of interest of the research. This is the meaning of the lines connecting the two sides (concepts and facts) of the map. (As a matter of fact, the overall connection has the shape of a 'V' linking events, at the base of the 'V', to concepts and facts.)

V-9 Afterword

We want to emphasize once more that this study was a first attempt to use Ausubel's theory as a theoretical framework to organize the teaching of physics and to investigate the effects of an Ausubelian approach in terms of concept learning. Certainly, many criticisms can be made to the study and alternative explanations for our findings might be found. However, considering that the experimental approach and the corresponding instructional materials were used for the first time (without any pretesting) and compared with a well established and classical approach, we believe that our findings at least justify more research in this area. Besides the intrinsic value that this research has for ourselves, we hope that it will also have the instrumental value of encouraging other people interested in physics education to do theory-based studies in this field.

APPENDIX I

CONTENT OF EACH UNIT - EXPERIMENTAL GROUP

CONTENT OF EACH UNIT -

EXPERIMENTAL GROUP

UNIT I - PHYSICS and CONCEPTS : A general view of physics and its evolutionary nature; method, concepts, theories, models; "truths" in physics; classical x modern physics; relevance of physics; the role of concepts, evolution of concepts, concept maps.

UNIT II - FORCES and FIELDS : Force; the basic forces of Nature; electric and magnetic forces; a concept map for forces; field; vector and scalar fields; gravitational, electric, magnetic and nuclear fields; superposition principle; lines of force; potential and energy; the electromagnetic field; are fields real?; a concept map for fields.

UNIT III - THE ELECTROMAGNETIC INTERACTION : An overview of electromagnetism; basic events and an event map; basic field events and a field event map; Maxwell Equations: introduction and semi-qualitative analysis; a general concept map of electromagnetism.

UNIT IV - MATHEMATICS REVIEW : Vectors: addition; scalar and vector products; Integrals: definite integrals; the fundamental theorem of calculus; the variable substitution technique; multiple integrals; integrals of vectors; line and surface integrals.

UNIT V - STATIC ELECTRIC AND MAGNETIC FIELDS : Electric and magnetic forces and definition of fields vectors; torque on electric and magnetic dipoles; action of electric and magnetic fields on moving charges; electric and magnetic fluxes; Gauss's Laws for electricity and magnetism; the gravitational analogy; a concept map for the electric and magnetic fields.

UNIT VI - CALCULATION OF ELECTRIC FORCES AND FIELDS : Electric forces between point charges; electric fields due to: 1) discrete distributions of charge 2) continuous distributions of charge (linear and superficial); the use of Gauss's Law and the principle of superposition in the calculation of electric fields.

UNIT VII - CALCULATION OF MAGNETIC FORCES AND FIELDS : Calculation of magnetic forces using the defining equation of $\vec{B}$ and the principle of superposition; magnetic force on a wire carrying an electric current; calculation of magnetic fields due to arbitrary distributions of current using Biot-Savart's Law; calculation of magnetic fields due to symmetric distributions of current using Ampère's Law.

UNIT VIII- POTENTIAL : The gravitational potential; the electric potential and the electric field; potential due to point charges and to continuous distributions of charge; electric potential energy of a system of point charges; calculation of $\vec{E}$ from the potential V; the magnetic scalar potential; a concept map for potential.

UNIT IX - STUDY OF AN ELECTROSTATIC FIELD (Laboratory) : Experimental determination of equipotential lines and lines of force of simulated electrostatic fields in a liquid layer.

UNIT X - ELECTRIC AND MAGNETIC PROPERTIES OF MATTER : Dielectrics and conductors; Gauss's Law in dielectric media; conductivity and resistivity; electric current and resistance; Ohm's Law; magnetic dipole in an external field; paramagnetism; diamagnetism; ferromagnetism; a concept map for electric and magnetic properties of matter.

UNIT XI - ELECTRIC CURRENT AND CIRCUITS : Single-loop circuits; electromotive force and potential difference; loop-theorem; electric power; magnetic circuits; reluctance; magnetomotive force; a concept map for circuits.

UNIT XII - LINEAR AND NONLINEAR RESISTORS (Laboratory) : Experimental study of the linearity or nonlinearity of several different resistors; verification of Ohm's Law.

UNIT XIII- RC CIRCUIT (Laboratory) : Experimental determination of the curves of charge and discharge of a capacitor in a single-loop RC circuit; determination of the time constant RC.

UNIT XIV - ELECTRIC AND MAGNETIC INDUCED FIELDS : Electromagnetic induction: historical and qualitative aspects, applications; Faraday's Law; calculation of induced electromotive forces and currents; induced electric fields; induced magnetic fields; Ampère-Maxwell Law; displacement current; a concept map for induced fields.

UNIT XV - ELECTROMAGNETIC INDUCTION (Laboratory) : Experimental study of the electromagnetic induction using magnets, coils, iron bars, and a galvanometer; intensity of induced current as a function of the distance between primary and secondary coils.

UNIT XVI - ENERGY IN THE ELECTROMAGNETIC FIELD : A concept map for energy; kinetic energy, field energy, mass energy; review: potential energy of an electric dipole in an external field; electrostatic potential energy of a system of point charge; electric energy density; potential energy of a magnetic dipole in an external field; magnetic energy density; energy of an electromagnetic wave.

UNIT XVII- MAXWELL EQUATIONS : Basic equations of classical physics; statics versus dynamics in electromagnetism; review: basic electromagnetic phenomena and their description through Maxwell Equations; review: basic electromagnetic concepts; review: a concept map for the electromagnetism; Maxwell Equations in the differential form.

APPENDIX II

CONTENT OF EACH UNIT -

CONTROL GROUP

CONTENT OF EACH UNIT -

CONTROL GROUP

UNIT I - ELECTRIC CHARGE AND ELECTRIC FORCE : Positive and negative charges; attraction and repulsion; conductors and insulators; Coulomb's Law; forces between point charges; quantization and conservation of the electric charge.

UNIT II - MATHEMATICS REVIEW : Vectors: addition; scalar and vector products; Integrals: definite integrals; the fundamental theorem of calculus; the variable substitution technique; multiple integrals; integrals of vectors; line and surface integrals.

UNIT III - THE ELECTRIC FIELD : Definition of the electric field vector $\vec{E}$; lines of force; calculation of electric fields due to discrete and continuous distributions of charge using Coulomb's Law and the superposition principle; acceleration of a point charge in an electric field; electric torque on an electric dipole; electric potential energy of a dipole in an electric field.

UNIT IV - GAUSS'S LAW : Electric flux; application of Gauss' Law to calculate electric fields due to distributions of charge with spherical, cylindrical, and plane symmetry.

UNIT V - ELECTRIC POTENTIAL : Potential and the electric field; potential due to discrete and continuous distributions of charge; potential due to a dipole; electric potential energy of a system of point charges; calculation of $\vec{E}$ from the potential V; (the scalar magnetic potential: complement, not required on tests).

UNIT VI - STUDY OF AN ELECTROSTATIC FIELD (Laboratory) : Experimental determination of equipotential lines and lines of force of simulated electrostatic fields in a liquid layer.

UNIT VII CAPACITORS AND DIELECTRICS : Calculation of capacitances; dielectrics: an atomic view; Gauss's Law in dielectric media; claculation of electric fields inside dielectrics; electric energy stored in a capacitor; electric energy density.

UNIT VIII- CURRENT-RESISTANCE-ELECTROMOTIVE FORCE : Conventional and electronic current; current density; calculation of electric resistances; resistivity and condutivity; Ohm's Law; electric power; energy dissipated in a resistor; electromotive force; potential differences; single-loop circuits; loop-theorem; (magnetic circuits : complement, not required on tests).

UNIT IX - LINEAR AND NONLINEAR RESISTORS (Laboratory) : Experimental study of the linearity or nonlinearity of several different resistors; verification of Ohm's Law.

UNIT X - RC CIRCUIT (Laboratory) : Experimental determination of the curves of charge and discharge of a capacitor in a single-loop RC circuit; determination of the time constant RC.

UNIT XI - THE MAGNETIC FIELD : Definition of the vector field $\vec{B}$; lines of $\vec{B}$; magnetic flux; Lorentz force magnetic force on a current; torque on a magnetic dipole; magnetic potential energy of a magnetic dipole in a magnetic field; circulating charges in magnetic fields.

UNIT XII - AMPÈRE'S LAW : Calculation of magnetic fields due to symmetrical distributions of current using Ampère's Law.

UNIT XIII- BIOT-SAVART'S LAW : Calculation of magnetic fields due to arbitrary distributions of current using Biot-Savart's Law.

UNIT XIV - FARADAY'S LAW : Calculation of induced electromotive forces and currents using Faraday's Law; Lenz's Law; time-varying magnetic fields; induced electric fields.

UNIT XV - ELECTROMAGNETIC INDUCTION (Laboratory) : Experimental study of the electromagnetic induction using magnets, coils, iron bars, and a galvanometer; intensity of induced current as a function of the distance between primary and secondary coils.

UNIT XVI - INDUCTANCE : Self-induction; calculation of inductance; magnetic energy stored in an inductor; magnetic energy density.

UNIT XVII- MAGNETIC PROPERTIES OF MATTER - MAXWELL EQUATIONS : Gauss's Law for magnetism; inexistence of magnetic monopoles; paramagnetism; diamagnetism; ferromagnetism - Induced magnetic fields; Maxwell's generalization of Ampère's Law; displacement current; Maxwell Equations (summary).

APPENDIX III

EXAMPLE OF A STUDY-GUIDE FOR THE CONTROL GROUPS

EXAMPLE OF A STUDY-GUIDE FOR THE CONTROL GROUPS

Institute of Physics - UFRGS
Physics II - 1976

UNIT VII

CAPACITORS AND DIELECTRICS

I - INTRODUCTION

In previous units you have studied the properties of the electric field generated by static charges (the electrostatic field), in vacuum, and learned to describe it mathematically in two different but equivalent ways: the vector function intensity of the electric field vector $\vec{E}$ and the scalar function electric potential V. Thus, at this point you are ready to study a practical application of this knowledge which has a great importance for the electric technology.

In the first part of this unit, you will study capacitors (also called condensers). They are indispensable components of almost all electric or electronic devices from the simplest pocket radio to the most sophisticated electronic computer. In the second part, completing the study of electrostatic, you will use capacitors to study the properties of the electric field inside insulating materials like water, glass, paper, and oil which are also called dielectrics.

II - OBJECTIVES

At the end of this unit you must be able to:

1a) Calculate the capacitance of capacitors with plane, cylindric or spherical shape in the absence of dielectrics.

1b) Calculate the capacitance of parallel-plate capacitors with a dielectric filling totally or partially the space between the plates.

2) Calculate the equivalent capacitance of a parallel, series or mixed association.

3) Describe macroscopically what happens to the capacitance, the electric field, and the potential difference of a capacitor when a dielectric slab is slipped between the plates, and explain microscopically (i.e., under the atomic point of view) the reason of these changes.

4) Use Gauss's Law to calculate the electric field existing inside a dielectric slab which is slipped between a parallel-plate capacitor, and the induced charged in the surface of the dielectric.

5) Calculate the electric potential energy (and the electric energy density) stored in the space between the plates of a capacitor.

6) Use the concept of stored electric energy to calculate works and forces in parallel-plate capacitors with or without dielectrics.

III - SUGGESTED PROCEDURE

1. Objective 1a: (a) Read section 30-1. The long explanation provided in the first two pages and the unclear Fig. 30-1 are designed only to show that the potential difference between two charged bodies, one with charge +q and the other with -q, decreases when they are approximated, whereas the charge remains constant. That is, the capacitance ($C = q/V$) of the system is greater when the bodies are near to each other. This is the reason why in order to have large capacitance the plates of a capacitor must be as near as possible, but well insulated from each other.

(b) Read section 30-2.

(c) From equation 30-5 we get that $E = q/\varepsilon_0 A$; this field is due to the upper plate, due to the lower plate or due to both of them?

(d) Answer question 4.

(e) Carefully examine examples 1 and 2.

(f) Solve problems 7 and 10.

2. Objective 2: (a) Examine examples 3 and 4.

(b) Solve problems 11, 12, 15 and 16.

3. Objective 3: (a) Read sections 30-3 and 30-4.

(b) Answer questions 7, 9, 12 and 13.

4. Objectives 4 and 1b: (a) Read section 30-5.
(b) Make sure that you are able to explain the exact meaning of each term in equations 30-14 and 30-15.
(c) Carefully analyze example 5.
(d) Solve problems 17, 18, 19, 22 and 23.

5. Objective 5: (a) Read section 30-7.
(b) Answer question 14.
(c) Solve problem 27.

6. Objective 6: (a) Examine examples 7 and 8 in detail.
(b) Answer question 11.
(c) Solve problems 28 and 29.

7. Additional Problems: It is also suggested that, as exercise, the following problems should be solved: 6, 20, 24, 34 and 37.

8. Supplementary Reading: Chapter 25 of the "Student-Guide, Programmed Problems" will be extremely useful in helping you grasp the content and achieve the objectives of this unit, specially if you had trouble in reading Chapter 30 of the textbook. The examples and programmed problems are particularly recommended.

9. Answers to even numbered problems:

30-10: $1.8 \times 10^{-6}C$

30-12: a) $7.9 \times 10^{-4}C$; b) 79V

30-16: a) $q_1 = q_3 = 9 \times 10^{-6}C$, $q_2 = q_4 = 16 \times 10^{-6}C$

b) $q_1 = 8.4 \times 10^{-6}C$, $q_2 = 16 \times 10^{-6}C$

$q_3 = 10.8 \times 10^{-6}C$, $q_4 = 14.4 \times 10^{-6}C$

30-20: before) $C = \varepsilon oA/d$; after) $C = \varepsilon oA/(d-b)$

30-22: a) $q = 16 \times 10^{-10}$ b) $E_o \cong 1.8 \times 10^4$ V/m

c) $E = 0.25 \times 10^4$ V/m, d) $C = 16 \times 10^{-12}$ F

30-24: a) $k = 7.14$, b) $q' \cong 8 \times 10^{-7}C$

30-28: a) $V' = 2V$, b) $U = \varepsilon oAV^2/2d$, $U' = \varepsilon oAV^2/d$

c) $W = \varepsilon oAV^2/2d$

30-34: $U = 27 \times 10^{-2}J$

APPENDIX IV

EXAMPLE OF A STUDY-GUIDE FOR THE EXPERIMENTAL GROUPS

EXAMPLE OF A STUDY-GUIDE FOR THE EXPERIMENTAL GROUPS

Institute of Physics, UFRGS
Physics II - 1976

UNIT V

STATIC ELECTRIC AND MAGNETIC FIELDS

I - INTRODUCTION

This unit is concerned with the two instances of the electromagnetic field: the electric and magnetic fields. However, time dependent fields will not be considered, i.e., the unit deals only with static fields.

Differently from previous units, the content of this one will be rather specific. The basic properties and the action of those fields on charged particles will be studied in detail and some calculations will be made. However, calculations of $\vec{E}$ and $\vec{B}$ (the field vectors) will be made in the two following units. In this unit values of $\vec{E}$ and $\vec{B}$ will be used but not calculated.

The Maxwell Equations concerned in this unit are Gauss's laws, specially the one for electricity. The basic readings are Chapters 27, 28 and 33 of the textbook (excluding the sections corresponding to calculations of $\vec{E}$ and $\vec{B}$) plus section 37-2. Notes "V" emphasize the basic concepts and the similarities of the two fields. A suggested reading sequence is also provided in Notes "V".

This is quite a long unit but the subject is not complex either under a qualitative or quantitative point of view.

II - OBJECTIVES

The overall objective of this unit is a joint study of the static electric and magnetic fields in order to examine with some detail the basic features of these fields emphasizing the similarities. That is, on one hand, the idea is to differentiate between the electric and magnetic fields by studying the properties of each one and the action of each one on moving charged particles. On the other hand, the idea is to integrate them by emphasizing the similarities and the use of common basic concepts. They are different and you should be able to distinguish between them, but they are also only different manifestations of the same field (the electromagnetic field) and you should also be able to recognize this.

More specifically, at the completion of the unit you should be able:

1) To identify the basic concepts used in the description of the electric and magnetic static fields and their action on dipoles and moving charged particles.

2) To apply these concepts to describe the basic features of these fields and their action on dipoles and charged particles.

3) To give examples of the application of these concepts to the gravitational field.

4) To distinguish between the electric and magnetic fields in terms of their basic properties (e.g., field vectors and field lines).

5) To establish a correspondence between the electric and magnetic field phenomena studied in this unit (e.g., electric dipole x magnetic dipole under the action of external fields).

6) To explain the physical meaning of Gauss's law for electricity and to apply it to show that in an insulated conductor any excess of charge resides entirely on its outer surface.

7) To explain the physical meaning of Gauss's law for magnetism and to apply it to show that the magnetic field lines have no beginning and no end.

8) By analogy to the electric case, apply the Gauss law for the gravitational field.

9) Calculate: (a) the acceleration, velocity, kinetic energy and deflection of a charged particle in an electric field.

(b) the radius, the linear and angular velocities, the frequency, and the kinetic energy of a charged particle in a magnetic field.

(c) the torque and the potential energy of an electric dipole in an external electric field.

(d) the torque and the potential energy of a magnetic dipole (a current loop) in an external magnetic field.

(e) the resultant force on a charged particle moving through an electric and a magnetic field.

(f) the magnetic force on an electric current.

(g) electric and magnetic fluxes across open and closed surfaces.

10) Give examples of real situations or practical applications of the effect of electric and magnetic fields on charged particles.

III - SUGGESTED PROCEDURE

1) Skim the following sections of the textbook: 27-1, 2, 3, 5, 6;
28-1, 2, 3, 4, 5;
33-1 to 8 (all chapter);
37-2. These are the basic readings of this unit but for the time being just skim them in order to get the flavor of what this unit is all about.

2) Read Notes "V". They provide a framework for the study of this unit and a suggested sequence of readings.

3) The following steps are suggested to really go through the content of this unit. Whenever you feel it's necessary, do not hesitate coming to us for individual assistance, but first give a real thought to the points that you do not understand. Your interaction with the teacher and the proctors will be much more profitable if you have already studied the material and tried the questions and problems.

4) Read the sections corresponding to Item I of the sequence suggested in Notes "V". Answer questions: 27-3, 4, 6
and 33-1, 2, 3.
Solve problems: 27-2, 4, 8
and 33-1, 2.

5) Read the sections corresponding to Item II of the sequence suggested in Notes "V". Answer questions: 33-4, 5, 6, 7, 15, 16.
Solve problems: 27-26, 28, 30, 31
and 33-19, 20, 27, 28.

6) Read the sections corresponding to Item III of the sequence suggested in Notes "V". Answer questions: 27-12, 13
and 33-8.
Solve problems: 33-11, 12, 13 and the following problem:

An electric dipole consists of two point charges of opposite signs, and magnitude $q = 10^{-6}$C, separated by a distance $d = 2.0$ cm. This dipole is placed in an external field $E = 10^{5}$N/C.

(a) What is the maximum torque exerted by the field on the dipole?

(b) How much work must be done by an external agent to invert the position of the dipole starting at the position $\theta = 0$?

7) Read the sections corresponding to Item IV of the sequence suggested in Notes "V". Answer questions: 28-1 to 8.

The following study questions are also designed to help you achieve the objectives of this unit (you may use them as a self evaluation):

Besides Gauss's law, can the other Maxwell Equations be applied to the gravitational field? Why?

Why can't a magnetic static field change the kinetic energy of a moving charge?

What is the nature of the acceleration acquired by an electric charge under the action of a static magnetic field? (Hint: see section 6-3).

Magnetic field lines have no beginning and no end. Discuss.

What are the basic differences between the field lines of $\vec{g}$, $\vec{E}$ and $\vec{B}$?

How can uniform magnetic and electric fields be obtained? (Hint: Think in terms of uniformity only in a limited region of space.)

What are the basic concepts of this unit? Are they specific or general concepts in physics? Explain with examples.

Use Gauss's law to show that in an insulated conductor any excess of charge resides entirely on its outer surface.

Try to draw your own concept map for this unit.

Do you think that you are able to do what is specified in the objectives of this unit? Have you discussed with the teacher and/or proctors the points in which you have doubts?

If so, you are prepared for the unit-test. Ask for if from a proctor. The purpose of this test is to see if you have achieved the objectives of this unit.

IV - ANSWERS TO EVEN NUMBERED PROBLEMS (odd numbered are provided in the textbook):

27-26) a) 1.7×10^{17} m/s^2; b) 1.7×10^{-10} s;

c) After reaching .1c the relativistic effects start to be relevant.

27-28) a) Yes; b) Strikes the upper plate at 2.7 cm from the left extremity.

27-30) a) q = 5e; b) Electrons cannot be seen; the field should be very weak.

33-2) a) 1 MeV; b) .5 MeV.

APPENDIX V

EXAMPLE OF NOTES (I)

EXAMPLE OF NOTES (I)

Institute of Physics, UFRGS
Physics II - 1976

NOTES (II)

FORCES & FIELDS

I - FORCES

I.1 - Newton's Laws

Isaac Newton developed a theory of motion, according to which the changes of motion of any object are the result of forces acting on it. In so doing he created the subject called classical or Newtonian mechanics, which was the central portion of your Physics I course. By the way, the enormous success of classical mechanics made it seem, at one stage, that nothing more was needed to account for the whole of physical phenomena. However, the discovery of radioactivity, of the electron and the nucleus, and the progress of electromagnetism, called for fundamentally new ideas. Newtonian mechanics, like every physical theory, showed to have its fundamental limitations. The analysis of motions at extremely high speeds required the use of the modified concepts of space and time, proposed by Einstein in his special theory of relativity. But this does not alter the fact that Newtonian mechanics holds in an enormous range and variety of situations. The point is that physical theories have their limits of validity and predictive power.

The concept of force comes initially from subjective experiences, the muscular effort involved in applying a push or a pull. (Thus, force is used to describe what is common in the events of push and pull.) We must exert a "great force" in order to push an automobile but a similar "great force" applied to a large truck produces no motion at all. Knowing that a truck has a greater amount of matter, i.e., a greater mass than an automobile leads us to the conclusion that the "amount of motion" that is produced by a given force depends on the mass of the body.

Newton's first law provides a crude notion regarding force:

> "If the net force on an object is zero, then the acceleration of the object is zero and the object moves with constant velocity."

In fact we have here only a definition of zero force. However, there is the implication that force is somehow intimately connected with the acceleration. The second law states this connection:

> "The accelerated motion of a body can only be produced by the application of a force to that body. The acceleration is proportional to the impressed force and the constant of proportionality is the inertia or mass of the body."

As you probably know in mathematical terms this law is expressed by the following equation:

$$\vec{F} = m\vec{a} \qquad \text{(Eq. 1)}$$

This equation is a general statement regarding force (but not the most general However, it is not a definition unless mass is uniquely defined.

Newton's third law states that:

"If an object 1 exerts a force on an object 2 then object 2 exerts an equal force, oppositely directed, on object 1."

In mathematical terms:

$$\vec{F}_{12} = - \vec{F}_{21} \qquad \text{(Eq. 2)}$$

(It can be shown that this law provides a method of uniquely defining mass; then the equation $\vec{F} = m.\vec{a}$ gives a definition of force.)

I.2 - The Basic Type of Forces

All forces arise from interactions between objects. One of the most remarkable features in the development of modern science has been the growing realization that only a very few basically distinct kinds of interaction are at work. The following are the only forces that we know of at present:

Gravitational forces, which arise between objects because of their masses.

Electromagnetic forces, due to electric charges at rest or in motion.

Nuclear forces, which dominate the interaction between subatomic particles if they are separated by distances less than about 10^{-15}m.

It may even be that this degree of categorization will prove to be unnecessarily great; theoretical physicists are looking for a unifying idea that would allow us to recognize all these forces as aspects on one and the same thing. However, at present time the assumption of these three primary types of forces seems meaningful as well as convenient. Of course, there is always the possibility that Nature is more complicated than we are aware of, but at the present time there seems to be no need to invoke any additional type of force to account for any observed process. All forces that we know are instances of these three types.

All our experience suggests that a gravitational interaction between material objects is a universal phenomenon. It is always an attractive interaction. The general law of gravitational interaction arrived at by Newton states that:

> The force with which any particle attracts any other is proportional to the product of the masses of the particles, inversely proportional to the square of their distance, and directed along the line separating the two particles.

In mathematical terms:

$$F_{12} = G \frac{m_1 m_2}{r_{12}^2} \qquad \text{(Eq. 3)}$$

where G is a constant of proportionality called the universal gravitational constant.

Although electric forces are responsible for holding atoms together, they would, by themselves, prevent the existence of atomic nuclei. Nuclei contain protons electrically repelling one another and not stabilized by a compensating negative charge. Nuclei are held together by nuclear forces. The force that acts at the small distances within nuclei and maintains the stability of nuclei, in spite of their tendency to fly apart because of electric repulsion, is called <u>strong nuclear force</u>. The strong nuclear force acts between nucleons (protons and neutrons) but it is effective only over distances of up to 10^{-13} cm. (nuclear dimensions). It is said to be a short range force.

Another type of nuclear force is so-called weak force. The range of this force is even less than that of the strong forces and it acts between nuclear and elementary particles.

I.3 - Electric and Magnetic Forces

These are the forces more directly relevant to this course. The forces that electrically charged particles exert on one another are of fundamental importance in nature. Although gravity is always present, the electrical force is overwhelmingly the most significant agent in all chemical and biological processes and in the interactions between physical objects of everyday size (gravitational forces play a prime role in most astronomical systems, and nuclear forces at very small distances). It holds atoms together, provides the rigidity and tensile strength of material objects, and is the only force involved in chemical reactions.

The basic law of electric force is that found by the nineteenth-century French physicist, C. A. Coulomb, and known by his name. Coulomb's law states that:

> "A charged particle at rest will attract or repel another charged particle at rest with a force proportional to the product of the charges, inversely proportional to the square of their separation and directed along the line separating the two particles."

The force is attractive when the charges are unlike and repulsive when they are alike in sign.

In mathematical terms:

$$F_{12} = k \frac{q1q2}{r_{12}^{2}} \qquad \text{(Eq. 4)}$$

where q1 and q2 denote the charges carried by the particles and k in the proportionality constant. This type of force is usually called Coulomb force.

So far, the electric force that we are talking about is between stationary charged particles. Moving charges also exert electric forces on each other. But an additional force arises in this case which we call the magnetic force. It has the interesting property that it depends on the velocity of the charges and always acts on a given charged particle at right angles to the particle's motion.

Actually, from the standpoint of relativity theory, the magnetic force is not something new and different. Charges that are moving with respect to one observer can be stationary with respect to another. Thus, if one accepts the basic idea of relativity, one may expect to be able to relate a magnetic force, as observed in one reference frame, to a Coulomb force, as observed in another frame. Ultimately, magnetic forces can be considered a relativistic effect that arises when electric charges are not stationary. The "magnetic force of a magnet," which certainly is familiar to you, is a macroscopic effect of the magnetic forces originated in the motion of the electrons (electric charges in motion). Magnetic forces arise exclusively from charges in motion.

In future units, calculations of electric and magnetic forces will be made. Now, before speaking of fields let's try a concept map for forces (it may help you to have a general view of forces). See Fig. 1.

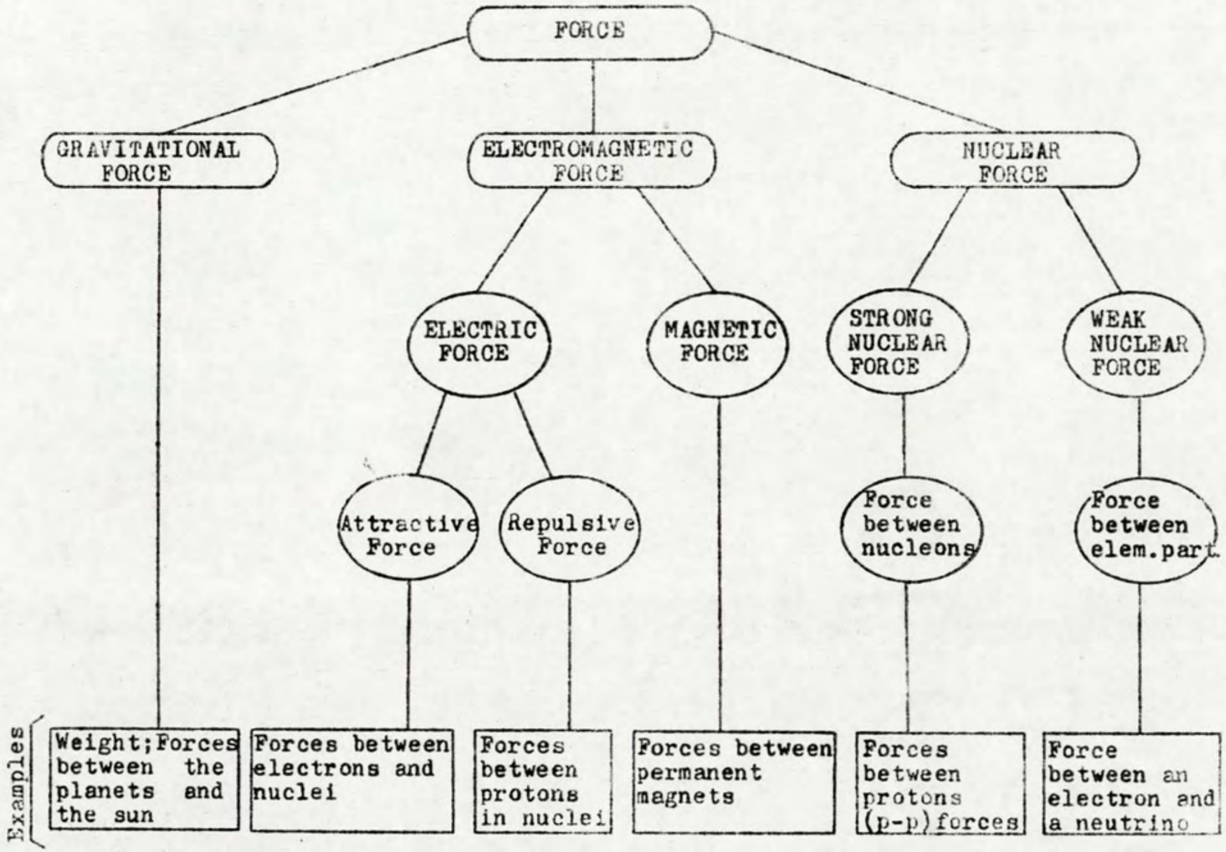

Fig. 1. (Notes II). A Concept Map for Forces.

In this map, force is at the top as the most inclusive (superordinate) concept. The three basic types of forces are at a lower level and are considered less inclusive concepts than force itself. On the other hand, the primary types are more inclusive than their subdivisions (electric, magnetic, strong and weak) which in turn are more inclusive than further subdivisions and then the specific examples at the bottom of the map. This map presents the concepts hierarchically organized, but notice that they are connected by lines not arrows pointing down which means that such organization is not in only one direction. Sometimes a more general concept emerges from more specific instances of such a concept; sometimes a more general concept is used to look at particular instances of that concept.

Another remark is that this is just a map--not the only map of forces. You can try your own map, for example, starting with force at the center of the map.

II - FIELDS*

In spite of the fact that most of the forces that we encounter in everyday experience are of the contact type, we push or pull on something or one object strikes another, the forces that we have labeled gravitational, electric, and magnetic are "action -at-a-distance" forces. To ancient men, contact forces were the only real forces. However, an entirely new concept emerged when Newton established the theory of universal gravitation. According to this theory, the Earth, Moon, Sun and planets all exert forces on one another without contact or any material medium between them. The term "action-at-a-distance" was used to describe such an interaction.

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*Based on Physics and the Physical Universe, J.B. Marion, Chapter 8.

This conception, however, was not easily accepted and something called "ether" was invented to serve as a "medium" to transmit action-at-a-distance forces. The ether was supposed to be a tenuous substance that filled all space and was required to have a vanishingly small density to account for the fact it could not be observed by any known means in an evacuated space. Although the ether concept was used for many years it did not survive the test of experiment. In particular, careful attempts to measure the speed of the Earth through the ether always gave the result of zero. Physicists were not willing to believe that the Earth was permanently at rest in the ether and that all other bodies in the universe were in motion through it. The ether theory had been forced to include so many ad hoc assumptions to explain so many facts that it finally collapsed. In its place came the field theory approach to all action-at-a-distance forces. The idea of a field of force is most useful for this type of interaction.

The above description is an example of what we discussed in Notes "I": the evolutionary nature of concepts in physics and the interplay between theory and experiment.

II.1 - What is a Field?

Any physical quantity that has a well-defined value at any point in space can be considered a field quantity. That is, we can imagine measurements being made of a certain physical quantity (the field quantity) at every point in space. We must obtain a unique value of the field quantity at every point. And furthermore, there must be a smooth variation of the field from point to point. This smooth variation from point to point in space is an essential feature of a field.

A meterological map is actually a representation of the pressure field for the particular area. Such maps are prepared by measuring the atmospheric pressure at a larger number of points throughout a region and then plotting curves (called isobars) to connect points of equal pressure. The pressure at a given point is specified by a single number. That is, pressure is a scalar quantity and the pressure field is a scalar field.

Weather maps also show the variation of temperature across a region. In this case, curves (called isotherms) are drawn connecting points at which equal temperatures have been measured. The temperature field is also a scalar field since temperature is a scalar quantity.

When water flows in a river or stream, generally the flow velocity is not the same at all points but varies in a smooth way from surface to bottom and from midstream to bank. Because the flow velocity varies smoothly from point to point in the river, we can describe the situation in terms of a velocity field. A velocity field differs in an essential way from pressure or temperature fields because velocity requires both magnitude and direction for its specification. The velocity field is a vector field.

Any physical quantity that has a well-defined magnitude and direction at every point in space can be considered a vector field quantity. Most of the interesting field quantities that we encounter in physics are vectors. What about forces? Forces are vectors; the gravitational, electric and magnetic forces have well-defined magnitude and direction in every point of space. Consequently, it makes sense to speak about fields of forces or, more specifically, the gravitational field, the electric field and the magnetic field.

II.2 - Field Vectors*

Consider the gravitational attraction of the Earth for a particle outside it. The pull of the Earth depends on the mass of the attracted particle and on its location relative to the center of the Earth. This attractive force divided by the mass of the particle being pulled depends only on the Earth and the location of the attracted object. We can, therefore, assign to each point of space a vector, of magnitude equal to the Earth's pull on a particle divided by its mass, and of direction identical with that of the attractive force. Thus we imagine a collection of vectors throughout space, in general different in magnitude and direction at each point of space, which define the gravitational attraction of the Earth for a test particle located at an arbitrary position. The totality of such vectors is called a field, and the vectors themselves are called the field strengths or intensities of the field.

In this example, the gravitational field strength $\vec{g}$ at point P is:

$$\vec{g} = \frac{\vec{F}}{m} \qquad \text{(Eq. 5)}$$

One can generalize this for the field produced by any distribution of matter, for which the field strength $\vec{g}$ as a function of position describes quantitatively the gravitational field. The gravitational force exerted on an object of mass m by this field is then given by:

$$\vec{F} = m\vec{g} \qquad \text{(Eq. 6)}$$

This field description of forces is especially useful in specifying electromagnetic forces. The electric field produced by a charged particle

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*Based on Newtonian Mechanics, A.P. French.

or by a collection of such charged particles is described by the electric field strength vector or intensity vector $\vec{E}$, where: $\vec{E} = \frac{\vec{F}}{q}$ (Eq. 7)

$\vec{F}$ is the vector force acting on a positive test charge of magnitude q, and $\vec{E}$ depends on position.

For electric fields produced by charges at rest, the situation is similar to the gravitational case. The magnetic field can be also described by a magnetic field strength vector or intensity vector. This vector is denoted by $\vec{H}$, however, the expression: $\vec{H} = \frac{\vec{F}}{m}$ (Eq. 8)

where m in this case would stand for "magnetic charge" or "magnetic mass" is not useful because the concept of magnetic mass (or magnetic charge) is not useful. So far, no magnetic monopole has been detected in Nature. That is, no magnetic south or north pole isolated has been detected. Magnetic poles are always in pairs. So, the use of Equation '8' is dependent upon considerations about long magnets in order to consider the effect of only one pole. These approximations are interesting in order to carry the analogies further between the three fields (Coulomb's law would hold for magnetic masses:

$$F_{12} = k\, m_1 m_2 / r_{12}^{\;2} \quad), \qquad \text{(Eq. 9)}$$

but not for practical calculations, for example. In subsequent units, when the magnetic field will be studied in detail, another vector will be defined to describe the magnetic field. This will not be the only time that the magnetic analog of certain electric quantities will not be useful due to the absence of magnetic monopoles. As a matter of fact, the absence of magnetic monopoles is the central phenomenon described by one of the four basic equations of electromagnetism (Maxwell Equations).

Some physicists are not convinced of this at all and are still carrying out experiments to detect a magnetic monopole. So far, it is not likely, but if some day a magnetic monopole is discovered some of the present concepts and equations should be at least modified to account for the new experimental evidence. That's the way physics works.

II.3 - The Principle of Superposition

One fact that makes the field concept so useful is that the force vectors and the field vectors obey the principle of superposition. That is, if we wish to calculate the force or the field in a certain point of space due to many objects or charges, the net force or field is the vector sum of all individual forces or fields; each of these individual forces or fields can be calculated as if the other objects (or charges) were not present.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots \qquad \text{(Eq. 10)}$$

$$\vec{g} = \vec{g}_1 + \vec{g}_2 + \vec{g}_3 + \ldots \qquad \text{(Eq. 11)}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots \qquad \text{(Eq. 12)}$$

The superposition principle holds for waves too. It is an experimental fact that, for many kinds of waves, two or more waves can traverse the same space independently of one another. The fact that waves act independently of one another means that the displacement of any particle at a given time is simply the sum of the displacements that the individual waves alone would give it. This process of vector addition of the displacements of a particle is called superposition.

The importance of the superposition principle is that, where it holds, it makes possible to analyze a complicated wave motion (or field configurations) as a combination of simple waves (or fields).

II.4 - Lines of Force*

A diagram or map of a vector field is more complex than that of a scalar field because both magnitude and direction must be specified. Suppose that we begin to map the gravitational force field around a certain source mass M by measuring the force on a small test mass. The results of such measurements can be represented by a series of arrows, as in Figure 2. The length of each arrow is proportional to the gravitational force at the end of the arrow and the direction of the force given by the direction of the arrow. Alternatively, we can construct around the source mass a set of continuous lines, called lines of force, so that at any point the direction of the force is the direction of the line of the force passing through that point. The magnitude of the force at any point in such a diagram is proportional to the density of lines in the immediate vicinity of that point. Figure 3 is the representation of the lines of force for the electric force field of a point charge +q. A simple inspection of the lines of force diagram reveals where the force is greatest (where the lines bunch together) and where the force is least (where the lines spread out to low density).

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*Based on Physics and the Physical Universe, J.B. Marion.

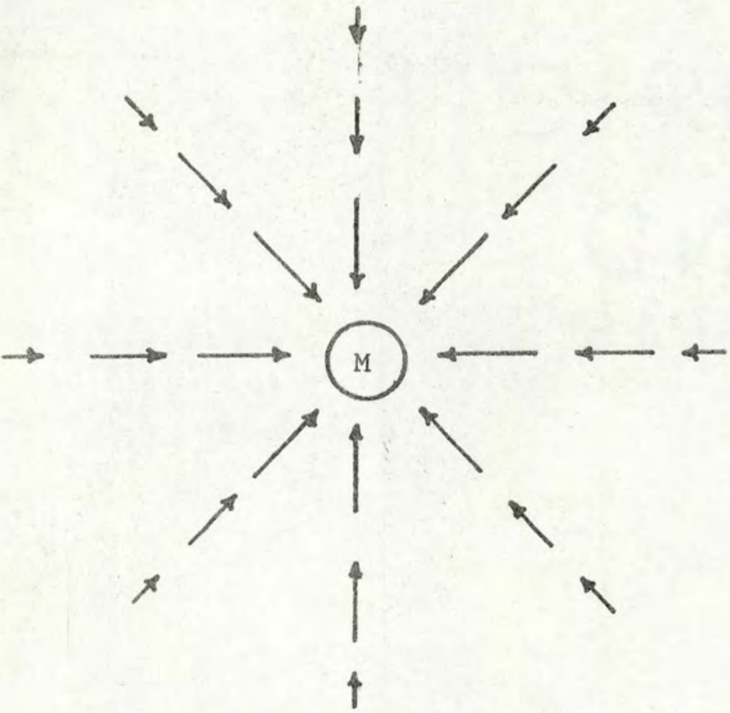

Fig. 2. (Notes II)

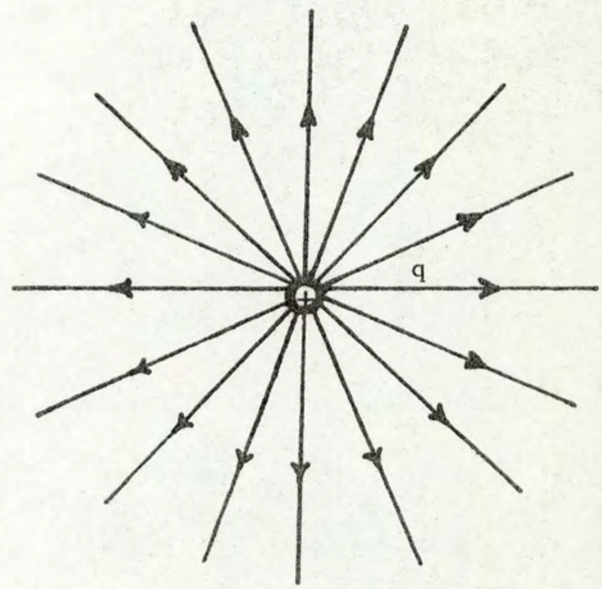

Fig. 3. (Notes II)

This type of representation can be used for the gravitational, electric, magnetic fields and other vector fields. However, although the lines of force scheme is useful in visualizing the force field surrounding an object or charge it is important to realize that this picture is only an invention--there are no rubber-band-like lines that extend through space and exert forces on other objects. Lines of force are not real--they serve only to provide a crutch for our thinking when we consider force-field problems.

Lines of force will be used throughout the course and further properties of them will be introduced later on.

II.5 - Potential and Energy

Fields can be described not only by vectors but also by scalar quantities called scalar potentials, or simply potentials, e.g., the scalar gravitational potential and the scalar electric potential. The vectors and potentials describing fields are intimately related, and often it is only a matter of convenience which one is used in a given problem.

Potentials can be defined in terms of energy. As a matter of fact, the concept of potential energy may sound familiar to you. When we raise an object to the height h above the surface of the Earth, we say that the object possesses a gravitational potential energy mgh relative to its initial position. The gravitational potential energy per unit mass is the gravitational potential. Similarly, the electric potential is the electric potential energy per unit of charge.

Mathematical expressions can be deduced for potentials. This will be done later on for the case of the electric potential. For the time being the point is that potentials are an alternative way (a scalar way) of describing fields of force. In practice, potential differences are of fundamental concern (the "voltage" is just a common term for electric potential differences).

What about the magnetic scalar potential? Well, by analogy with the electric field, it is possible to introduce a magnetic potential, or a magnetic potential difference. However, it is not so useful a quantity because it is not as easy to place a physical interpretation on the magnetic potential as it is with electric and gravitational potential. Again, this is related to the nonexistence of magnetic monopoles. If single magnetic poles really existed the magnetic potential could be defined as a potential energy per unit pole. This is not the case, however, and the concept of magnetic potential is not very useful. We will come back to this point in future units.

In the above paragraphs we have implicitly admitted that a field can possess energy. Let's explore a little bit this assumption. When we raise an object to a height h above the surface of the Earth, we say that the object possesses a gravitational energy mgh relative to its initial position. But does the object really possess this potential energy? Or does the Earth share in the energy? According to the field description of the gravitational interaction we should not ascribe the increase in potential energy to either body. An amount of work mgh has been done <u>on the field</u> by changing the relative position of the two bodies and it is the gravitational field that has acquired this energy. The energy can be recovered from the field to set the objects into motion. Similar comments also apply for the electrical and nuclear force fields. (Usually however, we speak about the potential energy of a body or charge or of a system of bodies or charges.)

If we choose to group together all types of potential energy under the heading "field energy," we would have three types of energy:

Kinetic Energy
Field Energy
Mass Energy.

II.6 - The Electromagnetic Field*

We have seen previously that electric and magnetic forces are very intimately associated with each other. In fact, together they account for that interaction between two charged objects which is due to their charge. Therefore, instead of considering the forces separately, we should properly speak of the electromagnetic interaction, which includes them both. If the two charged objects are at rest, then the magnetic forces are zero, and only electric forces need to be considered. If the charged objects are in motion, however, then both electric and magnetic forces should be considered.

Since the electric and magnetic forces are intimately related, their fields are also intimately related, and we should properly speak of the electromagnetic field. A charge at rest gives rise to an electromagnetic field comprised of an electric field alone. A charge in motion gives rise to an electromagnetic field which includes both an electric and a magnetic field. According to the special principle of relativity, a charge can be considered either at rest or at motion, depending on the reference frame, despite the fact that the fields are different in the two cases. The two parts of the electromagnetic field are very intimately associated indeed.

Thus far we have emphasized two types of vector fields, the gravitational field and the electromagnetic field. The gravitational and electromagnetic interactions and their fields account for nearly all forces we are ordinarily aware in daily life. The gravitational interaction keeps us on the Earth. Nearly all other forces we are aware of arise (directly or indirectly) from the electromagnetic interaction. We know, of course, that much of our machine and instrumentation are electrical. However, even the ordinary pulls and

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*Based on Conceptual Physics, J.R. Ballif and W.E. Dibble.

pushes of one object against another involve electrical forces. The atoms are held together by the electrical attraction between the positive protons and the negative electrons. However, that same force would prevent the existence of atom nuclei (for we know that nuclei contain protons, positively charged particles, electrically repelling one another). As we have mentioned before, short range forces called nuclear forces maintain the stability of nuclei in spite of their tendency to fly apart because of the electrical repulsion. But, what about nuclear fields?

II.7 - The Field of the Nuclear Force

Well, the field concept has been applied to the strong nuclear force, but the nature of the force in this case requires a departure from our previous reasoning regarding fields. Unlike the gravitational and electromagnetic forces, the effect of the nuclear force does not extend to infinity, but is instead confined to extremely small distances. Thus, what is the nature of the nuclear field? The basic new idea that paved the way for our present (and still incomplete) understanding of the nuclear force was provided by the Japanese physicist, Yukawa, in 1935. Yukawa hypothesized that two nucleons (protons or neutrons) experience an attractive force at small distances because of the exchange between them of a new elementary particle (which had not yet been observed at that time) called a meson. A meson is a particle with mass intermediate between the mass of an electron and that of a proton. Thus, physicists talk about a meson field and a meson exchange force.

This exchange force when treated in detail requires complicated mathematics, but a qualitative description can be made. Suppose that two boys are grappling for control of a basketball. One boy grabs the ball from the other boy and the second boy snatches it back again; the process is repeated over and over. This continual exchange of the ball results in each boy being pulled toward the other; that is, there is an attractive "basketball exchange force." This example gives you a rough idea of what is meant by exchange force.

Here again we have a nice example of the role of concepts or of a conceptual structure in physics. The existing conceptual structure (forces, fields, elementary particles, etc.) which accounted for many experimental facts led to a new conception. It was used to hypothesize a model for the nuclear interaction. A new particle (meson) was predicted, new concepts (exchange force, meson field) were introduced. This conception in turn led to new research directions (e.g., detection of the meson), which could determine the revision of the conceptual structure or the discarding or refinement of the new model. That's the way progress is made in physics.

> "A fresh line of scientific research has its origin not in objective facts alone, but in a conception, a deliberate construction on the mind. On this conception, all else depends. It tells us what to look for in the research. It tells us what meaning to assign these facts." (Schwab, 1962)

Several brands of mesons are now known, but the meson that is responsible for the strong nuclear force has been found to have a mass approximately 273 times that of an electron and is called the π meson or pion. (The pion was not discovered until a 1947 experiment revealed the presence of these mesons in cosmic rays; in the following year, pions were first produced artificially in an accelerator and since that time we have had available beams of pions for use in detailed studies of their properties and interactions.) Consequently, we speak about the "pion exchange force" and the "pion field" to describe the nuclear interaction. (Mesons are very short-lived, 10^{-8} seconds or less, and the exchange of the pion is an extremely rapid process, requiring only about 10^{-23} seconds.)

By using the pion field concept of the strong nuclear force, a great deal of progress has been made, although we still have much to learn before we can claim to understand completely this basic force of Nature. The situation with regard to the weak force is even more bleak. Presumably, there is a similar elementary particle that mediates the force between electrons and neutrinos, but as yet we have no clear conception of the nature of this particle.

II.8 - Are Fields "Real"?*

Energy, as you may have learned in your previous courses, is a very important, a key concept, in physics. If a field can contain energy, then must we conclude that the field is indeed a real entity? In physics we attribute reality exclusively to those quantities that are measurable. Distance, mass, velocity, and momentum are surely real. But we never measure the electric field vector $\vec{E}$ or the gravitational field vector $\vec{g}$. We

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*Based on Physics and the Physical Universe, J.B. Marion.

always measure the effect of these field quantities; that is, we always measure a force. Therefore, the fields are only mathematical constructions that enable us to interpret in a consistent and experimentally verifiable way the actions of the gravitational and electromagnetic forces. Are fields real? It is almost too fine a distinction to make. Whether or not they are real, the concept of field has been one of the most fruitful ideas in physics, and it is clear that we shall continue to reap the benefits of field theories in many areas of science and technology.

II.9 - A Concept Map for Fields

In the previous sections we have introduced the concept of field quantity as any physical quantity that has a well-defined value at any point in space. If the quantity was a scalar we talked about scalar field, and about vector field if the quantity was a vector quantity. As forces are vectors, the concept of fields of force was introduced to account for the "action-at-a-distance" interactions. Then we discussed the gravitational, electromagnetic and nuclear fields of force. We also introduced concepts like lines of force, potential, potential energy and the superposition principle which are connected to fields. We finished these Notes discussing the "reality" of fields.

After reading all this don't you think that a concept map for fields would help to give you a general picture of the subject and clarify some points? Why don't you try to draw your own map before looking at Figure 4 of these Notes. It is not difficult, just identify the key concepts and put them together in a diagram. Discuss your map with your teacher, your proctor, or your colleagues. Compare your map with Figure 4. Remember that there is not only one way to draw such a map.

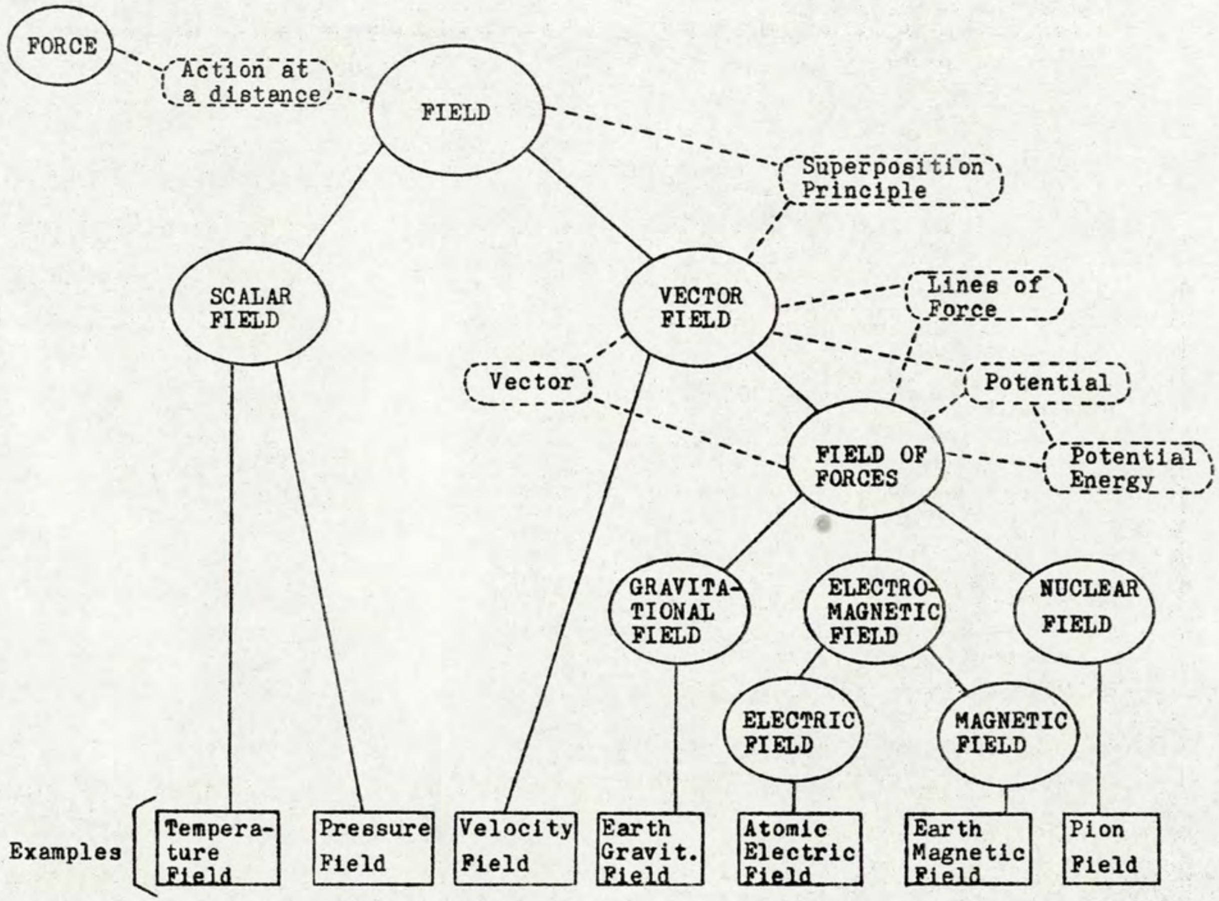

Fig. 4. (Notes II). A Concept Map for Fields.

At the bottom of the concept map presented in Figure 4 are specific examples of fields. Instead of specific examples we could use the basic events related to the concepts (remember that concepts describe regularities in events. For example, the basic event related to the gravitational field concept is that forces arise between objects because of their masses. Somewhere, in following units we may put the basic events in the map too.

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APPENDIX VI

EXAMPLE OF NOTES (II)

EXAMPLE OF NOTES (II)

Institute of Physics, UFRGS
Physics II - 1976

NOTES VIII
(Preliminary Version)

POTENTIAL

I - INTRODUCTION

In Figure 9 of Notes III, we presented a concept map for electromagnetism, and we said that it would serve as a frame of reference for this course. Well, looking at this map, we will see that, so far, we were concerned mainly with concepts and laws situated in the upper half (and in part of the right side of the lower half). As a matter of fact, only one concept of the upper half was not yet considered in detail, i.e., the concept of potential.

There are two types of potential, the scalar potential and the vector potential. In this course, only the first type will be considered in detail and we will call it just potential. We will start these notes with some brief remarks about the gravitational potential because it is used to describe a field quite familiar to you, i.e., the gravitational field. After this, most of the following pages will be dedicated to the electric potential because this is the case where the concept of potential is most useful within the scope of this course. As a complement, some attention will be paid to the magnetic potential and the vector potential. The Notes will end with a concept map for potential.

It must be emphasized, however, that the reading assignments of Unit VIII include not only these Notes but Chapter 29 of Halliday-Resnick as well.

II - THE GRAVITATIONAL POTENTIAL

As we have seen before, the potential is a scalar function that can be used to describe vector fields. However, this does not mean that a potential exists for any vector field. The potential exists only when the field vector meets certain mathematical requirements which we will not specify because they involve mathematical concepts above the level of this course, and because the important point for us is that the potential does not necessarily exist for any vector field.

In the gravitational field, a potential does exist: the gravitational potential. Taking the sea level as level of zero potential, the gravitational potential in a point at a height h above the sea level is defined as the work that is necessary in order to displace a body of unitary mass up to this point, or the gravitational potential energy per unit of mass at this position.

A surface of constant height relatively to the sea level is an equipotential surface; a body can be displaced from one point to another within this surface without a variation in its potential energy. Figure 1 illustrates this situation.

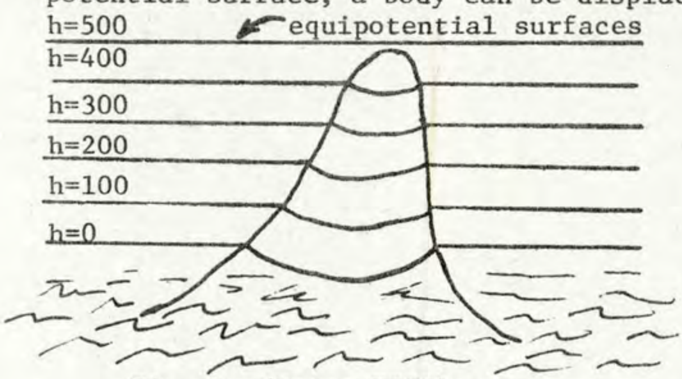

Fig. 1. (Notes VIII).

At great distances from earth the equipotential surfaces of the earth's gravitational field are spheres concentric with earth. The gravitational field is a field of force characterized by a field vector $(\vec{g})$ which, by definition, at each point is equal to the gravitational force exerted per unit of mass on a test body placed at that point $(\vec{g} = \vec{F}/m)$ and is directed toward the earth's center.

Considering that the gravitational potential is defined in terms of work and the gravitational field vector is defined in terms of force, a simple relationship should, therefore, exist between them. Such a relationship is as follows: $\vec{g} = -\nabla P$ (1)*, that is, the field is equal to the negative value of the potential gradient.

The gradient of a scalar function ϕ which may be represented by $\nabla\phi$ or grad ϕ is given by:

$$\nabla\phi = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\phi = \frac{\partial\phi}{\partial x}\vec{i} + \frac{\partial\phi}{\partial y}\vec{j} + \frac{\partial\phi}{\partial z}\vec{k}$$

The gradient is a vector which has the direction of the largest variation of ϕ. In the gravitational case, therefore, $\vec{g}$ has the direction of largest variation of P (or Vg). Consequently, the gradient is always perpendicular to the equipotential surfaces: the largest variation is obviously obtained by passing from one surface to another in the shortest possible path, i.e., perpendicularly. (If you have already tried to climb a steep hill on a bike you certainly know that going up along a straight line is harder, although the work is the same, because this is the direction of largest variation in potential energy, i.e., the direction of the potential gradient vector.) In addition, the closer are the equipotential surfaces the higher is the gradient because of the greater rate of change of the potential relatively to the distance between the surfaces. This situation

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*In spite of being written for the gravitational field, this is a general relationship between field and potential.

is illustrated in Figure 2: if S_1, S_2 and S_3 are equipotential surfaces such that $P_3 > P_2 > P_1$, the field vector $\vec{g}$ will have larger magnitude at point 2 than at point 1 because at 2 the surfaces are closer than at 1. In both cases, however, $\vec{g}$ is perpendicular to the equipotential surfaces and points in the direction opposing the increase of P ($\vec{g}$ = - grad P).

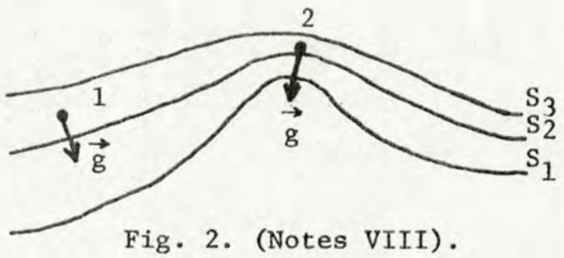


Fig. 2. (Notes VIII).

III.1 - THE ELECTRIC POTENTIAL (Electrostatic Potential)

In the case of the electrostatic field the situation is analogous to the gravitational field: the potential is also defined in terms of a work and the electric field vector is defined in terms of a force ($\vec{E} = \vec{F}/q$). The work, in this case, is the work performed when displacing an electric charge q in the electric field $\vec{E}$. Assuming zero potential at infinity, the electric potential at a point P of an electrostatic field is equal to the work that must be done by an external agent to bring a test charge (or the work per unit of charge) from the infinity to that point:

$$V = \frac{W}{q_o} \qquad (2)$$

We must realize, however, that in both cases the level of zero potential is arbitrarily defined, i.e., the selection of the sea level for the gravitational potential, nd the infinity for the electric potential as levels of zero potential is arbitrary (although convenient for analytic purposes). But, on the other hand, if only potential differences are taken into account this problem is avoided because they do not depend on the level of reference.

This is the reason why in many instances we use only potential differences. In electric circuits, for example, we usually deal with potential differences instead of potentials.

Using equation (2) for two arbitrary points A and B of an electric field:

$$V_B - V_A = \frac{W_{AB}}{q_o} \qquad (3)$$

It can be shown (see section 29-2 of Halliday-Resnick) that in an electric field $\vec{E}$ the following equation expresses the relationship between the electric potential V and the electric field vector $\vec{E}$:

$$V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dl} \qquad (4)$$

If point A is at an infinite distance from all charges, that is, if $V_A = 0$, this equation will give us the potential at point B, or, without the subscript B:

$$V = -\int_{\infty}^{B} \vec{E} \cdot \vec{dl} \qquad (5)$$

These two equations (4 and 5), therefore, allow us to calculate the potential difference between any two points, or the potential at any point when $\vec{E}$ is known as a function of distance. It must be remarked, however, that the expression $V_B - V_A = Ed$ which might be familiar to you is a particular case of equation 4 in which $\vec{E}$ is constant, i.e., the field is uniform. In general, $\vec{E}$ is a function of distance and to solve the integral $\int \vec{E} \cdot \vec{dl}$ we must know this function. By the way, this integral is also a line integral as that one of Ampère's law, but not closed.

As in the gravitational case, the field vector is equal to the potential gradient with a minus sign: $\vec{E} = -\text{grad } V$ (6)

While equation 5 allows the calculation of V from $\vec{E}$, equation 6 allows us to calculate $\vec{E}$ from V. This reciprocity supports an earlier statement saying that the field vector and the potential are intimately related quantities.

Further details about the subject of this section are in Chapter 29 of Halliday-Resnick. In the next section we will give some examples on how to calculate the potential due to continuous and discrete distributions of charge.

III.2 - CALCULATION OF THE POTENTIAL

(a) <u>Discrete distributions of charge:</u> In order to calculate the potential at any point due to a group of point charges, we first calculate the potential V_n due to each charge, as if the other charges were not present, and, then, we add <u>scalarly</u> the quantities so obtained:

$$V = \sum_n V_n \tag{7}$$

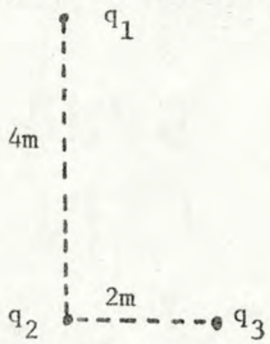
It is possible to show (see section 29-3 of Halliday-Resnick) that the potential due to a point charge is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \tag{8}$$

Thus, the potential due to a group of point charges is given by:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_n \frac{q_n}{r_n} \tag{9}$$

Let's see an example: Suppose that we have to calculate the potential at point P due to three point charges q_1, q_2 and q_3 as shown in Figure 3.



$q_1 = 3 \times 10^{-6}C$

$q_2 = -2 \times 10^{-6}C$

$q_3 = 4 \times 10^{-6}C$

Fig. 3. (Notes VIII).

In order to do this, we will first calculate the potential due to each charge: $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$ where r_1 is the distance between q_1 and P.

$$V_1 = 9 \times 10^9 \frac{Nm^2}{C^2} \cdot \frac{3 \times 10^{-6}C}{2m}$$

$$V_1 = 13.5 \times 10^3 \ V$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q2}{r_2}$$

$$V_2 = 9 \times 10^9 \frac{Nm^2}{C^2} \cdot \frac{(-2 \times 10^{-6} C)}{(2^2 + 4^2)^{1/2}} \cong -4.0 \times 10^3 \text{ V}$$

$$V_3 = \frac{1}{4\pi\varepsilon_0} \frac{q_3}{r_3} = 9 \times 10^9 \frac{Nm^2}{C^2} \cdot \frac{4 \times 10^{-6} C}{4m} = 9.0 \times 10^3 \text{ V}$$

$$V = V_1 + V_2 + V_3 \cong 13.5 \times 10^3 - 4.0 \times 10^3 + 9.0 \times 10^3 \cong 18.5 \times 10^3 \text{ V}$$

Obviously, we could have used equation 9 directly:

$$V = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \right),$$ the result would be the same.

It must be remarked that this sum is just an algebraic sum because the potential is a scalar quantity.

Now, we will use this opportunity to calculate the potential energy of this distribution of charges: the electric potential energy of a system of point charges is defined as the work required to assemble this system of charges by bringing them from an infinite distance to the position they occupy in the system. It is assumed that the charges are all at rest when they are infinitely separated, i.e., they have no initial kinetic energy.

For a system of two point charges q_1 and q_2, the electric potential energy U is given by (see section 29-6 of Halliday-Resnick):

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} \qquad (10)$$

where r_{12} represents the distance between q_1 and q_2.

For a group of point charges, we compute the potential energy for every pair of charges separately and add the results algebraically. Thus, in our case:

$$U = U_{12} + U_{13} + U_{23}$$

$$U_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} = 9 \times 10^9 \ \frac{Nm^2}{C^2} \cdot \frac{3 \times 10^{-6}C\ (-2 \times 10^{-6}C)}{4m} = -\ 13.5 \times 10^{-3}\ J$$

$$U_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{r_{13}} = 9 \times 10^9 \ \frac{Nm^2}{C^2} \cdot \frac{3 \times 10^{-6}C\ (4 \times 10^{-6}C)}{(4^2 + 2^2)^{1/2}\ m} \cong 24.2 \times 10^{-3}\ J$$

$$U_{23} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 q_3}{r_{23}} = 9 \times 10^9 \ \frac{Nm^2}{C^2} \cdot \frac{(-2 \times 10^{-6}C)\ 4 \times 10^{-6}C}{2m} = -\ 36.0 \times 10^{-3}\ J$$

$U = -\ 13.5 \times 10^{-3}\ J + 24.2 \times 10^{-3}\ J - 36.0 \times 10^{-3}\ J \cong -\ 25.3 \times 10^{-3}\ J$ (What is the physical meaning of this minus sign?)

In order to complete the study of this topic, you must also examine examples 5, 8 and 9 of Chapter 29 of Halliday-Resnick and solve some of the problems concerning discrete distributions of charge proposed at the end of this Chapter (e.g., 7, 8, 13).

(b) <u>Continuous distributions of charge</u>: In this case, we divide the distribution into differential elements of charge dq, calculate the potential dV established by dq at the point at which V is to be calculated, and integrate. In other words, the sum of equation 7 must be replaced by an integral:

$$V = \int dV = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} \qquad (11)$$

where r is the distance between dq and the point at which V is to be calculated and $dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r}$ is the potential established by dq at that point. Let's apply this equation to a problem: Suppose that we want to calculate the potential established by a nonconducting rod of length L, uniformly charged with a linear density of charge λ, at a point P located on the vertical line that passes at the middle of the rod as shown in Figure 4. The first step is to divide the distribution into elements of charge dq.

In order to do this we divide the length L into elements of length dx; each of these elementary lengths has a charge dq that produces a potential dV at point P. Thus, let's consider the element dx indicated in the Figure. (Notice, however, that dx is a differential element of charge located at any point on length L and, therefore, at a varying distance from P; elements of length or elements of charge at the extremities of a distribution of charge must be avoided because they usually lead to errors as, for example, considering constant the distance between these elements and point P.) This element will establish a potential dV at point P:

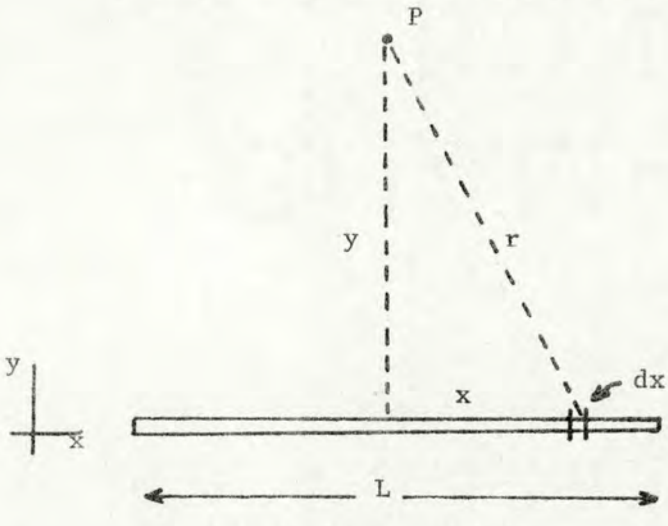


Fig. 4. (Notes VIII).

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\, dx}{(x^2 + y^2)^{1/2}}$$

(Notice that dq was replaced by $\lambda\, dx$; if λ were not given, dq could be replaced by $q/L \cdot dx$ which is the same thing because $q/L = \lambda$ by definition; however, it would be a mistake to replace dq by $\lambda\, q/L \cdot dx$.)

At this point, we can obtain the potential V established by the rod at point P by integrating dV from $-L/2$ to $+L/2$, or, from 0 to $L/2$ and multiplying the integral by 2; it would be wrong, however, to use $L/2$ and L or 0 and L as integration limits because, in this case, the origin would be at the extremities of the rod. Thus:

$$V_P = \int dV = 2\int_0^{L/2} \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{(x^2 + y^2)^{1/2}}$$

$$V_P = \frac{\lambda}{2\pi\varepsilon_0}\int_0^{L/2} \frac{dx}{(x^2 + y^2)^{1/2}} = \frac{\lambda}{2\pi\varepsilon_0} \ln\left[x + (x^2 + y^2)^{1/2}\right]_0^{L/2}$$

(The integral $\int \frac{du}{(u^2 + a^2)^{1/2}} = \ln[u + (u^2 + a^2)^{1/2}] + C$ was obtained in an integral table; it is neither an immediate integral nor easily solved by substitution of variables.)

$$V_P = \frac{\lambda}{2\pi\varepsilon_o} \ln[\ L/2 + (\ L^2/4 + y^2)^{1/2} - y]$$

$$= \frac{\lambda}{2\pi\varepsilon_o} \ln[\ L/2 + \frac{1}{2}\ (\ L^2 + 4y^2)^{1/2} - y]$$

$$= \frac{\lambda}{2\pi\varepsilon_o} \ln[\ \frac{L/2 + (\ L^2 + 4y^2)^{1/2}\ /\ 2}{y}]$$

$$= \frac{\lambda}{2\pi\varepsilon_o} \ln[\ \frac{L + (\ L^2 + 4y^2)^{1/2}}{2y}]$$

Thus, $V_P = \frac{\lambda}{2\pi\varepsilon_o} \ln[\ \frac{L + (\ L^2 + 4y^2)^{1/2}}{2y}]$ is the expression of the potential V at a point P located on the median (V is a function of y). From this expression we can calculate $\vec{E}$ at point P using the relation $\vec{E}$ = -grad V. In this case:

$$\vec{E} = -[\ \frac{\partial}{\partial x} V(y)\ \vec{i} + \frac{\partial}{\partial y} V(y)\ \vec{j} + \frac{\partial}{\partial z} V(y)\ \vec{k}\ ]$$

As V is a function of y only:

$$\vec{E} = -\frac{\partial}{\partial y} V(y)\ \vec{j} \qquad \text{or} \qquad Ey = -\frac{d}{dy} V(y)$$

$$Ey = -\frac{d}{dy}\ (\ \frac{\lambda}{2\pi\varepsilon_o} \ln[\ \frac{L + (\ L^2 + 4y^2)^{1/2}}{2y}])$$

$$Ey = -\frac{\lambda}{2\pi\varepsilon_o}\left[\frac{2y}{L + (L^2 + 4y^2)^{1/2}} \cdot \frac{1/2(L^2+4y^2)^{-1/2} 8y\ .\ 2y - 2[L+(L^2+4y^2)^{1/2}]}{4y^2}\right]$$

(Remember that $d\ (\ \ln u) = u^{-1}\ du$) In order to simplify this expression let's call $(L^2 + 4y^2)^{1/2} = k$:

$$Ey = -\frac{\lambda}{2\pi\varepsilon_o}\left[\frac{1}{L+k}\cdot\frac{4y^2k^{-1}-(L+k)}{y}\right]$$

$$Ey = -\frac{\lambda}{2\pi\varepsilon_o}\left[\frac{\frac{k}{L+k}\cdot 4y^2k^{-1} - \frac{k}{L+k}\cdot(L+k)}{ky}\right]$$

$$Ey = -\frac{\lambda}{2\pi\varepsilon_o}\left[\frac{\frac{4y^2}{L+k} - k}{ky}\right] \qquad \text{but} \quad 4y^2 = k^2 - L^2$$

$$Ey = -\frac{\lambda}{2\pi\varepsilon_o}\left[\frac{\frac{k^2-L^2}{L+k} - k}{ky}\right] = -\frac{\lambda}{2\pi\varepsilon_o}\left[\frac{k-L-k}{ky}\right]$$

$$Ey = \frac{\lambda L}{2\pi\varepsilon_o ky} \qquad \text{or} \qquad Ey = \frac{q}{2\pi\varepsilon_o y}\cdot\frac{1}{(L^2+4y^2)^{1/2}}$$

since $\lambda = q/L$ and $k = (L^2 + 4y^2)^{1/2}$

This last expression gives the magnitude of the electric field $\vec{E}$ at a point P on the median. (Notice that this result is the same obtained with the expression $\vec{E} = \int d\vec{E}$; see problem 27-15 of Halliday-Resnick) Such a result was obtained from V but this was possible because we knew V as a function of V. A common mistake in the calculation of $\vec{E}$ from V is the following: V is known at a specific point (not a generic point) and $\vec{E}$ is determined simply by multiplying V by a distance. <u>This is wrong because instead of knowing the function V, what is known is the value of this function at a certain point. The relation between $\vec{E}$ and V is a relation between functions</u>.

In order to complete the study of the preceding topics, you must read Chapter 29 of Halliday-Resnick. Carefully examine the examples and solve some of the problems proposed at the end of this Chapter (e.g., 18, 19, 20).

The last section of these Notes is designed to provide some complementary notions about the magnetic potentials.

IV - THE SCALAR MAGNETIC POTENTIAL AND THE VECTOR POTENTIAL

(a) The Scalar Magnetic Potential. In the electric field, the potential difference between any two points A and B was defined in terms of the line integral of the vector $\vec{E}$ along a path of integration connecting these points:

$$V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dl}$$

Notice that $V_B - V_A$ does not depend on the path because the electrostatic field is a conservative field, i.e., a field where the work required to displace a charge q from one point to another does not depend on the path taken between those points; the potential difference between two points, as we said before, is exactly this work per unit of charge. Notice also that, when $V_A = V_B$, $\oint \vec{E} \cdot \vec{dl} = 0$, i.e., the integral of $\vec{E}$ along a closed path is always zero in an electrostatic field. This result provides additional support for the assertion that the electrostatic field is conservative (a field of forces is conservative when the work done by the force on a particle describing a closed path is zero).

By analogy, it is possible to define a magnetic potential difference in terms of the line integral of the vector $\vec{B}$ along the path connecting two points. However, we have already seen that usually $\oint \vec{B} \cdot \vec{dl} \neq 0$ because according to the Ampère-Maxwell's law

$$\oint \vec{B} \cdot \vec{dl} = \mu_o \left(i + \varepsilon_o \, d\phi_E/dt\right)$$ which means that the magnetic field $\vec{B}$ is nonconservative.* But in the case of the magnetostatic field $(d\phi_E/dt = 0)$ it is possible to say that at regions external to a conductor where $i = 0$, $\oint \vec{B} \cdot \vec{dl} = 0$ and the field is conservative. In this case, it is possible to define a difference of scalar magnetic potential:

$$V_m = V_B - V_A = -\frac{1}{\mu_o} \int_A^B \vec{B} \cdot \vec{dl}$$

As in the electrostatic field, the relation $\vec{B} = -\mu_o \,\mathrm{grad}\, V_m$ is true. From this relation follows that the component of $\vec{B}$ along an arbitrary direction ℓ is given by: $\vec{B} = -\mu_o \dfrac{\partial V_m}{\partial \ell}$

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*When studying time dependent fields we will see that electric fields associated with a changing magnetic flux are also nonconservative because $\oint \vec{E} \cdot \vec{dl} \neq 0$ ($\oint \vec{E} \cdot \vec{dl} = - d\phi B/dt$, Faraday-Lenz's law)

However, it is difficult to find a physical interpretation for the magnetic potential difference, similar to that one used for the electric potential difference, because it does not represent the work required to displace any real entity along a path between A and B. There are no magnetic monopoles that could be displaced between A and B.

In addition, contrarily to the electrostatic case where the large usefulness of V lies on its path independence, which is related to the fact that

$\oint \vec{E} \cdot \vec{dl} = 0$, $\oint \vec{B} \cdot \vec{dl} = 0$ is not generally true. Thus, the magnetic potential V_m is a concept of very limited usefulness.

(b) The Vector Potential.* There is a theorem in vector analysis according to which any field vector $\vec{F}$ that does not have divergence is equal to the curl of some other field vector $\vec{A}$; that is, if

$$\vec{\nabla} \cdot \vec{F} = 0 \qquad \text{then} \qquad \vec{F} = \vec{\nabla} \times \vec{A}.$$

In other words, if the field vector $\vec{F}$ has no divergence then another field vector $\vec{A}$ may, in principle, be found in such a way that the curl of $\vec{A}$ is equal to $\vec{F}$. This vector $\vec{A}$ is called the vector potential.

In the case of the magnetic field, we have already seen that $\oint \vec{B} \cdot \vec{ds} = 0$ which means that $\vec{\nabla} \cdot \vec{B} = 0$ and therefore, a vector potential exists. However, we will not go into further details because this potential is relevant only in an advanced course in electromagnetic theory. In the case of the electric field, on the other hand, in general

$\oint \vec{E} \cdot \vec{ds} = q/\epsilon_o$ and $\vec{\nabla} \cdot \vec{E} = q/\epsilon_o$. Thus, the curl of $\vec{E}$ is different from zero and the vector potential does not exist.

NOTE: This section (b) was included in these Notes just to remark that besides the scalar potential there is also a vector potential.

III A CONCEPT MAP FOR POTENTIAL

To close these Notes a concept map for potential is presented in Figure 5. As usual, this is a map, and not the map. It is designed to provide some sense of order and to give a general view of the content of this unit.

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*This topic will not be asked in tests because it requires advanced concepts of vector analysis.

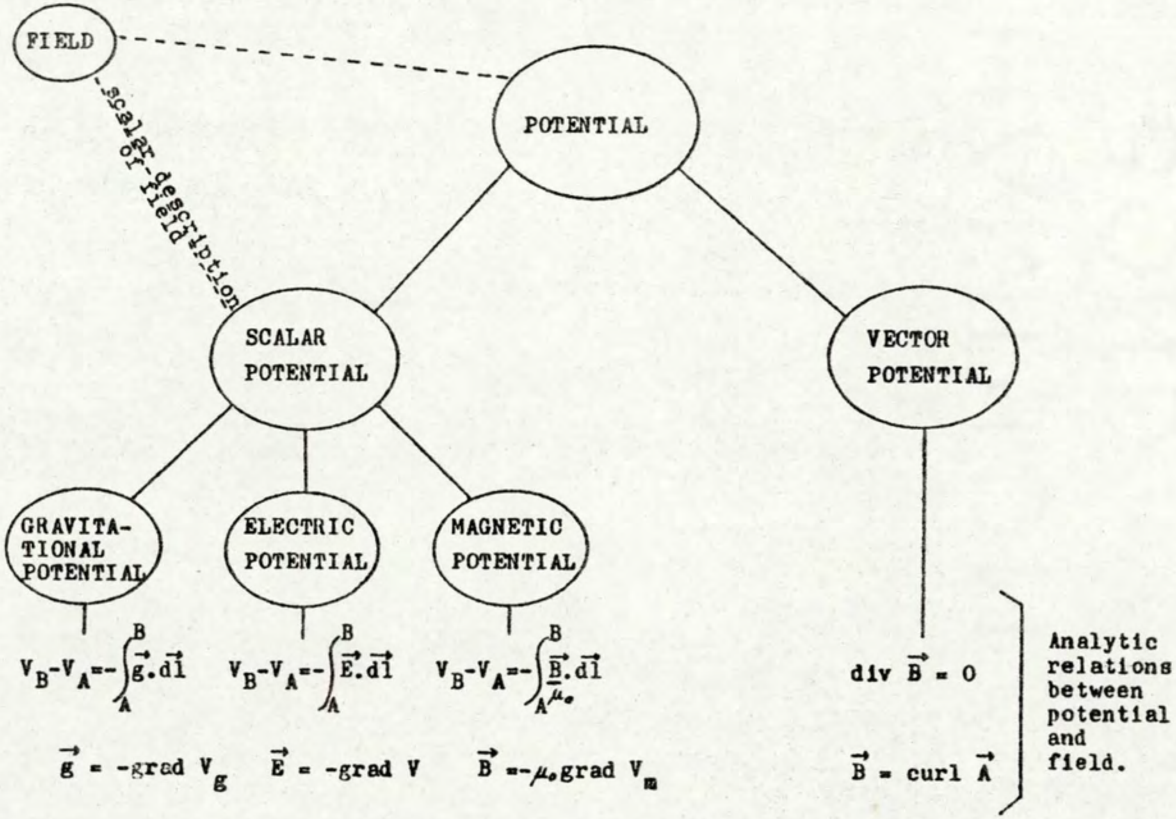

Fig. 5. (Notes VIII). A Concept Map for Potential.

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APPENDIX VII

EXAMPLE OF "NOTES" (III)

EXAMPLE OF "NOTES" (III)

Institute of Physics, UFRGS
Physics II - 1976

NOTES XVII
(Preliminary Version)
MAXWELL'S EQUATIONS

I - INTRODUCTION

As a matter of fact, we have always been dealing with Maxwell's Equations, either directly when they were introduced in Unit III and when we used them to calculate electric and magnetic fields or, indirectly, when we studied phenomena described by them. Now, however, we will look at them under a global point of view, i.e., focusing on all of them at the same time.

Since this is the last unit, a general review of the course will also be made in the light of Maxwell's Equations and of the general concept map of electromagnetism introduced at the beginning of the course.

II BASIC EQUATIONS OF CLASSICAL PHYSICS*

Before discussing Maxwell Equations once more, let's see what position they occupy within the content of classical physics. The following equations are the basic equations of classical physics:

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*Adapted from "All of Classical Physics," Section 18-3, Vol. II, The Feyman Lectures on Physics.

1) $\varepsilon_o \oint \vec{E} \cdot d\vec{s} = q$ — Gauss's Law for Electricity

2) $\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$ — Faraday-Lenz Law

3) $\oint \vec{B} \cdot d\vec{s} = 0$ — Gauss's Law for Magnetism

4) $\oint \vec{B} \cdot d\vec{l} = \mu_o(i + \varepsilon_o \frac{d\phi_E}{dt})$ — Ampère-Maxwell's Law

5) $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ — Lorentz's Force

6) $\vec{F} = \frac{d\vec{p}}{dt}$ — Second Newton's Law

$(p = \frac{mv}{(1 - v^2/c^2)^{1/2}}$ — Einstein's relativistic correction)

7) $\vec{F} = - G \frac{m_1 m_2}{r^2} \vec{u}_r$ — Newton's Law of Universal Gravitation

Basically, these equations represent the fundamentals of classical physics, that is, the physics that was known around 1905. These equations describe all classical physics.

First, we have Maxwell's Equations which give us information about electric and magnetic fields. However, these equations don't say anything about the action of these fields on electric charges. This is the concern of the next equation, the Lorentz Force, which says that these fields exert forces on electric charges. But, knowing how to calculate these forces doesn't mean too much, unless we know what happens when a force acts on something. That is, we need a law of motion. This law, discovered by Newton, says that the force is equal to the rate of change of the momentum. (With Einstein's correction, relativistic effects are also included in this law.)

As in terms of classical physics there are only two fundamental types of force, namely, the electromagnetic and gravitational forces (the nuclear force belongs to the domain of modern physics), the list of equations will not be complete unless it includes a law of force for the gravitational interaction. This law is Newton's Law for the universal gravitation.

At this point, you should realize that, in the same way that in the first unit we emphasized that the number of key concepts of classical physics was quite small, we are now emphasizing that the number of basic equations is also relatively small. That is, as most of classical concepts are subordinated to a reduced number of key concepts, most "formulas" that you know are derived directly or indirectly from a few basic equations. Isn't it surprising that physics is so simple when seen in the light of its key concepts and fundamental equations?

But, what about Maxwell's Equations? Well, from what was said, we can infer that they are not only the basic equations of the electromagnetism but constitute the majority of the basic equations of classical physics as well. Isn't this surprising too?

III- STATICS VERSUS DYNAMICS IN ELECTROMAGNETISM*

Although we started this course at a very general level and introduced Maxwell's Equations at the beginning, later on, in several opportunities we

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*Adapted from "What is True for Statics is False for Dynamics," Section 15-6, Vol. II, The Feynman Lectures on Physics.

focused on particular cases, i.e., without general validity. This section is designed to distinguish once more between the general and the particular. By and large, what is valid for static electromagnetism is not valid in electrodynamics. This is summarized in the following Table:

TABLE 1 (Notes XVII)

Valid Only for Static Fields	Always Valid
$F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2}$ (Coulomb's Law)	$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ (Lorentz Force) $\varepsilon_o \oint \vec{E} \cdot \vec{ds} = q^*$ (Gauss's Law for electricity)
$\oint \vec{E} \cdot dl = 0$ $\vec{E} = -\text{grad } V$ For conductors, $E = 0$, $Q = CV$ $V = \text{constant}$	$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}^*$ (Faraday's Law) $\vec{E} = -\text{grad } V - \frac{\partial \vec{A}}{\partial t}$ ($\vec{A}$ is the vector potential) In a conductor, $\vec{E}$ causes electric currents
$\oint \vec{B} \cdot \vec{dl} = \mu_o i$ (Ampère's Law) $dB = \frac{\mu_o i}{4\pi} \frac{dl \sin \theta}{r^2}$ (Biot-Savart's Law)	$\oint \vec{B} \cdot \vec{dl} = \mu_o (i + \varepsilon_o \frac{d\phi_E}{dt})^*$ $\oint \vec{B} \cdot \vec{ds} = 0^*$ (Gauss's Law for Magnetism)
$U = \frac{1}{2} CV^2 + \frac{1}{2} Li^2$	$U = \int (\frac{1}{2} \varepsilon_o E^2 + \frac{1}{2} \frac{B^2}{\mu_o}) dv$

* = Maxwell's Equations.

In this Table, Maxwell's Equations are indicated with an asterisk and, as we could expect, all of them are in the right side of the Table, i.e., they are always valid. What is in the left side of the Table is true for static electromagnetic phenomena but false for dynamic electromagnetic phenomena. Coulomb's law and Ampère's law, for example, are valid for static fields, but in electrodynamics these laws must be substituted for those in the right side of the Table, and so on.

IV - BASIC PHENOMENA DESCRIBED BY MAXWELL'S EQUATIONS

In this section, a summary of the basic phenomena described by each of Maxwell's Equations will be made.

IV.1 - Gauss's Law for Electricity: $\oint \vec{E} \cdot \vec{ds} = q/\varepsilon_0$

This law describes the electric charge and the electric field that it produces. It says, essentially, that in an electric field the electric flux through any closed surface is proportional to the net charge enclosed by such a surface. That is, the law says simply that the electric flux depends only on the internal charges in relation to the surface under consideration. (By the way, this law does not say that the field $\vec{E}$ also depends only on internal charges.)

From this law we can derive Coulomb's law ($F = 1/4\pi\varepsilon_0 \cdot q_1 q_2/r^2$) which describes the attraction and repulsion between point charges. With Gauss's law we can also show that any excess of charge placed on an insulated conductor resides entirely on its outer surface.

Gauss's law for electricity also says that, in general, the lines of force of the electric field $\vec{E}$ are not closed because usually $\oint \vec{E} \cdot \vec{ds} \neq 0$ which means that there are points where lines of force "are born" (positive charges) and points where they "die" (negative charges). In other words, there is a "source" of $\vec{E}$ if q is positive and a "sink" of $\vec{E}$ if q is negative; if $q = 0$, the flux of $\vec{E}$ over the surface is also zero.

This law allows the calculation of electric fields either directly, when depending on the conditions of symmetry of the problem, $\vec{E}$ can be put outside the integral symbol, or, indirectly, through Coulomb's law.

As far as static fields are concerned, both the phenomena and the problems are described and formulated in terms of the concepts of force, field, flux, and potential. Considering that Gauss's law provides an expression for the force (Coulomb's law) and allows the calculation of the flux, of the field and of the potential (because it can be obtained from $\vec{E}$), the previous statement saying that this law describes the electric field generated by electric charges is justified.

On the other hand, in the case of induced electric fields, Gauss's law is not useful because these fields are associated with a variation of the magnetic flux and not directly with electric charges. If $q = 0$, Gauss's law would say that $\oint \vec{E} \cdot \vec{ds} = 0$ which would mean that the lines of force of the induced electric field are closed, i.e., contrarily to the case of the electric field associated with electric charges, they will have no beginning and no end. Indeed, these lines of force are closed, but with Gauss's law we would stop at this point. That is, we could not, for example, calculate the induced electric field (which can be done with Faraday's law).

IV.2 - Gauss's Law for Magnetism: $\oint \vec{B} \cdot \vec{ds} = 0$

This law describes the magnetic field. As a matter of fact, it describes a fundamental feature of the magnetic field and, for that reason it does not provide as much information as its counterpart for electricity. This fundamental feature is that isolated magnetic poles do not exist.

In other words, this law says that the lines of force of the magnetic field are closed and the flux of $\vec{B}$ through any closed surface is always zero.

Comparing this law to the similar one for electricity, we can say that in magnetism there is no counterpart to the free charge q in electricity. The simplest magnetic structure known in magnetism is the magnetic dipole which is characterized by its magnetic moment $\bar{\mu}$.

As a consequence, the "Coulomb's law for magnetism" and "the scalar magnetic potential" are not meaningful, except in rough approximations. In addition, the fact that the integral $\oint \vec{B} \cdot \vec{ds}$ is always equal to zero makes this law useless to calculate $\vec{B}$.

IV.3 - Ampère-Maxwell's Law: $\oint \vec{B} \cdot \vec{dl} = \mu o(i + \varepsilon o \frac{d\phi_E}{dt})$

This law describes the magnetic effect of an electric current or of time varying electric field. In words, we could say that this law states that a magnetic field can be produced by an electric current or by the time variation of an electric field.

Not considering time varying fields, the equation of this law is reduced to $\oint \vec{B} \cdot \vec{dl} = \mu o\ i$ which is known as

Ampère's law and describes the production of magnetic field by an electric current. It says that in a static magnetic field the integral of $\vec{B}$ over a closed linear path is proportional to the net current that passes through the area bounded by the closed path. Thus, the integral depends only on the currents that are internal to the path of integration. (By the way, the law does not say that $\vec{B}$ also depends only on internal currents.)

With Ampère's law we can calculate magnetic fields produced by steady currents when the conditions of symmetry of the problem allow us to put $\vec{B}$ outside the integral symbol. (When this is not possible, Biot-Savart's law must be used.)

The term $\varepsilon_o d\phi_E/dt$ which was introduced by Maxwell says that a magnetic field may also be produced by a time varying electric field (this phenomenon is the symmetrical counterpart of Faraday's law of induction). For example, in a capacitor that is being charged or discharged the electric field between the plates is changing with time (in the RC experiment we have seen that V is a function of t): having a very sensitive instrument we could detect the presence of a magnetic field between the plates as if they were connected by a wire carrying an electric current.

This term, proposed by Maxwell, is known as displacement current and allows us to keep the idea of continuity of the electric current. For example, in an AC circuit containing capacitors, it is possible to show that the magnitude of the displacement current between the plates of the capacitors is exactly the same of the conduction current that passes through the other components of the circuit.

Considering only displacement currents, we have

$$\oint \vec{B} \cdot \vec{dl} = \mu_o \, i_d = \mu_o \, \varepsilon_o \, \frac{d\phi_E}{dt}$$

which describes one of the most important electromagnetic phenomena, i.e., the production of a magnetic field by a changing electric field. Depending on the conditions of symmetry of the problem, this equation can be used to calculate induced magnetic fields.

The Maxwell-Ampere's law also says that, in general, the magnetic field is nonconservative because $\oint \vec{B} \cdot \vec{dl} \neq 0$. This fact implies that the concept of scalar potential is not useful in this case, that is, the concept of scalar magnetic potential is not meaningful.

IV.4 - <u>Faraday - Lenz's Law:</u> $\oint \vec{E} \cdot \vec{dl} = - \frac{d\phi_B}{dt}$

In the same way that the equation $\oint \vec{B} \cdot \vec{dl} = \mu_o \, \varepsilon_o \, d\phi_E/dt$ describes the production of a magnetic field by a changing electric field, the equation $\oint \vec{E} \cdot \vec{dl} = - d\phi_B/dt$ describes the production of an electric field by a changing magnetic field. This is also one of the most important electromagnetic phenomena.

The integral $\oint \vec{E} \cdot \vec{dl}$ is called electromotive force and is represented by the symbol $\mathcal{E}$. Many times the Faraday-Lenz's law is written simply as $\mathcal{E} = - d\phi_B/dt$ and the fundamental phenomenon which is the production of an electric field by a changing magnetic field is often overlooked.

The minus sign which appears in the equation indicates that the effect of the induced electromotive force (i.e., of the induced electric field) always opposes the change that produced it. (This effect is just a consequence of the law of conservation of energy.) For example, when the magnetic flux through a coil is changing with time an electromotive force is induced in the region where the coil is, and, in turn, will cause the appearance of an induced current in the coil. The magnetic field of this current will have such a direction that opposes the change of the magnetic field that produced the induced electromotive force (if not, it would reinforce it and increase the induced current indefinitely).

Faraday's law not only says that an electric field is produced by a changing magnetic flux, but also that the magnitude of the induced field depends on how much and how fast the magnetic flux changes, that is, it depends on the rate of change of the magnetic flux. Considering that $\oint \vec{E} \cdot \vec{dl} \neq 0$, this law also says that the induced electric field is non-conservative and the concept of potential is not applicable to such a field.

In this section we tried to summarize the basic phenomena described by each of Maxwell's Equations. It is clear that several phenomena were overlooked, however, a careful analysis will show that directly or indirectly they are also described by these equations.

V SUMMARY OF THE STUDY OF ELECTROMAGNETISM: EQUATIONS, CONCEPTS, SEQUENTIAL ORGANIZATION, AND GENERAL CONCEPT MAP

V.1 - Equations:

Obviously, the basic equations studied in this course were Maxwell Equations. However, in addition, the following equations were also emphasized:

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \qquad \text{(Coulomb's law)}$$

$$dB = \frac{\mu_o i}{4\pi} \frac{dl \sin\theta}{r^2} \qquad \text{(Biot-Savart's law)}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \text{(Lorentz's Force)}$$

V.2 - Concepts:

The following concepts formed the conceptual basis of the study of electromagnetism in this course:

Force Field Potential Flux Energy Induction Changing Field	Superordinate concepts: each of them applicable to both the electric and magnetic cases (with the possible exception of potential). As a matter of fact, the concept of changing field could be considered subordinated to the general concept of field. But, given its relevance for the description of fundamental electromagnetic phenomena and considering that it is applicable to both instances of the electromagnetic field, this concept was included among the superordinates.

Obviously, many other concepts were also used in the description of electromagnetic phenomena studied in this course, but they are concepts specific to electricity or to magnetism or are subordinated to the above concepts. It is the case of current, resistance, electromotive force, capacitance, inductance, permeability, and others. Electric charge seems to be a special case because, on one hand, it is a very specific concept, but, on the other hand, the electric charge is at the roots of all electromagnetic phenomena.

V.3 - Sequential Organization:

The study of electromagnetism, from Units I to XVII, was sequentially organized according to the scheme shown in Figure 1.

As shown in this Figure, the course started at a very general level dealing with physics and physical concepts. After this, it became a little more specific when just two concepts were emphasized: forces and fields. Then, on one hand, the particularization continued because the focus was only on the electromagnetic interaction, but, on the other hand, the subject was approached at a general level again and Maxwell's Equations were introduced almost qualitatively.

At this point, i.e., after the introduction of the basic phenomena and equations, Maxwell Equations started to be "unpacked" to show how they describe electromagnetic phenomena and how they can be used to calculate electric and magentic fields. In the following units, the topics became more and more detailed, but the basic concepts and the Maxwell's Equations involved in each unit were always emphasized in order to distinguish between the general and the particular. This was done until all phenomena and equations had been studied with a degree of detail compatible with the scope of the course. At that point, however, an integrative review of all topics studied in the course was necessary and this is the goal of this unit.

V.4 - A General Concept Map of Electromagnetism:

At the end of unit III we introduced a concept map of electromagnetism and we said that this map would serve as a frame of reference for the entire

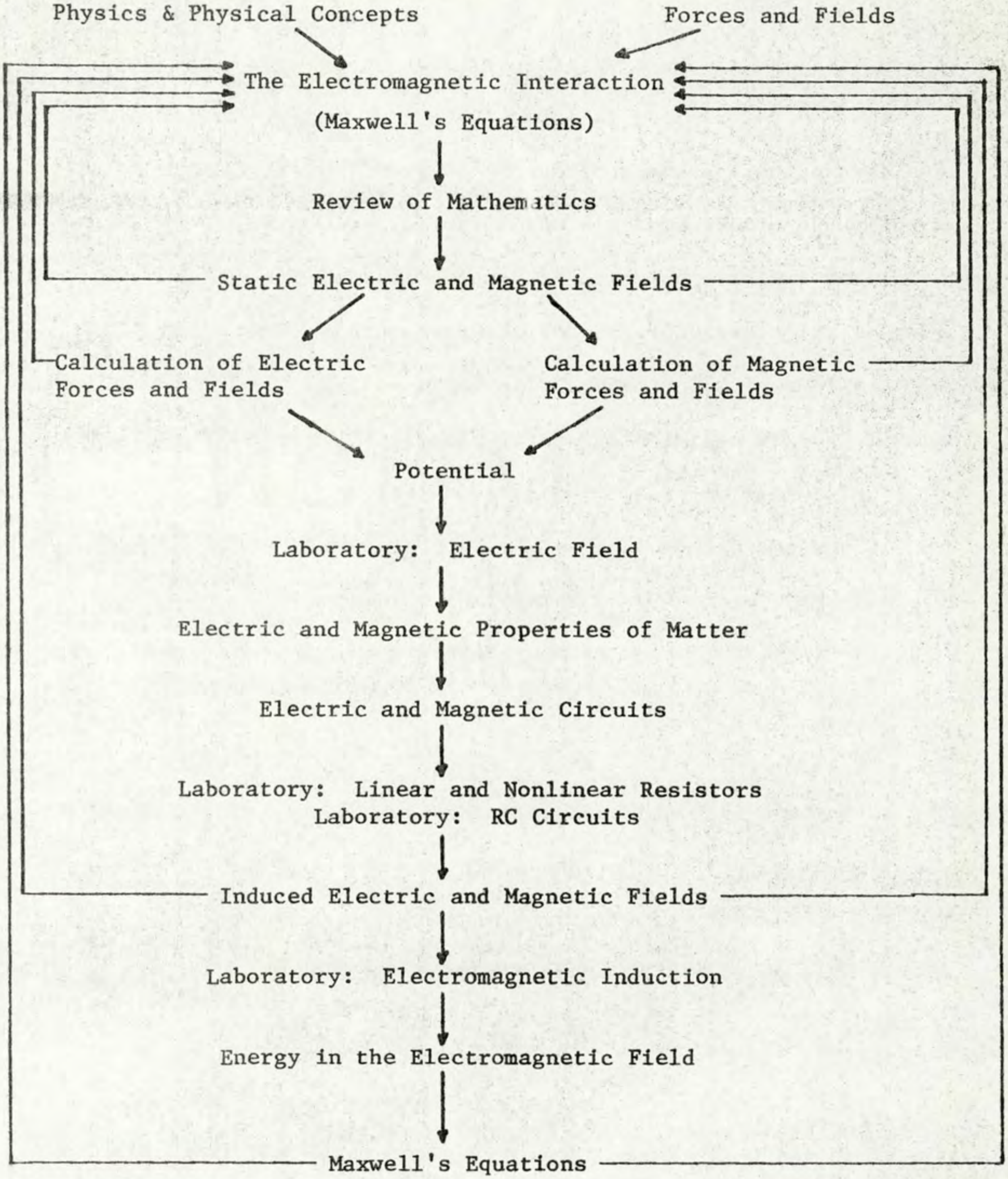

Fig. 1. (NOTES XVII)

course. Now, we will retake this map (see last page of these Notes) and justify such a statement.

In this map, the four Maxwell's Equations are inside rectangles. The concepts used in the description of the electromagnetic phenomena are inside ellipses. In the central column, also inside ellipses but in capital letters, are the superordinate concepts, the key concepts. (By the way, the concept of field is at the center of the map.) The laws of Biot-Savart and Coulomb which had a relevant role in the calculation of the electric and magnetic fields are also inside ellipses. (As a matter of fact, the equation of the Lorentz Force should be also included in the map.)

The map tries to emphasize the symmetry or analogy between electricity and magnetism as instances of electromagnetism. In order to avoid an excessively complicated map, not all possible associations were represented by connecting lines.

Throughout the course, when static phenomena were studied we were at the upper half of the map; when we discussed induced fields we were in the lower half; when we spoke only about electric fields we were at the left side; and when we were dealing only with magnetic fields we were at the right side.

VI - DIFFERENTIAL FORM OF MAXWELL EQUATIONS

This section was included in these Notes in order to give you an idea about the mathematical treatment that would be given to Maxwell's Equation in a more advanced course in electromagnetism. In case you are interested, the subject of this section may also be found in "Supplementary Topic V" of Halliday-Resnick.

In this course we have used Maxwell's Equations in its integral form.

$$\mathcal{E}_o \oint \vec{E} \cdot \vec{ds} = q$$

$$\oint \vec{B} \cdot \vec{ds} = 0$$

$$\oint \vec{B} \cdot \vec{dl} = \mu o(i + \mathcal{E}o \frac{d\phi_E}{dt})$$

$$\oint \vec{E} \cdot \vec{dl} = - \frac{d\phi_B}{dt}$$

The variables $\vec{E}$ and $\vec{B}$ which are usually the unknown variables appear in the integrands. Only in a few cases, depending on the conditions of symmetry, they can be placed outside the integral symbol. In this case, the problem of calculating $\vec{E}$ and $\vec{B}$ is easily solved, but, in more general problems, this cannot be done.

However, changing Maxwell's Equations from its integral form to its differential form, we will have equations applicable to <u>each point of the space</u> instead of integrals valid only in <u>regions of the space</u>. In this way, it will be possible to relate $\vec{E}$ and $\vec{B}$ at a certain point with the charge density and current density (which are microscopic quantities as we have seen) at this point.

In order to make such a transformation in the mathematical form of Maxwell's Equations, we have to use more advanced concepts of vector analysis, in particular, the operator

$$\vec{\nabla} = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

But, as this section has only a complementary and illustrative purpose, we will not go into the details on how to obtain the differential form of Maxwell's Equations. In addition, another system of units should be used instead of the MKS system.

Thus, we will just present the equations in its differential form:

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_o}$	(or, $\operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_o}$)	Gauss's law for electricity
$\vec{\nabla} \cdot \vec{B} = 0$	(or, $\operatorname{div} \vec{B} = 0$)	Gauss's law for magnetism
$C^2 \vec{\nabla} \times \vec{B} = \frac{\vec{j}}{\varepsilon_o} + \frac{\partial E}{\partial t}$	(or, $C^2 \operatorname{rot} \vec{B} = \frac{\vec{j}}{\varepsilon_o} + \frac{\partial E}{\partial t}$)	Ampère-Maxwell's law
$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$	(or, $\operatorname{rot} \vec{E} = -\frac{\partial B}{\partial t}$)	Faraday-Lenz's law

In this form, the constants that appear in Maxwell's Equations are ε_o and C^2 . ε_o can be experimentally determined by measuring the force between two point charges and using Coulomb's law. The constant $\varepsilon_o C^2$ that appears in the equation $\vec{\nabla} \times \vec{B} = \vec{j}/\varepsilon_o C^2$ (when $\partial E/\partial t = 0$) by measuring the force between two currents. Through these measurements, we obtain $C = 3 \cdot 10^8$ m/s . Maxwell carried out this calculation for the first time and said that "packages" of electric and magnetic fields should propagate with such a speed. He also called attention to the puzzling coincidence that this was the speed of light: "It is hard to avoid the inference that light consists of transverse oscillations of the same medium that is the cause of the electric and magnetic phenomena."

Maxwell made one of the great unifications of physics. Before him, there was light and there was electricity and magnetism. The last two were unified by the experimental work of Faraday, Oersted and Ampere. Then, all of a sudden, light was not "something else" but electricity and magnetism in their new form: small "packages" of the electric and magnetic field propagating throughout the space.

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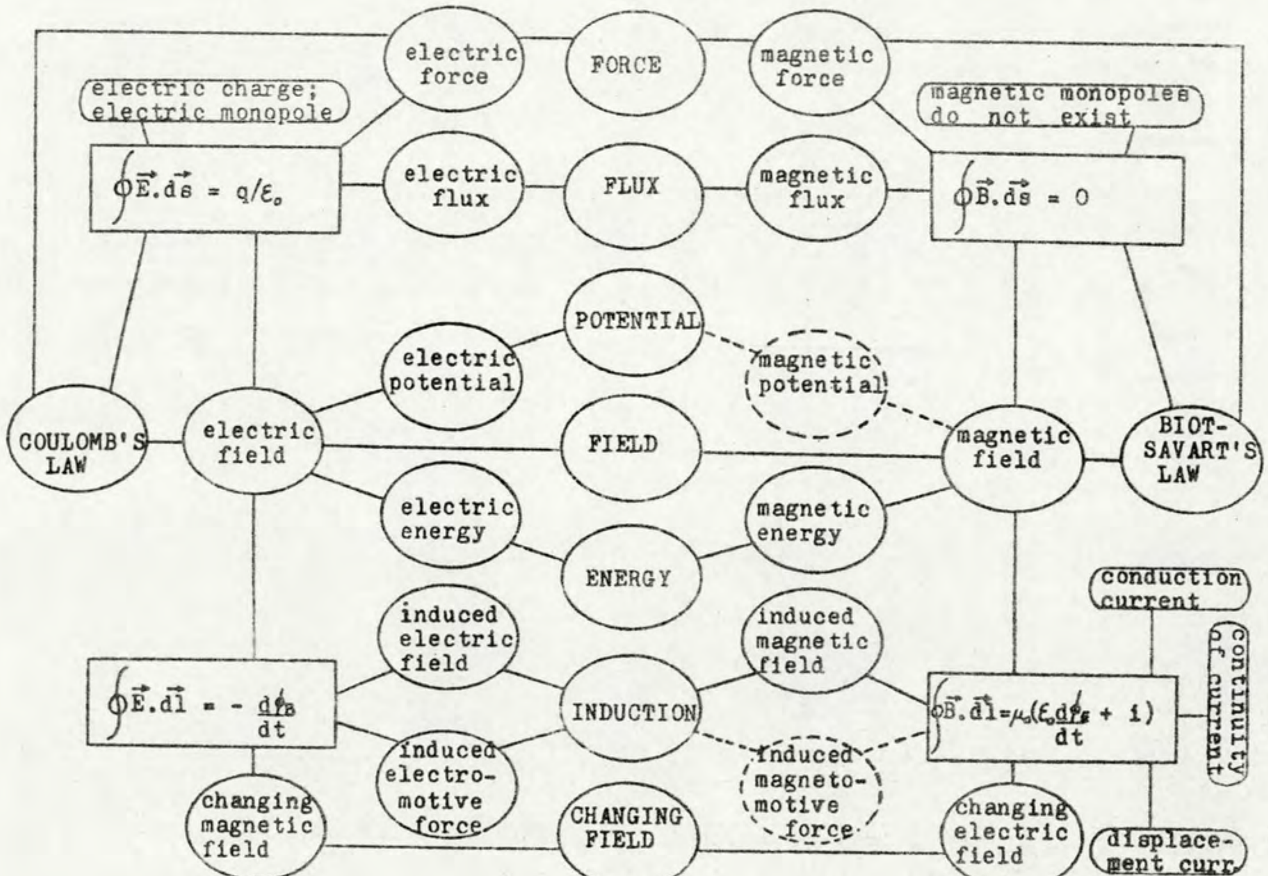


Fig. 2. (Notes XVII). A General Concept Map of Electromagnetism.

APPENDIX VIII

EXAMPLES OF UNIT-TESTS FOR THE PSI GROUPS

EXAMPLES OF UNIT-TESTS FOR THE PSI GROUPS

[(a)- Experimental Group]

IFUFRGS
Physics II - 1976

UNIT X - Test 2

Name: ______________________ Number: ______________________

Date: ______________________ Group: ______________________

1. Distinguish between insulators and conductors in terms of polarization and induced electric field.

2. A capacitor is charged by using a battery, which is then disconnected. A dielectric slab is then slipped between the plates. Describe qualitatively (saying if increases, decreases or remains constant) what happens to:

   (a) The real electric charge
   (b) The electric field
   (c) The potential difference
   (d) The capacitance
   (e) Explain under the atomic point of view what happened in (b).

3. A cylindrical capacitor consists of two coaxial cylindric tubes of radii R1 and R2 and length L (being $R1 < R2 < L$). Calculate its capacitance.

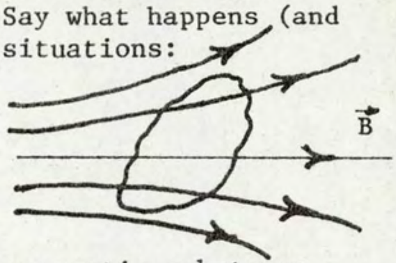
4. A 1.0 m length aluminum bar has a square cross-sectional area of 1.0 cm^2.

   (a) What is the value of the resistance when measured between the extremities of the bar ($\varrho = 2.8 \times 10^{-8} \Omega$ m)?
   (b) What should be the cross-sectional area of an iron bar ($\varrho = 1.0 \times 10^{-7} \Omega$ m) with the same length in order to have the same resistance of the aluminum bar?

5. What is the meaning of saying that a conductor is "non-ohmic"? Explain.

6. A small sample of a certain material is placed in a non-uniform magnetic field as indicated in the Figure. Say what happens (and explain why) to the sample in the following situations:

   (a) The material is paramagnetic
   (b) The material is diamagnetic
   (c) The material is ferromagnetic.



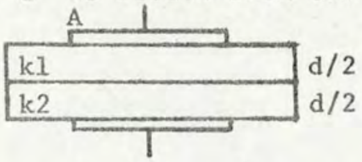
7. (a) Draw the magnetization curve for a ferromagnetic substance.
   (b) What is the hysteresis effect? Explain this effect in terms of the domain theory.

[(b) - Control Group]

IFUFRGS
Physics II - 1976

UNIT VII - Test 4

1. A parallel-plate capacitor with plates of area A separated by a distance d is filled with two different dielectrics as shown in the Figure. Calculate the capacitance of this capacitor, without regarding the arrangement as two capacitors in series.

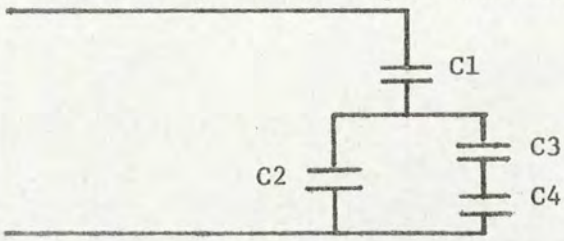


2. A capacitor is charged by using a battery, which is then disconnected. A dielectric slab is then slipped between the plates. Describe qualitatively (saying if increases, decreases, or remains constant) what happens to:

(a) The real electric charge; (b) The electric field;
(c) The potential difference; (d) The capacitance;
(e) Explain under the atomic point of view what happened in (b).

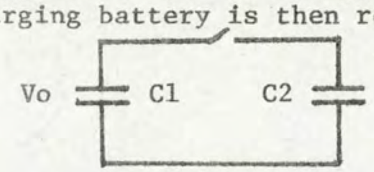
3. While a capacitor remains connected to a battery, a dielectric slab is slipped between the plates. Describe qualitatively what happens to the charge, the capacitance, the potential difference, the electric field, and the stored energy. Is work required to insert the slab?

4. Calculate the equivalent capacitance of the association. C1 = $5\mu F$



C2 = $5\mu F$
C3 = $5\mu F$
C4 = $10\mu F$.

5. A capacitor C1 is charged to a potential difference Vo. The charging battery is then removed and the capacitor is connected to an uncharged capacitor C2 as shown in the Figure.



(a) What is the final potential difference across the combination?
(b) What is the stored energy before and after the switch is thrown?

APPENDIX IX

THE CONCEPT ASSOCIATION TESTS

THE CONCEPT ASSOCIATION TESTS

Institute of Physics - UFRGS
Physics II - 1976 (1st Semester)

Name:______________________________ Number:_______________

Date:__________ Group:__________ Course:_______________

GENERAL INSTRUCTIONS

The data asked in the questionnaire below and in the tests that follow in the next pages will be used exclusively for research purposes. This is a research where we will try to investigate indirectly what you know about Physics II at the beginning, at the middle and at the end of the course. Thus, at the middle and at the end of the course you will be asked again to answer these tests to allow us to study the changes occurring in your answers.

We have done other research studies in previous semesters and we have always had a great amount of cooperation from the students. As this co-operation is vital for our research we expect to have yours too. Read the instructions of each test carefully and answer it with maximum seriousness. In these tests there is no right or wrong answer, just follow the instructions. By doing this you will be extremely helpful to us.

QUESTIONNAIRE
(Fill in All Items)

1.	Are you repeating Physics II?	[ ] Yes	[ ] No
2.	Do you work besides studying?	[ ] Yes	[ ] No
3.	If your answer was yes, approximately how many hours/week do you work?	______	
4.	How many courses are you taking this semester besides Physics II?	______	
5.	Did you repeat Physics I?	[ ] Yes	[ ] No
6.	What was your final grade in Physics I?	______	

WORD ASSOCIATION TEST

INSTRUCTIONS

This first test is designed to check how many words you are able to associate with a given concept in a certain period of time. We will give you a word meaning a physical concept and you will be asked to write down as many words associated with this concept as possible. These words should be from the field of physics, i.e., from the usual language of physics that you know from previous courses, from this course or from books where you have studied before or are studying now. We don't expect you to be able to fill in all the blanks in a given page of this test. What we expect is that you fill in as many blanks as possible. An example is given below:

TEMPERATURE

Temperature	Heat	Temperature	______
Temperature	Energy	Temperature	______
Temperature	Dilation	Temperature	______
Temperature	Specific Heat	Temperature	______
Temperature	Thermometer	Temperature	______
Temperature	______	Temperature	______
Temperature	______	Temperature	______
Temperature	______	Temperature	______
Temperature	______	Temperature	______
Temperature	______	Temperature	______

In each of the following pages you will find a single physical concept. Proceed as in the preceding example, writing beside this concept as many words as you can associate with it within the field of physics. Make sure of always thinking on the given concept and not on the words that you have already written. The concept before each blank is exactly for this reason.

You will have one minute per page. The teacher will tell you when to turn the page. Don't pass to the next page before you are told to do it.

[In the original, each of the next pages contained one of the following fifteen concepts: ELECTROMOTIVE FORCE, ELECTRIC RESISTANCE, INDUCTANCE, MAGNETIC FIELD, POTENTIAL, ENERGY, FORCE, ELECTRIC CURRENT, ELECTRIC FLUX, ELECTRIC FIELD, ELECTRIC CHARGE, MAGNETIC FLUX, TIME, WORK, CAPACITANCE. The concepts were given in this order which was randomly selected and displayed in each page as in the example.]

NUMERICAL CONCEPT ASSOCIATION TEST

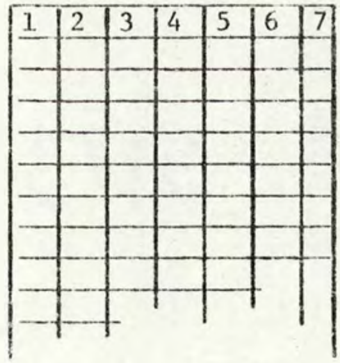
INSTRUCTIONS

This test is designed to verify to what extent you relate or associate certain physical concepts. Each of the following pages will contain a certain number of pairs of concepts. Beside each pair there is a numerical scale ranging from 1 to 7. In this scale, "1" corresponds to a high degree of relationship between the concepts (e.g., work and energy, force and acceleration) and "7" to almost no relationship (e.g., moment and volume). If you think that the concepts of a given pair are intimately related check number 1 in the scale. On the other hand, if in your opinion one concept of a pair has nothing to do with the other check number 7. These are the two extreme situations, the numbers between 1 and 7 reflect intermediate situations: the closer to 1, the greater the relationship; the closer to 7, the smaller the relationship.

Make sure not to skip lines. There is no time limit for this test.

[In the original, the 105 possible pairs between the 15 concepts of the preceding test were randomly ordered and displayed as illustrated below.]

Force - Potential
Work - Inductance
Force - Electric Charge
Time - Electric Resistance
Electric Charge - Electric Current
Time - Capacitance
Electric Current - Potential
. . . .
. . . .



GRAPHICAL CONCEPT ASSOCIATION TEST

INSTRUCTIONS

The rules of this test are quite simple. At the top of the next page you will find a list of concepts relevant to the study of electromagnetism which you are asked to display in a diagram, like a small map, according to the following rules:

1. More general concepts must be written inside rectangles.
2. Second level or intermediate concepts must be written inside circles or ellipses.
3. Least general or third level concepts must be just written without geometrical figures surrounding them.
4. The related concepts must be linked with lines of arbitrary size and shape.

Use your own judgment and make sure of using each concept only once. To avoid repetitions or omissions it is convenient to put a check on the concepts already used in the list.

ELECTRIC FIELD	ELECTRIC RESISTANCE	ELECTROMAGNETIC FIELD
ELECTRIC CURRENT	ELECTROMAGNETIC FORCE	DISPLACEMENT CURRENT
INDUCTANCE	CAPACITANCE	ELECTRIC ENERGY
MAGNETIC FLUX	MAGNETIC FORCE	MAGNETIC ENERGY
ELECTROMOTIVE FORCE	ELECTRIC FLUX	ELECTRIC CHARGE
MAGNETIC FIELD	ELECTRIC FORCE	ELECTRIC POTENTIAL
		ELECTRIC POWER

Using the rules given in the preceding page, make a diagram for these concepts in the space below:

[No Example was given in this Test]

APPENDIX X

EXAMPLE OF A LABORATORY-GUIDE FOR THE CONTROL GROUP

EXAMPLE OF A LABORATORY-GUIDE FOR THE CONTROL GROUP

Institute of Physics, UFRGS
Physics II - 1976

Name: ______________________ Number: ______________________

Date: ______________________ Group: ______________________

UNIT VI

EXPERIMENTAL STUDY OF A SIMULATED ELECTROSTATIC FIELD

INTRODUCTION

This is the first laboratory unit of this course. It is, essentially, a unit of application of what you have learned in previous units. You will have the opportunity to study, in practice, some of the topics already studied theoretically like electric fields, electric potential differences, lines of force, equipotential lines, and others. Besides, you will also have the opportunity to work with an instrument of measurement that has multiple purposes: the multimeter. In this experiment you will use the multimeter as a voltmeter.

PROCEDURE

In the following pages, you will find the objectives of the experiment and the suggested procedure to perform such experiment. After the experiment, when you feel that you are able to do what is stated in the objectives, present yourself to a proctor for an oral interview. At this occasion you must present him the results you obtained in this experiment.

Institute of Physics, UFRGS
Physics II - 1976

First Laboratory Experiment

EXPERIMENTAL STUDY OF A SIMULATED ELECTROSTATIC FIELD

OBJECTIVES

At the end of this experiment you must be able to:

1) Draw equipotential lines using the measured potential differences.
2) Draw lines of force of an electrostatic field when the equipotential lines are known.
3) Distinguish between lines of force and equipotential lines.
4) Identify the points of large and small intensity of an electrostatic field from its lines of force.
5) Say whether an electric field is uniform or not from its lines of force.
6) Determine the direction of an electric field from its equipotential lines.
7) Describe the experimental procedure necessary to study the electrostatic field existing in the vicinities of a charged conductor of arbitrary shape.

PROCEDURE

In the following pages you will find a programmed guide to perform the experiment. This guide contains questions that you must answer in the blanks. On the back of each page, you will find the answers to these questions. It is very important that you write all your answers to the questions of a given page before looking at the answer sheet. Once you have compared your answers with the answer sheet on the back of each page, correct your answers (if necessary) writing the right answer below yours and go to the next page. Obviously, sometimes you will have to decide about the correctness of your answer. As you proceed through the guide, you will realize that initially the questions are designed to provide a theoretical background for the experiment and, later on, they are concerned with the experiment itself. Always try to answer the questions without asking help from the teacher or proctors, and write your answer before checking the answer sheet.

[NOTE: In the original, the answers to each question are provided in the back of the same page, except the last ones which bear no answers.]

1. What do you understand by the electrostatic field surrounding a charged conductor?

2. Establish a fundamental difference between the electric and the gravitational field.

3. Is it possible to describe an electric field mathematically using a scalar function? If yes, say what function this is and whether it describes the field completely. If no, justify your answer.

4. As we have already seen, an electric field can be described by the electric potential function. Supposing that you can determine the potential in every point of an electric field, is it possible to draw lines of force of such a field? Is yes, explain how; if no, justify.

5. The following equipment is available to you: a small electrolytic tank, a multimeter, electrodes, a DC power supply, connecting wires, and conductors of various shapes and sizes. Draw a scheme showing how you will use this equipment to measure potential differences inside the liquid layer.

6. Connect the instruments according to the above scheme and ask the teacher or a proctor to check the connections before turning the power on.

7. What are you measuring when the terminal of the voltmeter is placed at a certain point inside the liquid layer?

8. Placing the terminal on different points, what is observed concerning the occurrence of identical measurements?

9. Does this experimental observation seem reasonable to you under the point of view of physics?

10. How will it be possible to draw an equipotential line of the electric field that you are testing with the terminal of the voltmeter?

11. Determine several different points with the same potential and draw on graph paper the corresponding equipotential line. Using this method draw several equipotential lines and some lines of force. If you did not attach any conductor to the extremities of the electrodes, they played the role of point charges and what you have represented graphically is the field of an electric dipole. The lines of force do not cross. Why?

12. To represent graphically a uniform field, repeat the previous procedure attaching two parallel metallic plane plates to the extremities of the electrodes. What do you observe concerning the equipotential lines and the lines of force in the region between the plates?

13. Place a metallic cylindrical tube between the parallel plates. Repeat the previous procedure. What do you observe concerning the potential inside the cylinder? Why?

14. Present to a proctor the graphical representations (in graph paper) of the electric fields obtained in steps 11, 12, and 13 above and be prepared to answer orally some questions on the objectives of the experiment.

APPENDIX XI

EXAMPLE OF A LABORATORY-GUIDE AND A LABORATORY-NOTES FOR THE EXPERIMENTAL GROUPS

EXAMPLE OF A LABORATORY-GUIDE AND A LABORATORY-NOTES FOR THE EXPERIMENTAL GROUPS

Institute of Physics, UFRGS
Physics II - 1976

Name: ______________________ Number: ______________________

Date: ______________________ Group: ______________________

UNIT IX

EXPERIMENTAL STUDY OF A SIMULATED ELECTROSTATIC FIELD

INTRODUCTION

This is the first laboratory experiment of this course. With this and the other three experiments that will follow on coming units we intend to give you an opportunity to acquire experimental abilities and familiarity with some instruments of measurement. But, above all, we intend to give you a better idea of the role of the laboratory in physics and of the interdependence between theory and experimentation, through the experimental study of a certain problem. We do not want just to illustrate topics already studied theoretically, neither give you a "recipe" to follow in order to obtain a predetermined result, i.e., a "right answer."

In accordance with such purposes, the instructions that you will receive will be intentionally brief in order to provide you the opportunity to find your own solutions to the problem. You will be quite free to plan, to carry out the experiment and to make your measurements. In this first experiment, the subject is static electric fields and we will provide you the necessary equipment to simulate such a type of fields and to study them experimentally.

OBJECTIVES

At the end of the experiment you must be able to:

1) Identify the basic question(s) under investigation as well as the basic phenomena and the key concepts involved in the experiment.

2) Draw lines of force of an electrostatic field when the equipotential lines are known.

3) Identify the points of large and small intensity of an electrostatic field from its lines of force.

4) Determine the direction of an electric field from its equipotential lines.

5) Describe the method used in the investigation of the basic question(s).

6) Report the answers (results) you found for the basic question(s) and discuss the importance and validity of these answers.

PROCEDURE

Read Notes "IX". They are designed to provide a theoretical background for the experiment and to pose the problem to be studied experimentally.

Identify the basic question(s) to be answered experimentally.

Identify also the basic phenomena and the key concepts involved in the basic question(s) to be investigated.

Plan and assemble the experimental setting necessary to data collection. Call the teacher or a proctor to provide you individual assistance if necessary or to check your experimental design. If the instruments of measurement are not yet familiar to you, ask for an explanation on how they work. The misuse of these instruments might cause permanent damage to them.

Record, analyze and interpret your data. Carry out the calculations and draw the graphs if necessary. Draw your conclusions

Present your results to a proctor and be ready to explain how you got them and discuss their importance and validity.

Institute of Physics, UFRGS
Physics II - 1976

NOTES IX
(Preliminary Version)

I - THEORY AND EXPERIMENTATION

In the first unit of this course, in Notes "I", we discussed the interdependence existing between theory and experimentation in physics. At that opportunity it was said:

> The scientist seeks to learn the 'truth' about nature. However, in physics we can never learn 'absolute truths' because physics is basically an experimental science; experiments are never perfect and, therefore, our knowledge of nature must be always imperfect. We can only state at a certain epoch in time the extent and precision of our knowledge of nature, with the full realization that both the extent and the precision will increase in the next epoch. Our understanding of the physical world has as its foundation experimental measurements and observations; on these are based our theories that organize our facts and deepen our understanding.
>
> Physics is not an armchair activity. The ancient Greek philosophers debated the nature of the physical world, but they would not test their conclusions, they would not experiment. Real progress was made only centuries later, when man finally realized that the key to scientific knowledge lay in observation and experiment, coupled with reason. . . .
>
> The mere accumulation of facts does not constitute good science. Certainly, facts are a necessary ingredient in any science, but facts alone are of limited value. In order to fully utilize our facts, we must systematize our information and discover how one event produces or influences another event. . . .
>
> When confronted with a set of facts, it is the scientist's task to find the simplest possible way to relate these facts one to another. A successful relationship is called a theory. An acceptable theory must account for all empirical information accumulated about the particular subject and, furthermore, it must be capable of predicting the results of any experiments that can be performed. (Frequently, a theory will predict effects that are beyond our capabilities to detect at present; in such cases, the tests must await the development of more sensitive techniques.) If any disagreement is found between theory and experiment, then the theory must be modified to account for the new information. Thus, theories evolve by successive refinements. . . . In its embryonic stages, a theory is often called a model. . . . (Excerpted from Physics and the Physical Universe, J.B. Marion)

Well, this is a laboratory unit and, as such, it must be at least coherent with the above discussion. Obviously, however, we must realize that several practical limitations will prevent us from proposing a real scientific experiment. (Besides, this is just an introductory physics course.) Instead, we will try to explore the interdependence between theory and experimentation by means of a very simple experiment. What we will do is to study electrostatic fields experimentally, based on the theory of the electrostatic field. Thus, we will start with a brief discussion about the theoretical background of the experiment followed by the identification of the problem. Next, we will proceed with some practical considerations and we will give you some suggestions. To perform the experiment is your responsibility.

II - THEORETICAL BACKGROUND

The electrostatic field, as we have seen before, may be described not only by a vector, the electric field intensity vector $\vec{E}$, but also by a scalar function, the scalar electric potential V. In addition, this field may be also described graphically or visualized through lines of force. These are three different ways of describing the same thing and, as such, one can expect that they are related. Indeed, this relationship exists and can be summarized as follows:

> The tangent to a line of force at any point gives the direction of the electric field vector $\vec{E}$ at that point. The lines of force are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of $\vec{E}$.

If $\vec{E}$ is known in every point of the field, the potential difference between any two points, or the potential at an arbitrary point, may be calculated through the equation

$$V_B - V_A = - \int_A^B \vec{E} \cdot \vec{dl}$$

Reciprocally, if V is known throughout a certain region, $\vec{E}$ may be calculated through the equation $\vec{E} = -\, \text{grad } V$. From this equation we infer that if we travel through an electric field along a straight line and measure V as we go, the rate of change of V with distance that we observe, when changed in sign, is the component of $\vec{E}$ in that direction. The minus sign implies that $\vec{E}$ points in the direction of decreasing V.

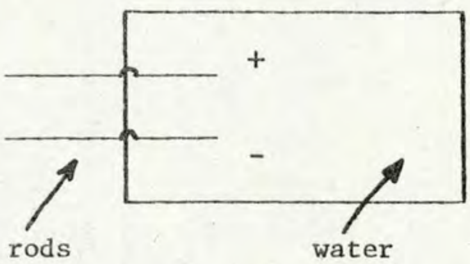
The lines of force and, consequently, the electric field vector $\vec{E}$, are perpendicular to the equipotential surfaces of the electric field.

These relationships provide the theoretical background necessary to the kind of experimental study of an electrostatic field to be done in this experiment: if the potential could be measured in a large number of points of an electric field, we could connect the points of same potential to obtain equipotential surfaces. Having these equipotential surfaces it would be possible to draw lines of force (perpendicular to the surfaces) and obtain the direction of the electric field. $\vec{E}$ would point in the direction of decreasing V and the density of lines of force would give information about the magnitude of this vector. Thus, we could, in principle, study in detail electrostatic fields due to arbitrary distributions of charge, e.g., the field of a dipole, the field of a capacitor or the field of a charged body of arbitrary shape.

III - CONSIDERATIONS OF PRACTICAL NATURE

In order to carry o· · the above outlined experiment, we immediately face a practical problem: the instruments of measurement (the voltmeter in this case) measure potential differences and not the potential at a certain point. In addition, these instruments work only when connected to an electric circuit through which an electric current passes. They measure the potential difference through the equation V = IR. However, if the field is static there is no motion of charges in such a field and, consequently, there is no electric current.

A way of overcoming this difficulty is to simulate an electrostatic field. This simulation can be achieved by placing the extremities of two



charged rods in a fine layer of chemically treated water (water from the faucet). An electric field will be established in the region surrounding the rods, however as the water layer will contain ions that will move under the action of his field, an electric current will be originated in the water. This current will be extremely weak but large enough to allow the use of a sensitive voltmeter to measure potential difference in the established field (which will be only slightly affected by the current). With these measures, equipotential lines, which will allow the drawing of lines of force and provide information about the field vector, might be determined. Attaching conductors of certain size and shape to the extremities of the rod that are inside the water, different field configurations can be simulated and experimentally studied.

IV - CONCLUSION

The necessary equipment to simulate several electrostatic field configurations will be available to you. Select some of them and study them experimentally. While performing the experiment, ask for individual assistance if necessary. At the end, present your results and conclusions to a proctor. The evaluation of this unit will be carried out through the presentation and discussion of these results.

Have in mind the fact that you will not be necessarily looking just for already known results. If you choose a well-known field configuration like the case of the dipole you, obviously, will get only already known results. But, if you select a distribution of charges with an unknown field configuration you will be getting new experimental results. In such a case, your results could be used to support the theory that served as basis of the experiment, or to provide evidences that this theory should be reformulated, or to establish its limits of validity.

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APPENDIX XII

RANDOMLY SELECTED EXAMPLES OF CONCEPT MAPS DRAWN BY

THE STUDENTS IN THE GRAPHICAL ASSOCIATION TEST

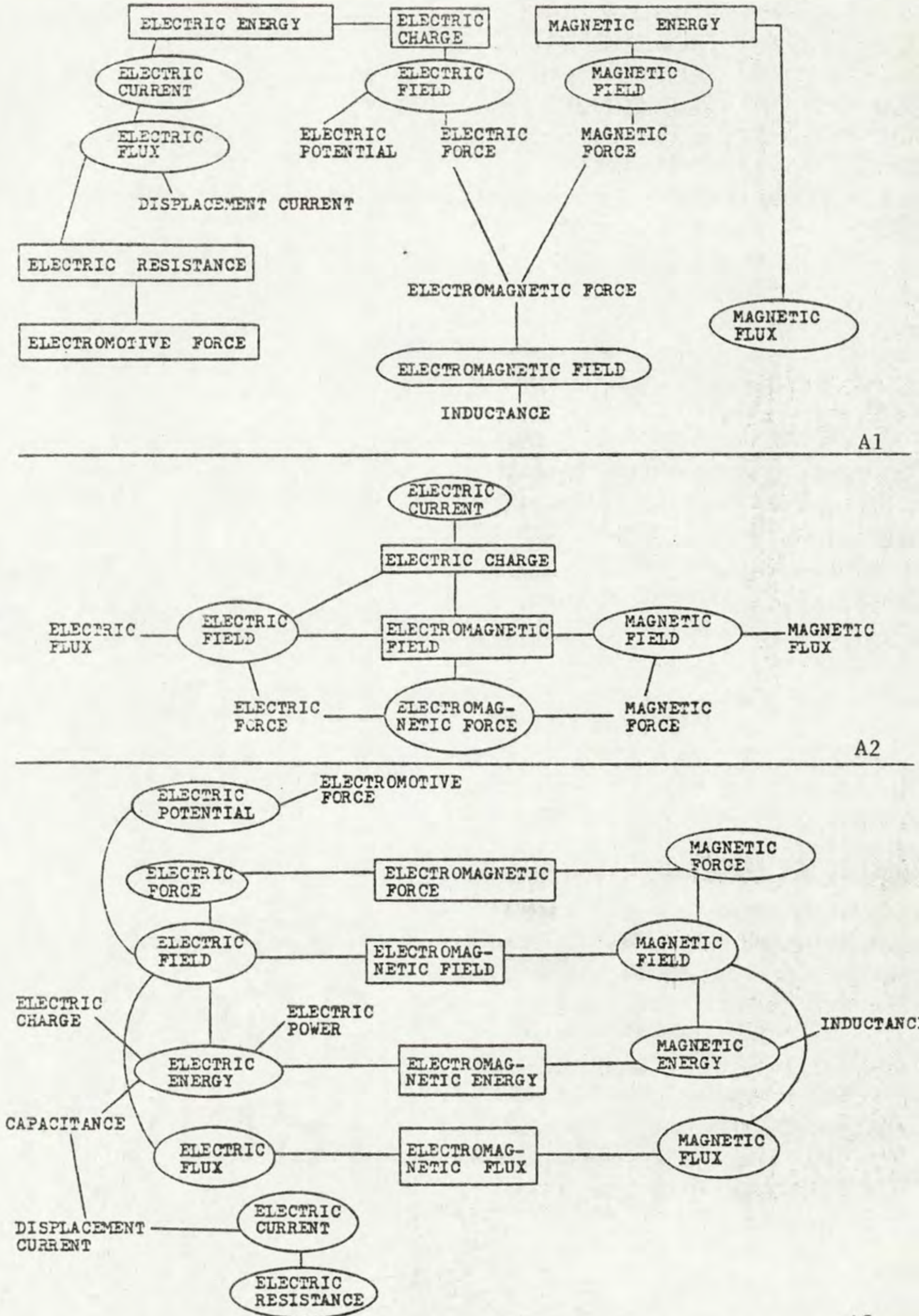

Fig. XII-1. Concept Maps, Student No. 27, Group E1.

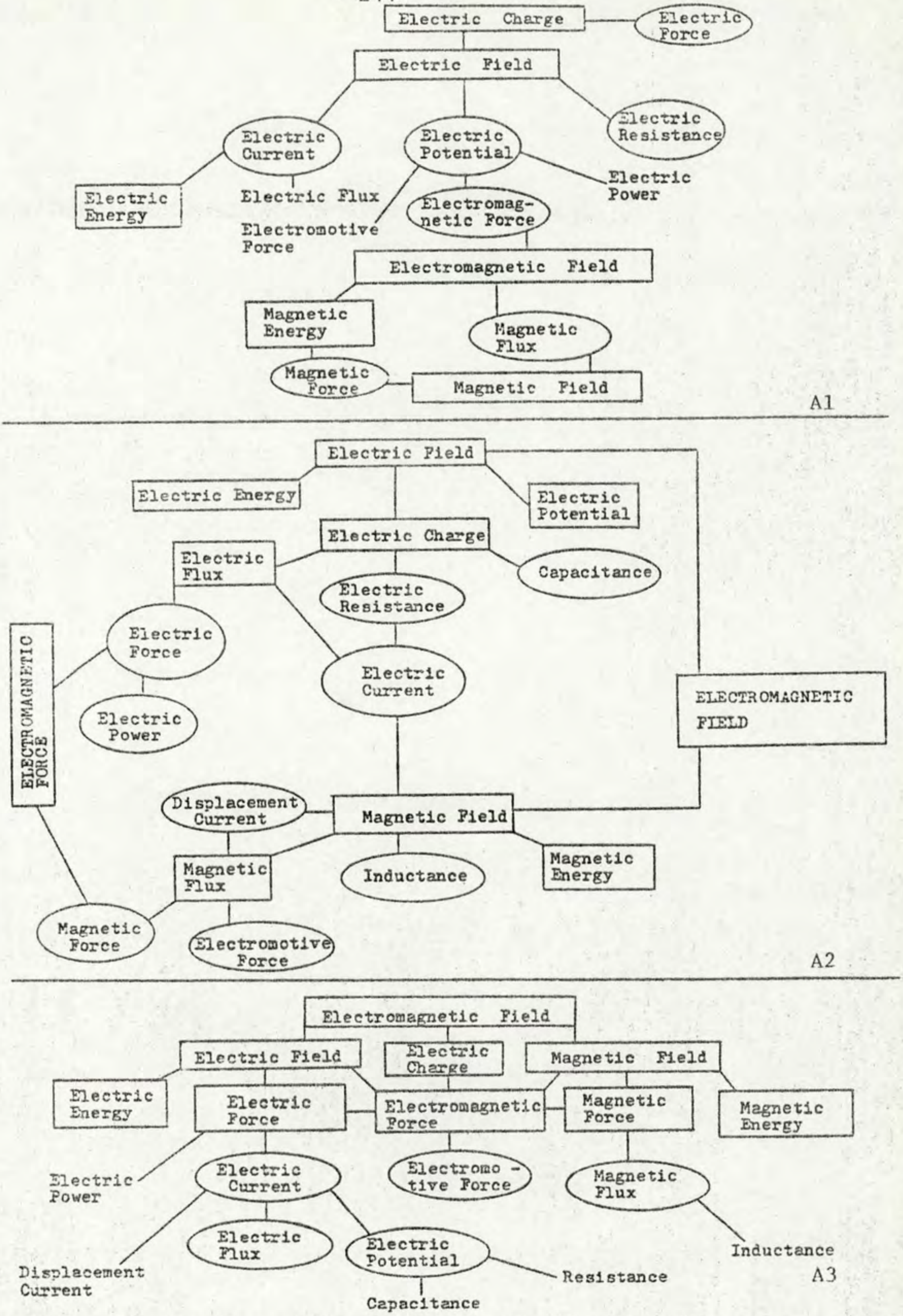

Fig. XII-2. Concept Maps, Student No. 28, Group E1.

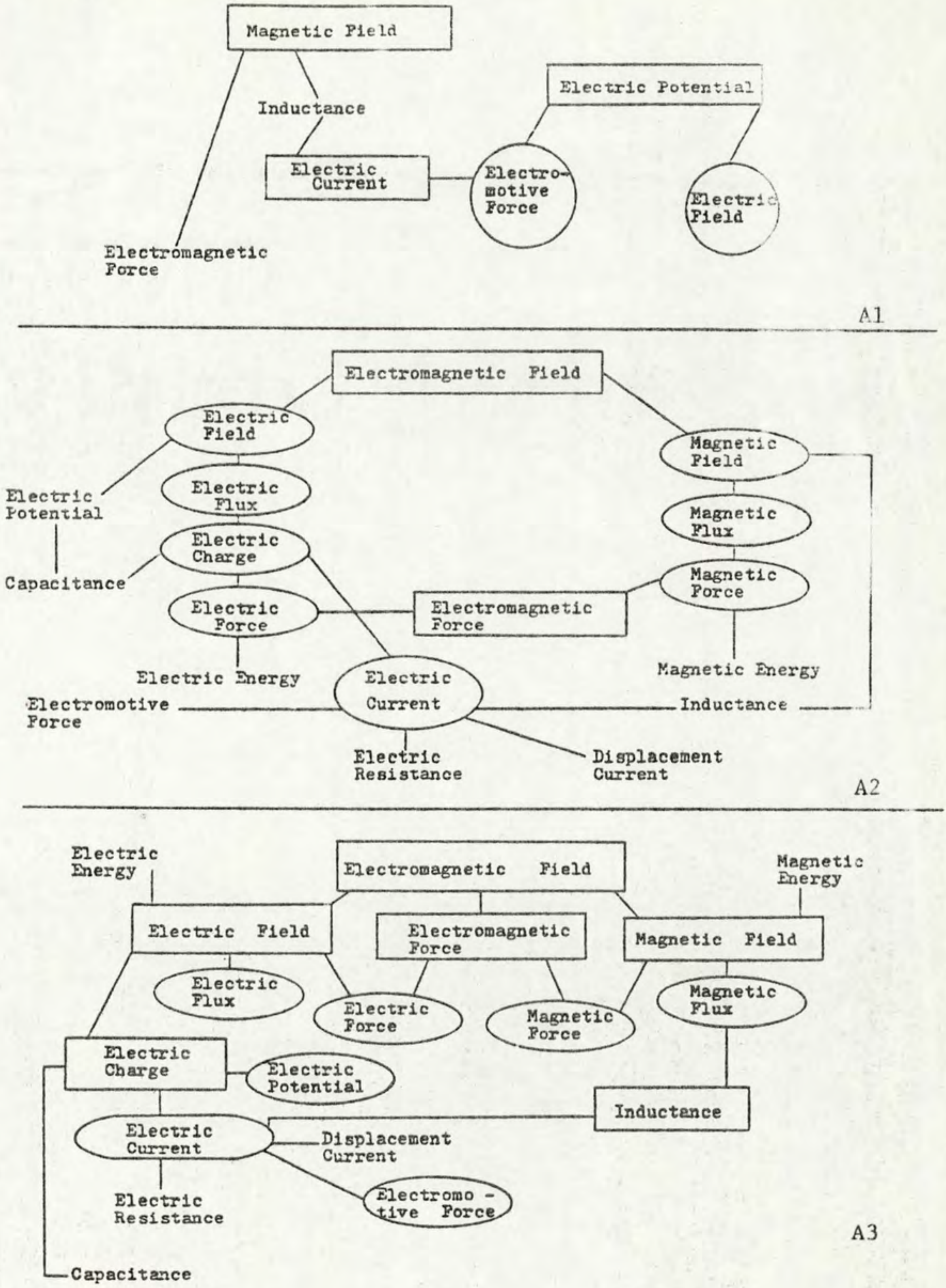

Fig. XII-3. Concept Maps, Student No. 37, Group E1.

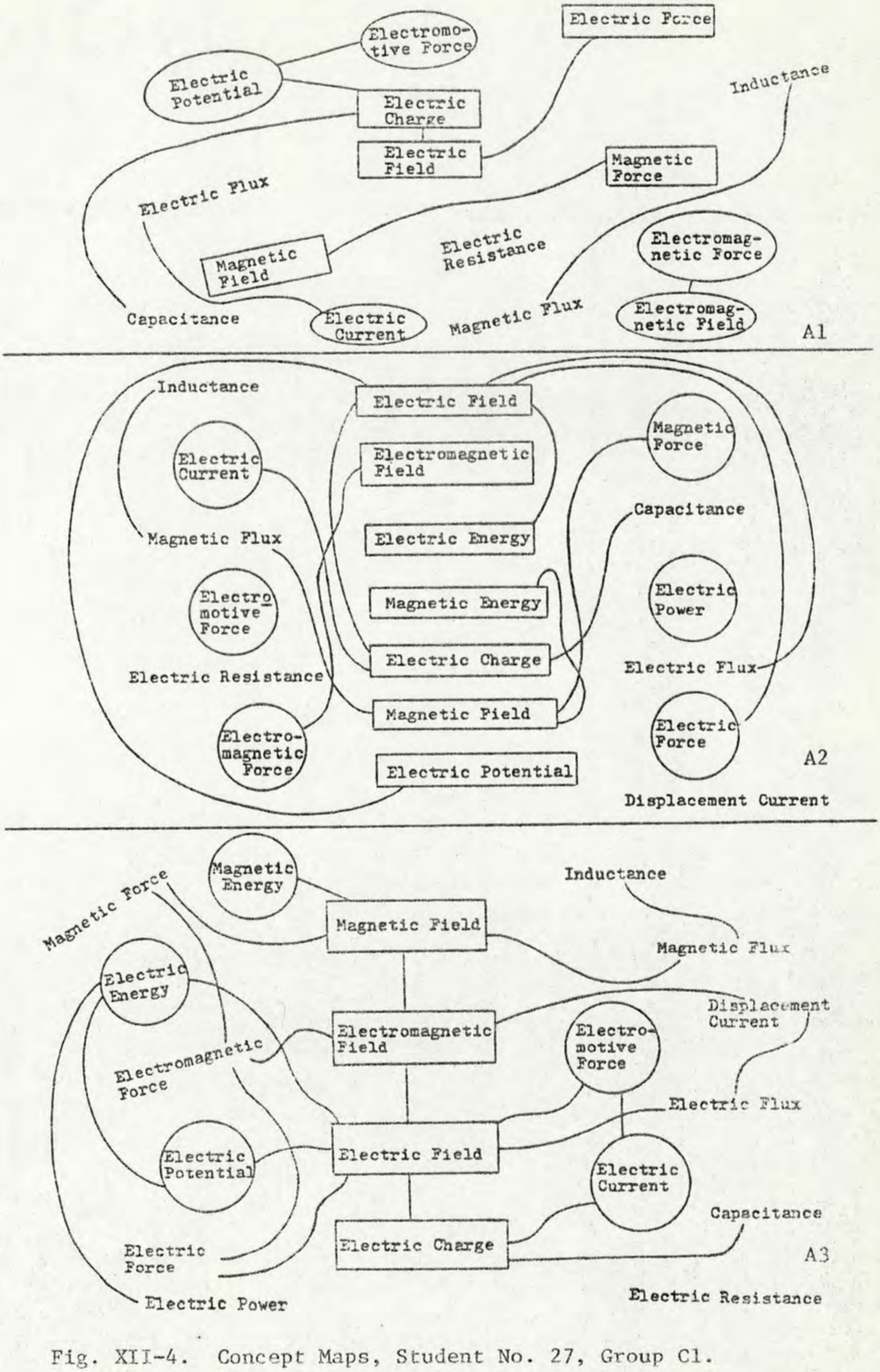

Fig. XII-4. Concept Maps, Student No. 27, Group C1.

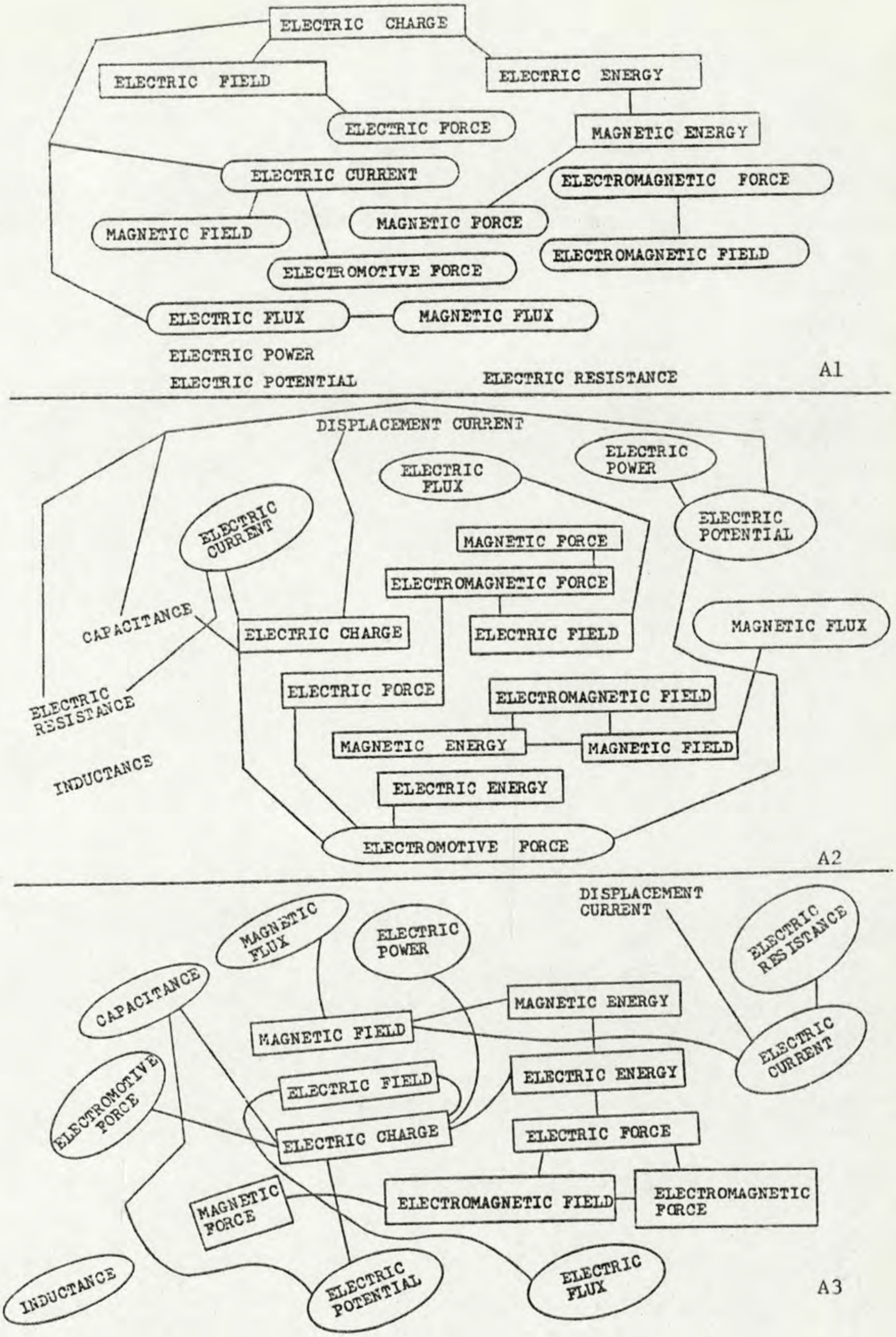

Fig. XII-5. Concept Maps, Student No. 22, Group C1.

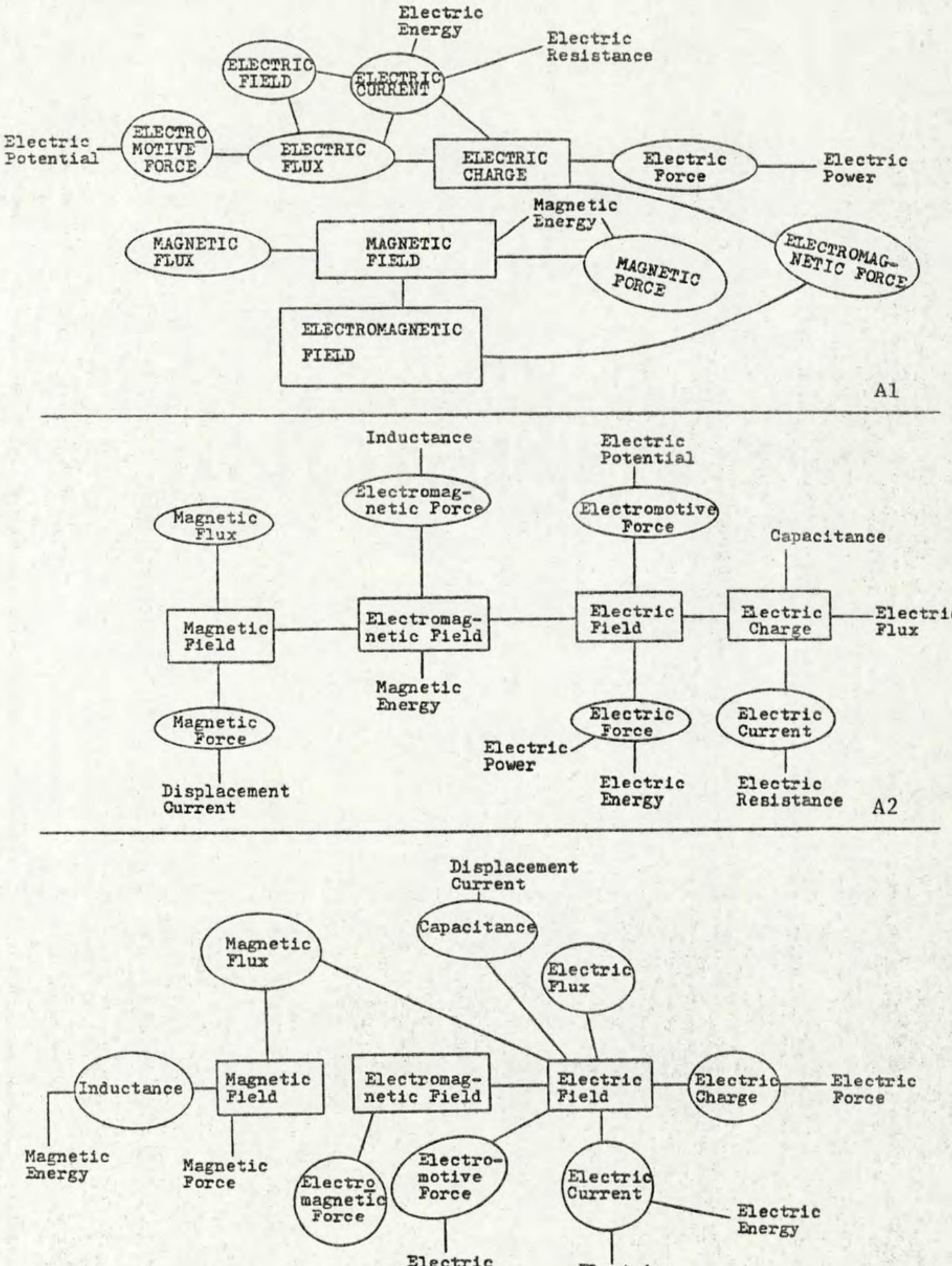

Fig. XII-6. Concept Maps, Student No. 11, Group C1.

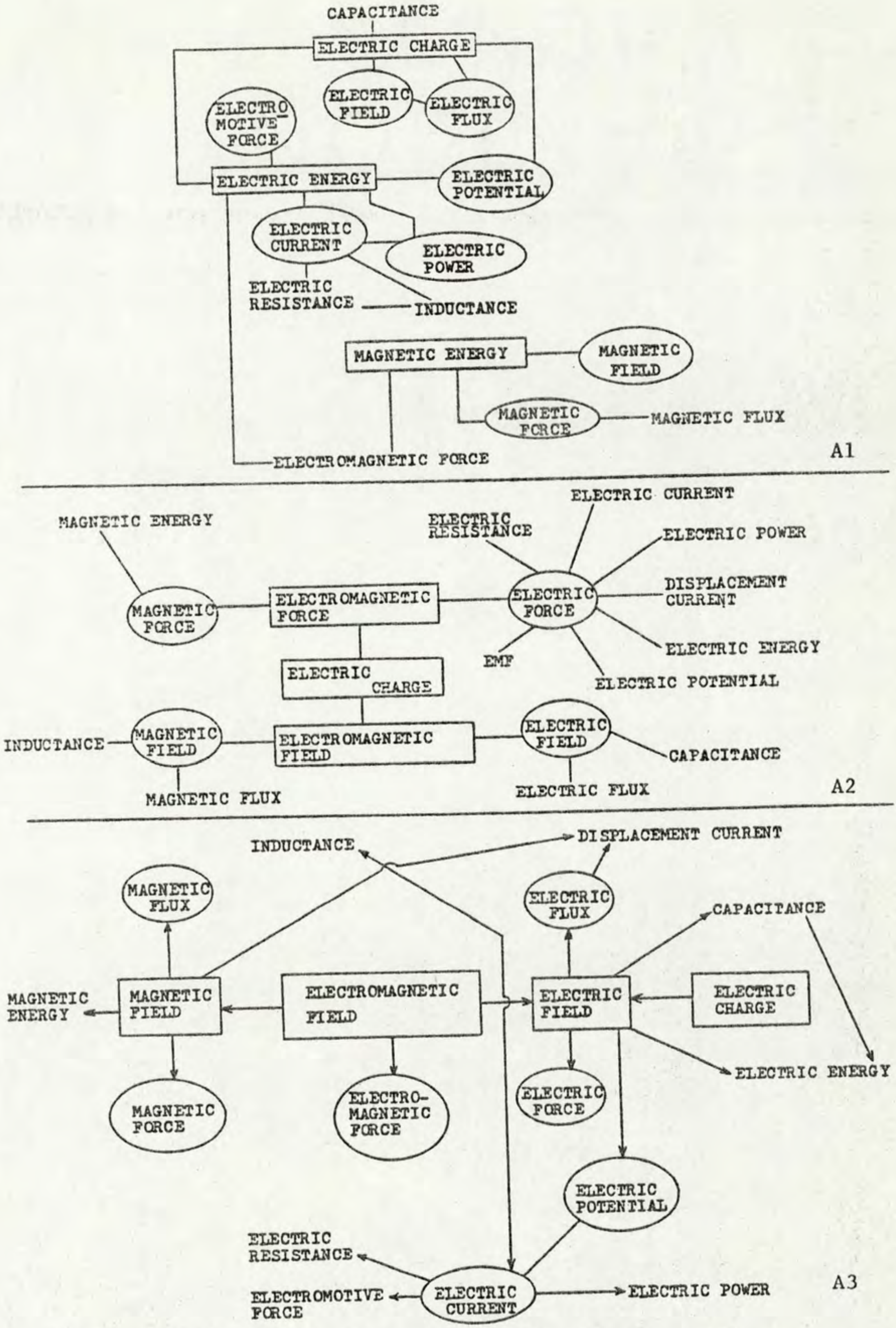

Fig. XII-7. Concept Maps, Student No. 34, Group E2.

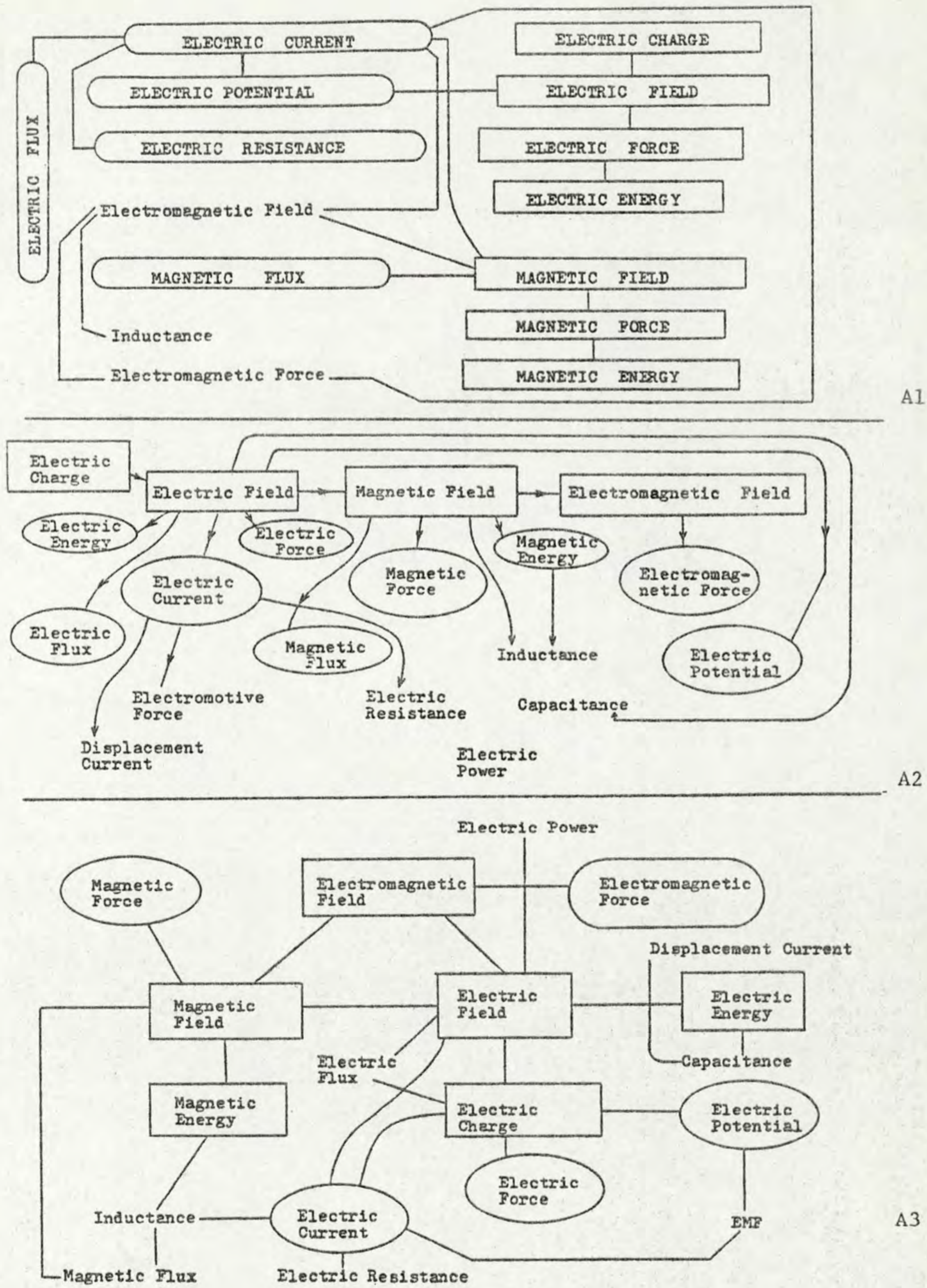

Fig. XII-8. Concept Maps, Student No. 2, Group E2.

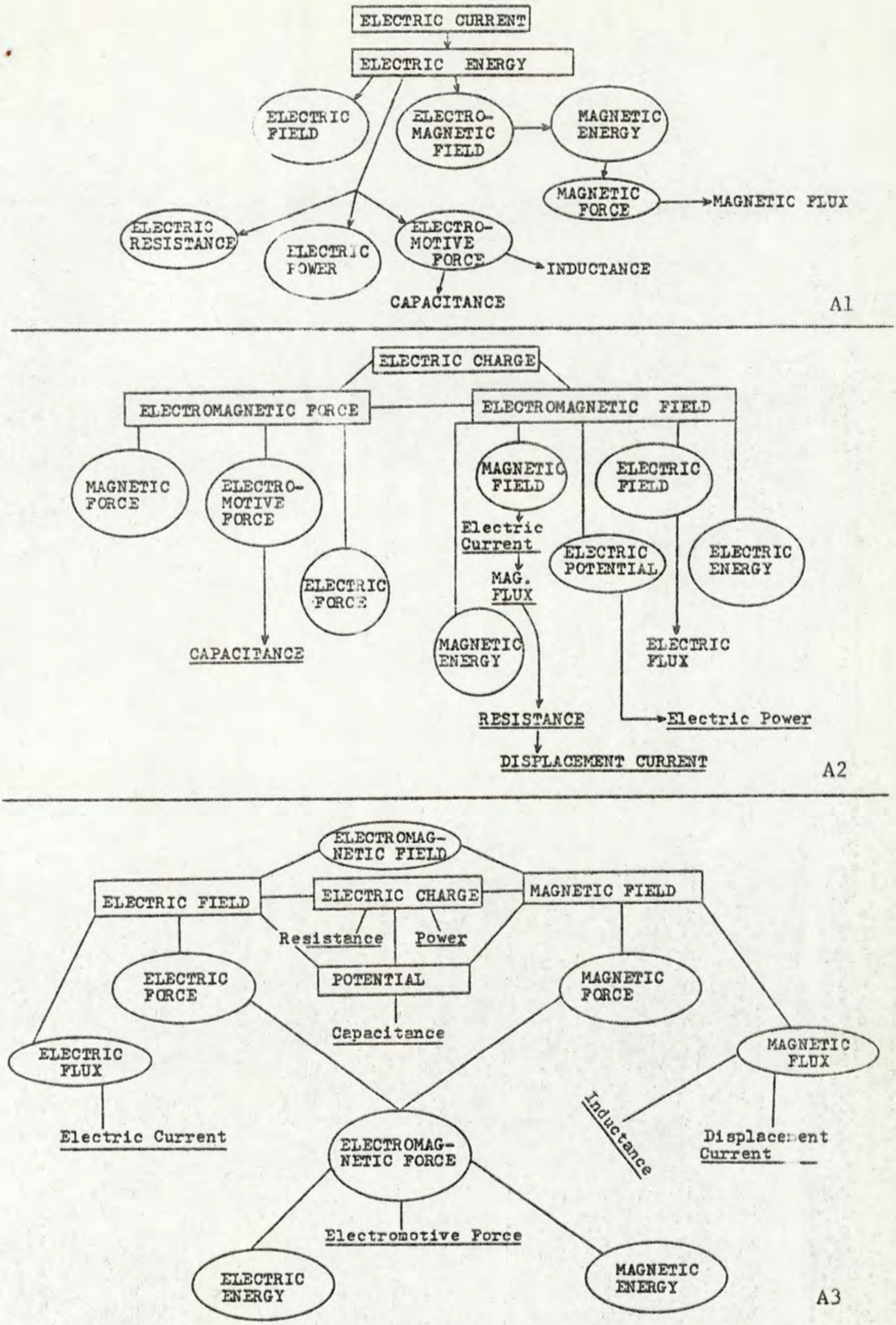

Fig. XII-9. Concept Maps, Student No. 26, Group E2.

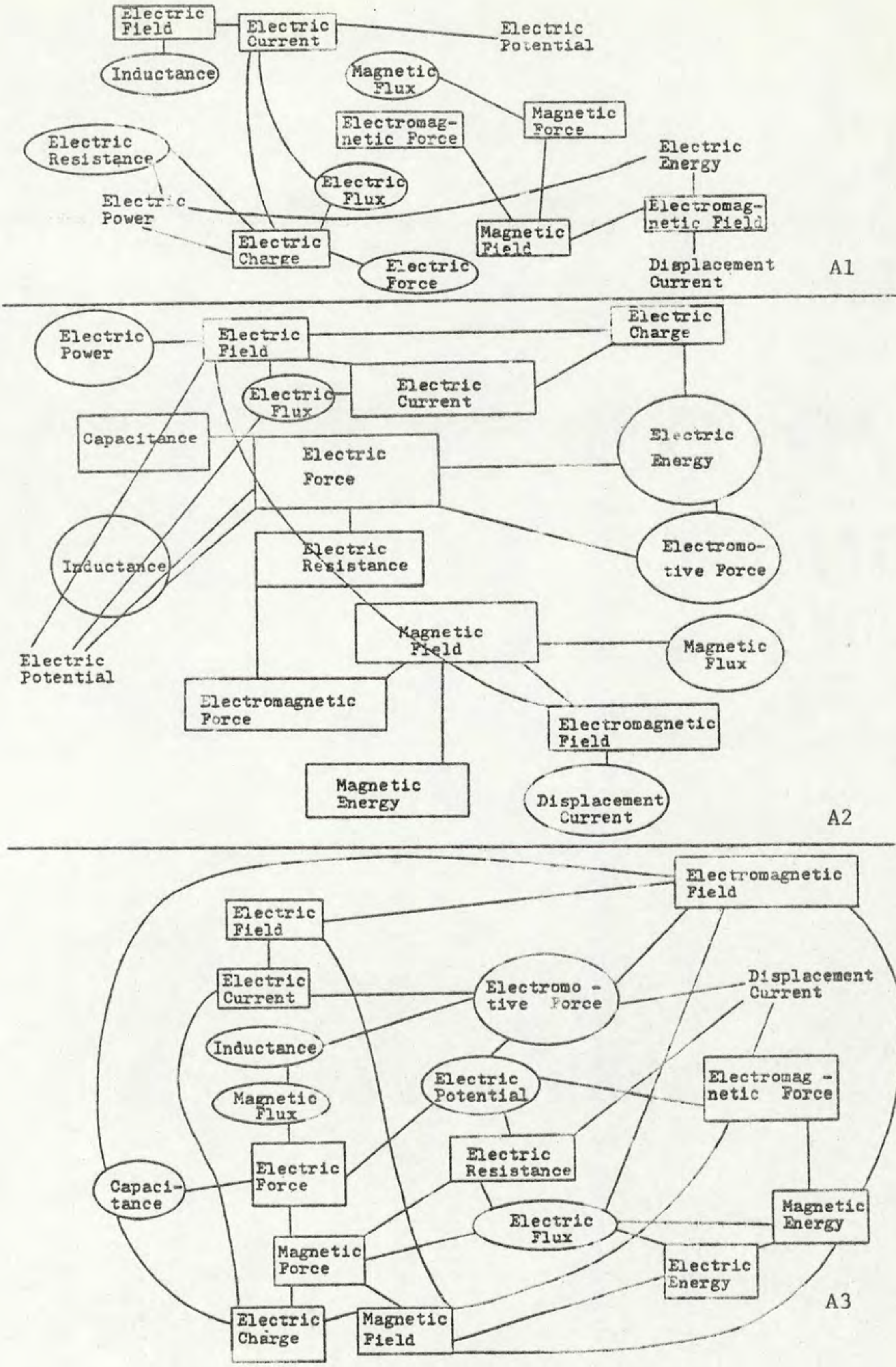

Fig. XII-10. Concept Maps, Student No. 13, Group C2.

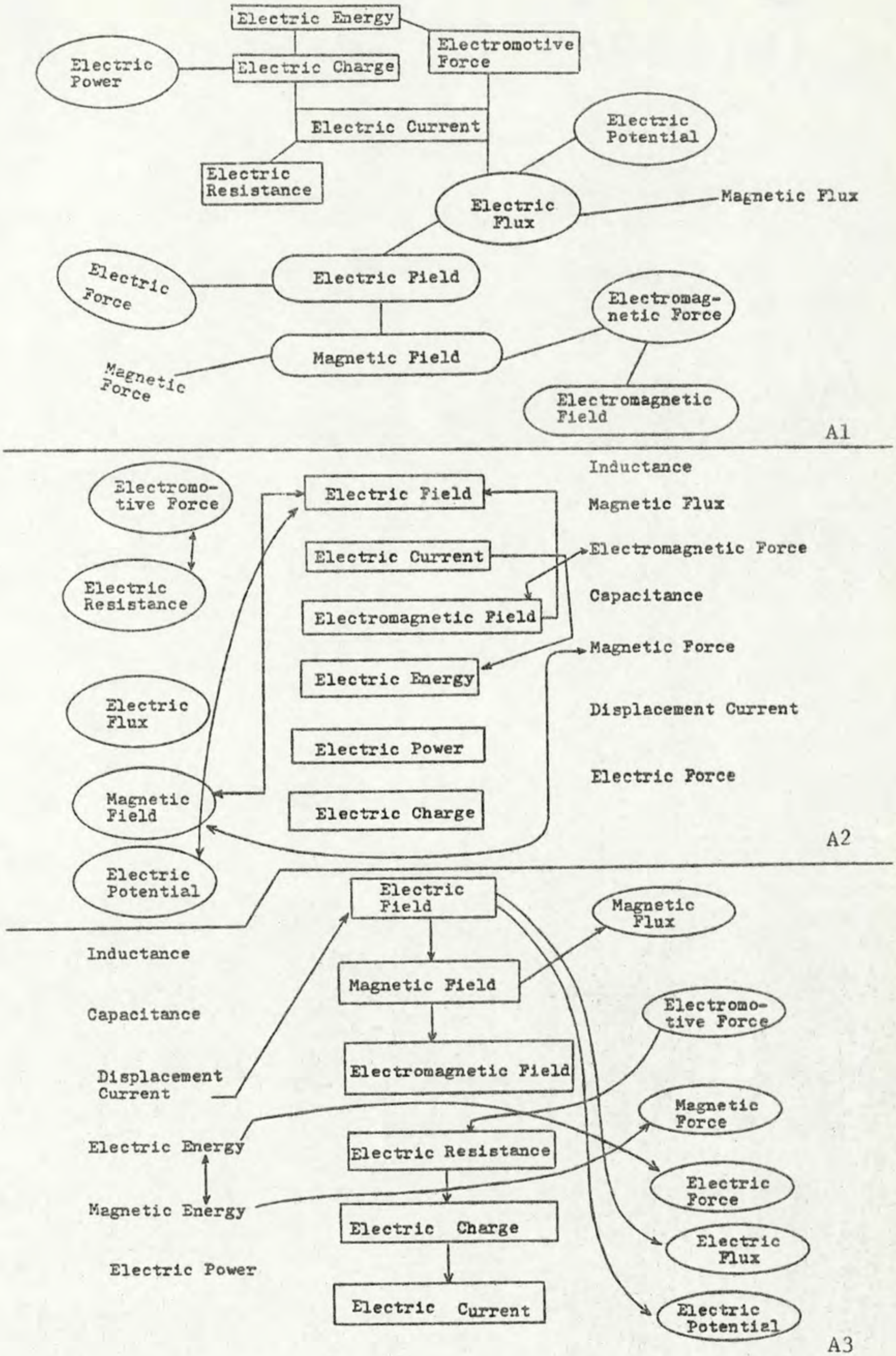

Fig. XII-11. Concept Maps, Student No. 23, Group C2.

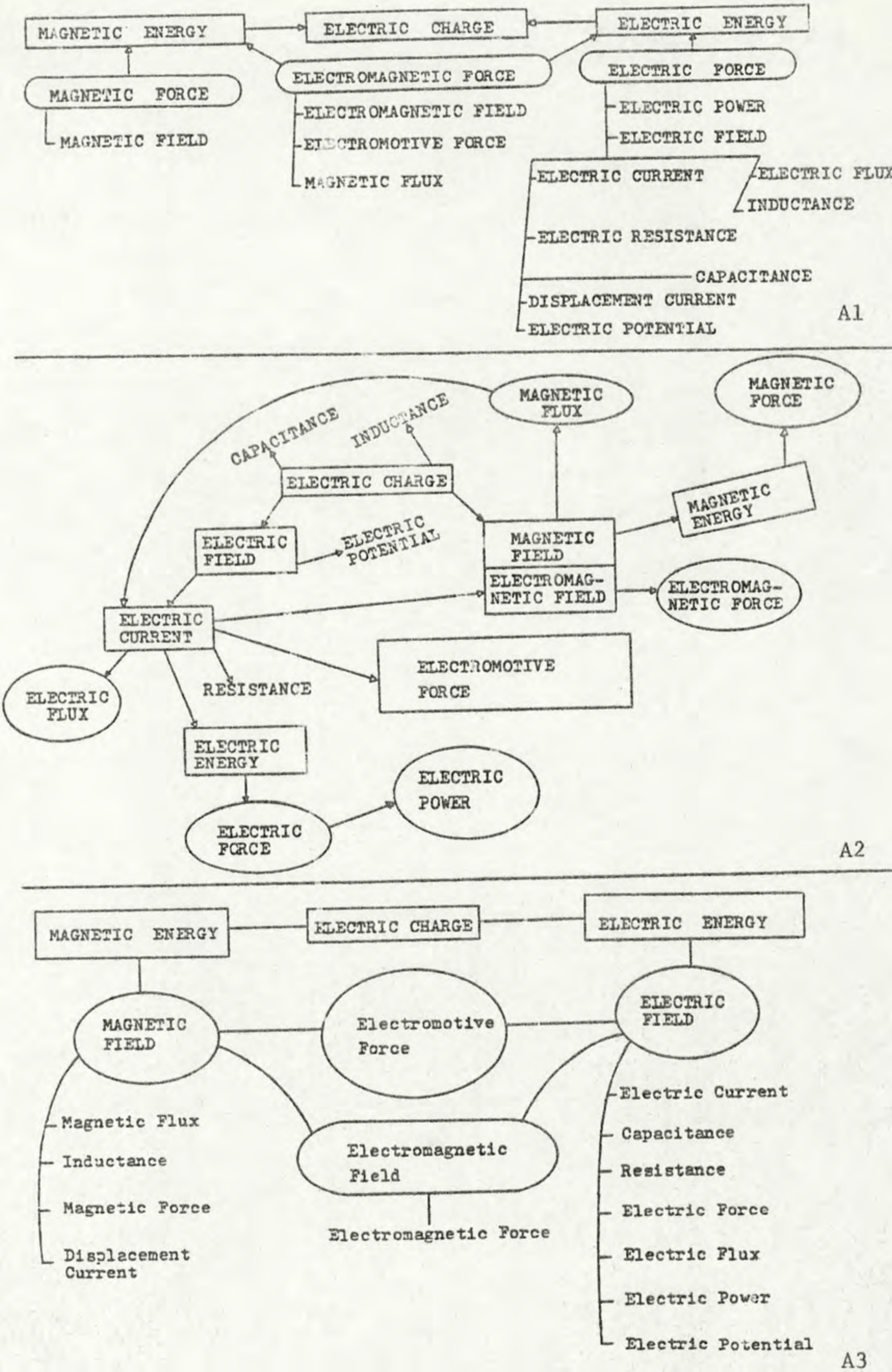

Fig. XII-12. Concept Maps, Student No. 15, Group C2.

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